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A METHOD OF EVALUATING TIME OF FATIGUE CRACK GROWTH TO LIMITING VALUE OF SOME SELECTED STRUCTURAL COMPONENTS – AN OUTLINE

The paper has been intended to introduce a method of evaluating time of fatigue crack growth to limiting value for some selected structural components during aircraft's operation process. The method, base on stress intensity factor and Paris' formula, which describe physical nature of the process. To describe the dynamics of the crack propagation was used a partial differential equation of the Fokker-Planck type. Having solved this equation enables the density function of the crack length to be found. This function, gave possibility of determination time density function when fatigue crack growth achieves limiting value.

***Keywords**: fatigue crack, stress intensity factor, limiting state.*

1. Introduction

Some selected aircraft's elements are destroyed during operation process as a result of fatigue crack growth. Usually, these elements are included in aircraft's structure and engine. Particularly dangerous damages are fatigue cracks in aircraft's engine. They are caused by fatigue crack of compressor blades, turbine and drive shaft. The damages occur rarely but have very serious effect in the form of aircraft's crash.

The base of prevention this damage is preventive service, worked out with use of prognosis specific symptom's growth. To this end, it's necessary to recognize fatigue crack process connected with flying time. The process of destruction is observed by special diagnostic systems, which are continuously improving.

In this work, authors base on previous publications [1-5]. An attempt has been made in this paper, to present a method of evaluating distribution of time, when fatigue crack growth achieves limiting value, assuming that:

1. Destructive factor is an element's load, taking vibrations into account.
2. Having spectrum of load, one can determine:
   - total number of load's cycle N_c during time of one flight,
   - maximum load in thresholds for given spectrum of load, amount to: $\sigma_1^{max}, \sigma_2^{max}, ..., \sigma_L^{max}$ (L – number of thresholds in given spectrum),
   - the number of repetition specific threshold's value during one flight, amount to n_i, where: $N_c = \sum_{i=1}^{L} n_i$.
3. Maximum load values for given thresholds can be defined:
$$\sigma_i^{max} = \frac{\sigma_i^{max} + \sigma_i^{min}}{2} + \sigma_i^a \qquad (1)$$
where: σ_i^{max} - maximum value of cyclic loading within i-th threshold; σ_i^{min} - minimum value of cyclic loading within i-th threshold; σ_i^a - amplitude of cyclic loading within i-th threshold, i=1,2,...,L.
4. Maximum load values: $\sigma_1^{max}, \sigma_2^{max}, ..., \sigma_L^{max}$ (for given thresholds), respond to frequency of theirs occurrences:
$\frac{n_1}{N_c} = P_1$, $\frac{n_2}{N_c} = P_2$, ... $\frac{n_L}{N_c} = P_L$.
In above mentioned way, probabilistic description of element's load was determined for every flight or assumed operational cycle of the aircraft.
5. Authors assumed that, within the confines of operational cycle, occurring minor load after major or inversely didn't require additional changes in created model.
6. Crack growth process has been described with the Paris' formula in the following form:
$$\frac{da}{d\ N_z} = C(\Delta K)^m \qquad (2)$$

where: ΔK - range of changes stress intensity factor value; C, m- material constants; a - crack length; N_z - the number of fatigue cycles.

2. Probabilistic model for density function of crack growth determination

Assumptions:

- Device's technical state was being determined by one dominant diagnostic parameter in the form of crack length "a";
- Crack length change can occur only during operating time;
- Paris formula described in the form (2), in this case has following form:

$$\frac{da}{d\ N_z} = C\ \ M_k^m (\sigma_{max})^m\ \ \pi^{\frac{m}{2}}\ \ a^{\frac{m}{2}} \qquad (3)$$

where: M_k - coefficient of the finiteness of the component's dimensions and position of the crack, σ_{max} - maximum value of cyclic loading, described in the form (1).

Relationship (3) can be expressed against time i.e. in more detail – against flying time of the aircraft. Hence, the following assumption is made:

$$N_z = \lambda t \qquad (4)$$

where: λ – intensity of the occurrence of fatigue load's cycles for structural component, t – flying time of the aircraft.

