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TERRESTRIAL MAINTENANCE SYSTEM FOR GEOGRAPHICALLY DISTRIBUTED CLIENTS

Clients (for instance, owners of ground equipment for satellite telecommunication network) are arbitrarily distributed on some territory. For maintenance/repair service, one uses Mobile Maintenance Stations (MMS) located at some Maintenance Bases (MB). The problem is to construct such maintenance zones which need minimum total number of MMS under condition that the Quality of Service (QoS) is not worse than required.
A heuristic mathematical model of optimal zoning is suggested. An illustrative numerical example of constructing service zone for Florida State (USA) is given.

***Keywords**: maintenance/repair service, maintenance zones, Mobile Maintenance Stations, Geographically Distributed Clients*

1. Brief description of the analyzed system

Assume that clients are arbitrarily distributed within some territory. Each client possesses equipment, for instance, a dish for receiving satellite signals. After the equipment failure, a client calls to a Maintenance Base (MB) and a Mobile Maintenance Station (MMS) is sent to serve client's request. If at the moment all MMS are busy then a current client has to wait until any MMS will be free to start moving to the waiting client. For the sake of simplicity, we assume that MMS always start to move to a client from the MB site.

The problem is to construct such zones that the total number of MMS on entire territory is minimum under condition that Quality of Service (QoS) is required. We will characterize the QoS by two indices: (1) client's waiting time of response from MB that MMS is directed for service, and (2) namely service time, which includes travel time from MB to the client site and time of repair/maintenance.

Let us give some qualitative arguments about existence of optimal solution of the problem. If the zone radius is chosen too small, it will be enough a single MMS within the zone. In this case, the number of service zones is huge, and for each zone, an adequate mathematical model will be M/G/1 queuing system. A moderate increase of the zone radius leads to decrease of the total number of MMS due to the well known fact that queuing system M/G/n with input of $n\lambda$ is more effective than n systems M/G/1 each with input λ. However, with the radius increase, the average total service time will significantly increase, and, actually, if the radius becomes larger than some value, it will be impossible to conduct maintenance service with required QoS at all.

2. Service zone with a single MMS

2.1. Maximum size of the zone

If call rate per square is low, the zone size (radius) is defined by the physical ability to reach a client for an admissible travel time. For instance, if service time equals 2 hrs, then for 8-hour working day, one has not more than 6 hours for round trip travel, even if the service starts in the very morning. (A factual working day usually is not defined in such strict terms, however, for the sake of simplicity of the solution, we will not take it into account.) If the average MMS speed is 35mph, then the radius of a service zone will be about 100 miles to satisfy the QoS for a remote client.

Speaking about a zone with low call rate, we keep in mind that the probability of appearance more than one request per a day is low enough (see Fig.1).

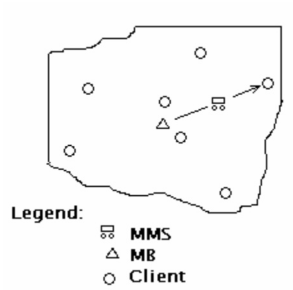

Fig 1. Service zone with a single MMS

Of course, if the request is obtained in the middle or at the end of the working day, it can be served only for some nearest clients. For remote clients service is postponed to the next day.

2.2. Queuing Model for a zone with a single MMS

Let Λ be a call rate within a zone, μ be a service rate, which is defined like:

$$\mu = (travel \quad time + repair \quad time)^{-1}$$

Then MMS can be described by queuing model of type M/M/1. From Queuing Theory (Gnedenko and Kovalenko 1989), one knows that the mean waiting time in such a system is:

$$W = \Lambda / \mu(\mu - \Lambda) \tag{1}$$

Notice that call rate, Λ, and service rate, μ, depend on service zone radius, r:

$$\Lambda(r) = \lambda S \tag{2}$$

where λ = call rate per sq. mile, S=zone square, and

$$\mu(r) = (\tau + r / v)^{-1} \tag{3}$$

where τ = mean repair time, r = zone radius, v = MMS velocity. Resulting expression for the waiting time can be written as:

$$W = \frac{\lambda \pi r^2 \left(\frac{r\alpha}{v} + \tau \right)}{\left(\frac{r\alpha}{v} + \tau \right)^{-1} - \lambda \pi r^2} \tag{4}$$

where α is some corrective coefficient depending on MB location within the service zone (in practical problems MB locates not I the center of the service zone but At some site with dense population).

3. Service zone with multiple MMS

Assume that it is not enough a single MMS for service all clients within the 100-mile zone. It means that the number of MMS should be increased (Fig.2).

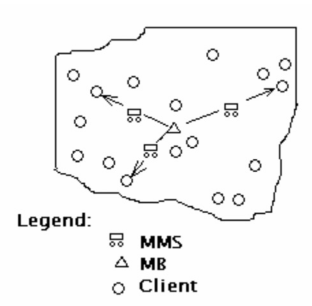

Fig. 2. Service zone with multiple MMS

In this case, an adequate mathematical model is queuing system *M/M/n* with service discipline FIFO. The mean waiting time can be written as:

$$W = \left[\frac{\rho^n \mu}{(n-1)!(n\mu - \lambda)^2} \right] p_0 \tag{5}$$

where ρ = the so-called loading coefficient $\rho = \Lambda / \mu$ and

$$p_0 = \frac{1}{1 + \sum_{1 \le j \le k} \frac{\rho^j}{j!} + \frac{\rho^{k+1}}{k!(k-\rho)}} \tag{6}$$

is the stationary probability that multi channel queuing system is not busy at all.

4. Construction of service zones

4.1. Brief description of the method

The suggested procedure is multi-step iterative procedure of finding a "current optimal" location and configuration of service zone. At each step of the procedure, one expands the service zone, and check QoS requirements. At each step, a current decision should be done with taking into account the results obtained at the previous step. In general terms, the procedure might be described as follows:

(1) Construct isolated optimal zone for an MB with a single MMS.
(2) Construct adjacent (neighbor) isolated optimal zone.
(3) Check if it is possible to aggregate these two zones into one with 2 MMS taking into account required QoS (namely, the service time).
(4) Construct the next adjacent zone. This zone expansion should such that allows the zone to be more or less spherical shape.
(5) Repeat the procedure from Step 3. Keep in mind that new aggregation might lead to a necessity of more than two MMS.
(6) Finishing constructing a zone, start to construct the next zone.
(7) Continue the procedure until service zones will cover entire territory.

Notice that the goal function for this optimization problem is multimode, i.e. the resulting solution might essentially depend on the initial "point of growth".

4.2. Constructing service zones with multiple MMS

Let us consider a situation, when some territory already has been covered with several service zones with a single MMS (Fig.3). Assume that a n aggregate zone with maximum admissible radius can cover all these zones. In this case, we can construct a zone with multiple MMS (Fig.4).

Notice that, as a rule, the number of MMS in the aggregated zone can be decreased.

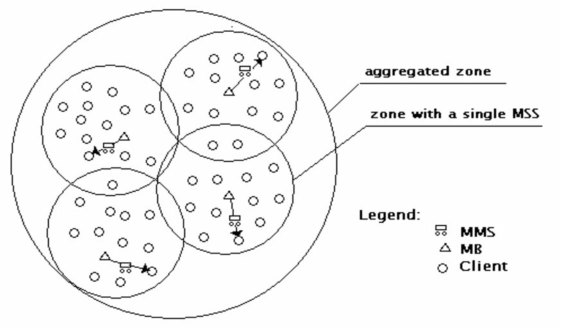

Fig. 3. Adjusted zones with a single MMS each

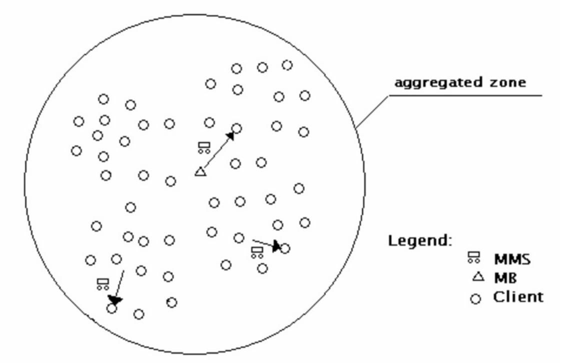

Fig. 4. Aggregated zone with multiple MMS

4.3. Comparison of service zone with a single MMS with aggregated zone with multiple MMS

Coverage of the territory by an aggregated service zone is more effective than use several zones with a single MMS. Below a comparison of several cases is given.

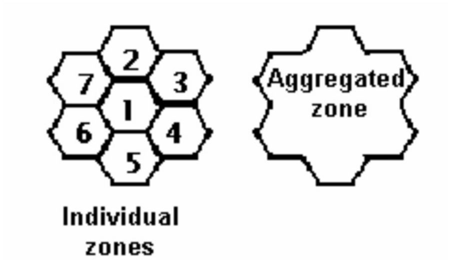

Fig. 5. Comparison of a group of adjusted individual zones with an equivalent aggregated zone

Numerical results for several different loading parameters are given in Table 1. The radius of an individual zone with a single MMS is assumed 35 miles (travel time = 1 hour). The radius of the aggregated zone is 105 miles (travel time = 3 hour). The number of MMS for individual zones is always equal to 7.

Naturally, the total loading for the aggregated zone is taken 7 times larger. From the comparison, one can see that for $\rho = 0.7$ the aggregate service zone decrease

Table 1. Comparison of the mean waiting time

Individual zones			Aggregated zone		
ρ	Waiting time	# MMS	7ρ	Waiting time	# MMS
0.7	3.7 hr.	7	4.9	2.5 hr.	6
0.8	5.1 hr.	7	5.6	1.8 hr.	7
0.9	9.1 hr.	7	6.3	5.1 hr.	7
0.99	-	7	≈7	3.2 hr.	8

the mean waiting time on 35% and, at the same time, the number of MMS decreases on 14% (6 MMS instead of 7). For $\rho = 0.99$, one should use 8 MMS and the mean waiting time becomes 3.2 hours but individual zones in this case do not work at all.

5. Case study (Zoning in Florida, USA)

We considered constructing service zones for user's equipment of a commercial satellite network in Florida (USA). The state is divided onto counties. Each service zone should include or expel entire county.

5.1. Input data

Real statistical data about call rates for different counties were used for constructing service zones. Squares of counties were taken from USA Atlas[1]. Corresponding input data for the numerical example are given in Table 2. We do not give data for all Florida counties, demonstrating the process only on the Southern part of the state.

Table 2. Example of input data for Florida counties

County name	Square (sq. miles)	Call rate (1/h)
Broward	1211	0,054
Collier	1994	0,010
Dade	1955	0,047
Hendry	1163	0,001
Martin	555	0,005
Monroe	1034	0,005
Palm Beach	1993	0,056
...	...	...

The QoS requirements are as follows: (1) the mean waiting time is to be less than 2 hours; (2) travel time is to be less than 3 hours.

5.2. Constructing a first service zone with a single MMS

Step 1. Dade is the first County chosen as initial for the further procedure (it is shadowed by dark gray in Fig. 6). The table with corresponding calculated results us given below.

[1] http://www.freac.fsu.edu/InteractiveCountyAtlas/Atlas.html

Fig. 6. The first choice is Dade County

From Excel program, which has been specially developed for this study, we find that waiting time = 0.5 hrs. Travel time is calculated by special program taking into account the MB location and population dispersion.

Step 2. Next expansion of the first service zone, we obtain by adding Monroe County (see Fig.7). In this figure, the county chosen at the first step is colored by light gray and the new one is again colored dark gray.

Fig. 7. First expansion: county Monroe is added

Calculation of travel time is performed by special sub-program, based on the Manhattan's metric that gives a possibility to take into account real road network configuration. From Excel program, we find that for this expanded zone is characterized by the mean waiting time = 0.8 hrs.

Step 3. Add adjacent county – Broward (see Fig. 8). As above, all already chosen counties are in light gray color and new one is darker.

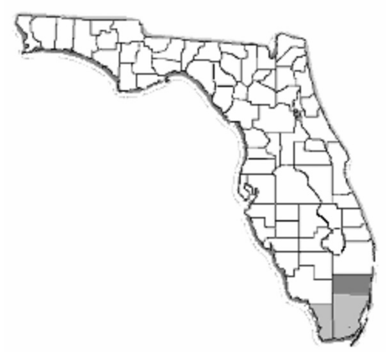

Fig. 8. Next expansion: added county is Broward

We assumed that waiting time should not excess 2 hours. Thus, this solution is unacceptable. At the next step, we will try another adjacent county – Collier instead of Broward.

Step 3a (second trial of step 3). At the second trial of step 3, let us add Collier County instead of Broward (Fig. 9).

Table 3. The 1st step of calculations (County Dade with MB at Miami)

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	*0.047*			*1,955*	*0.67*	*0.167*	*24.9*
Result		***1***	***0.5***				

Table 4. The 2nd step of calculations (Counties Dade and Monroe with MB at Miami)

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	*0.0471*			*1,955*	*0.67*	*0.167*	*24.9*
Monroe	*0.0046*			*1,034*	*0.48*	*0.016*	*18.1*
Total	*0.0517*	***1***		*2,989*	*0.82*	*0.182*	*30.8*
Results			***0.8***				

Table 5. The 3rd step of calculations (Counties Dade, Monroe and Broward, same MB)

Name	Call Rate	MSS Number	Waiting Time	Area	Travel time	Loading coefficient	Radius
Dade	*0.0471*			*1,955*	*0.67*	*0.166*	*24.9*
Monroe	*0.0046*			*1,034*	*0.48*	*0.016*	*18.1*
Broward	*0.0540*			*1,211*	*0.52*	*0.191*	*19.6*
Total	*0.1057*	***1***		*4,200*	*0.98*	*0.373*	*36.6*
Result			***2.3***				

Fig. 9. Second trial of Step 3: adding Collier County instead of Broward County

Since the average waiting time is still in acceptable limits, we are trying to add a next adjacent county.

Step 4. At this step, we add Hendry County (see Fig. 10). There were no calls registered in field statistics during the interval of observation, so we use conservative estimate, assuming 1 call, which in our case corresponds to call rate = 0.0008. (This assumption is marked by symbol "*" next to the name of the added county.)

Fig. 10. Step 4: addition of Hendry County to the first service zone

Step 5. Since the waiting time is still less than 2 hours, the next adjacent county (Gladis) can be added to this service zone (see Fig. 11). There also were no calls registered in field statistics during the interval of observation, as it was with Hendry County, so we do the same assumptions reflected in the Table 8.

Table 6. Results for step 3a.

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	*0.0470*			*1,955*	*0.67*	*0.166*	*24.9*
Monroe	*0.0046*			*1,034*	*0.48*	*0.016*	*18.1*
Collier	*0.0100*			*1,994*	*0.67*	*0.035*	*25.2*
Total	*0.0617*	***1***		*4,983*	*1.06*	*0.217*	*39.8*
Result			***1.1***				

Table 7. Results for step 4

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	*0.0471*			*1,955*	*0.67*	*0.166*	*24.9*
Monroe	*0.0046*			*1,034*	*0.48*	*0.016*	*18.1*
Collier	*0.0100*			*1,994*	*0.67*	*0.035*	*25.2*
Hendry*	*0.0008*			*1,163*	*0.51*	*0.002*	*19.2*
Total	*0.0625*	***1***		*6,146*	*1.18*	*0.219*	*44.2*
Result			***1.3***				

Table 8. Results for step 4.

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	*0.0471*			*1,955*	*0.67*	*0.166*	*24.9*
Monroe	*0.0046*			*1,034*	*0.48*	*0.016*	*18.1*
Collier	*0.0100*			*1,994*	*0.67*	*0.035*	*25.2*
Hendry*	*0.0008*			*1,163*	*0.51*	*0.002*	*19.2*
Gladis*	*0.0008*			*763*	*0.63*	*0.002*	*16.6*
Total	*0.0625*	***1***		*6,146*	*1.18*	*0.223*	*46.1*
Result			***1.5***				

Fig. 11. Step 4: addition Gladis County to the first service zone

The final solution for the first service zone with a single MMS is presented in Figure 12.

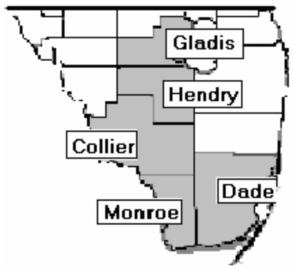

Fig. 12. First service zone

6. Constructing zone with two MMS

In the previous section, we considered only service zones with a single MMS. Here omitting details, we consider the results of constructing of a service zone with two MMS. Notice that actually, in this case, the limiting factor is the permissible travel time.

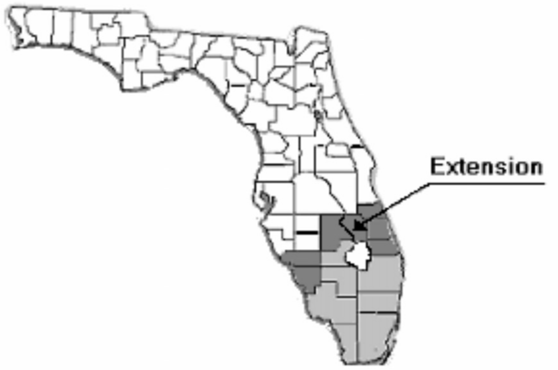

Fig. 13. Aggregated zone with two MMS.

Suggested method has been used at Hughes Network Systems, Inc. (USA). Evaluated amount of saved money exceeds $3,000,000.

Table 10. Results for service zone with two MMS.

Name	Call Rate	MSS Number	Waiting Time	Area	Travel Time	Loading Coefficient	Radius
Dade	0.0471			1,955	1.11	0.223573	24.9
Monroe	0.0046			1,034	0.81	0.021991	18.1
Broward	0.0540			1,211	0.87	0.256559	19.6
Collier	0.0100			1,994	1.12	0.047647	25.2
Palm Beach	0.0563			1,993	1.12	0.267554	25.2
Hendry*	0.0008			1,163	0.86	0.003665	19.2
Martin	0.0054			555	0.59	0.025656	13.3
Lee	0.0108			803	0.71	0.051	16
Glades*	0.0008			763	0.69	0.004	15.6
St. Lucie	0.0023			581	0.6	0.010	13.6
Charlotte*	0.0008			690	0.66	0.004	14.8
Highlands	0.0023			1,029	0.8	0.011	18.1
Okleehobee*	0.0008			771	0.7	0.004	15.7
Indian River	0.0046			497	0.56	0.022	12.6
Total	0.2006	2		15,039	3.08	0.952	69.2
Results			1.05				

7. References

[1] Gnedenko, B.V. and I.N. Kovalenko (1989). *Introduction to Queueing Theory*. Boston: Birkhauser.
[2] Gnedenko, B.V., and I.A. Ushakov (1985). *Probabilistic Reliability Engineering*. N.Y.: John Wiley & Sons.

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