In our case, $\lambda = \frac{1}{\Delta t}$, where Δt - load's cycle duration. Relationship (3) depending on time has following form:

$$\frac{da}{d\ t} = \lambda\ \ C\ \ M_k^m (\sigma_{max})^m\ \ \pi^{\frac{m}{2}}\ \ a^{\frac{m}{2}} \qquad (5)$$

Using notifications made earlier, one can set to describing – in probabilistic terms – the dynamics of crack growth. Thus, authors assumed: probability that for the flying time equal to "t", the crack length is "a" amount to $U_{a,t}$. For this, dynamics of crack growth was described by difference equation:

$$U_{a,\ t+\Delta t} = P_1 U_{a-\Delta a_1,\ t} + P_2 U_{a-\Delta a_2,\ t} + \ \cdots \ + \ P_L U_{a-\Delta a_L,\ t} \tag{6}$$

where: P_i – probability of stress σ_i^{max} occurring, described by the form (1), (i = 1, 2,..., L). These probabilities satisfy a condition: $P_1+P_2+...+P_L=1$, Δa_i - crack increment during Δt time for stress σ_i^{max} (i = 1, 2,..., L). These increments can be determined using forms (1) and (5).

Difference equation (6) has following sense: probability that for the flying time equal to "$t + \Delta t$" the crack length is „a", however for the flying time equal to "t", the crack length is "$a - \Delta a_{śr}$" and in the time range (t, $t + \Delta t$) raised by $\Delta a_{śr}$, where $\Delta a_{śr}$ is determined using assumed loading value.

Difference equation (6) in functional notation took the following form:

$$u(a,\ t+\Delta\ t) = \sum_{i=1}^{L} P_i u\ \ (a-\Delta\ a_i,\ t) \tag{7}$$

where: $u(a,t)$ - density function of crack length depending on flying time.

The following differential equation of the Fokker-Planck type has been obtained from equation (7):

$$\frac{\partial u(a,t)}{\partial t} = -\alpha(t)\frac{\partial u(a,t)}{\partial a} + \frac{1}{2}\ \beta(t)\frac{\partial^2 u(a,t)}{\partial a^2} \tag{8}$$

Solution of eq (8) is the requested crack length density function:

$$u(a,t) = \frac{1}{\sqrt{2\pi\ \ A(t)}}\ e^{-\frac{(a-B(t))^2}{2A(t)}} \tag{9}$$

where: $B(t)$ - an average value of crack length for the flying time "t", $A(t)$ - a variance of crack length for the flying time "t".

3. Determination of flying time distribution for the period of time when fatigue crack growth achieves limiting state for coefficient m=2

Probability that fatigue crack length achieves limiting value can be determined using density function (9). It required determination of the crack limiting value for assumed flight's safety. To determine some critical value of the crack length, the stress intensity factor in the following form can be used:

$$K = M_k\ \ \sigma\sqrt{\pi\ \ a} \tag{10}$$

where: M_k - coefficient of the finiteness of the component's dimensions and position of the crack. The stress intensity factor, determined with dependence (10), becomes a quantity of a critical value K_c, when the crack length and the stress take critical values a_{kr} and σ_{kr}, respectively. Then it is called "resistance of the material to cracking":

$$K_c = M_k\ \ \sigma_{kr}\sqrt{\pi\ \ a_{kr}} \tag{11}$$

With eq (11) applied, and for introduced the factor of safety, one can find the admissible value of the crack:

$$a_d = \frac{K_c^2}{kM_k^{\ 2}\sigma_{kr}^2\pi} \tag{12}$$

where: k - safety factor.

Connection between total flying time and number of the flights has been made by the form:

$$t_N = \sum_{i=1}^{N} t_i \tag{13}$$

where: N - the number of flights, t_i - time of i-th flight duration.

With eq (13) applied, density function can be described by the form:

$$u(a,t_N) = \frac{1}{\sqrt{2\pi A(t_N)}}\ e^{-\frac{(a-B(t_N))^2}{2A(t_N)}} \tag{14}$$

Coefficients $B(t_N)$ and $A(t_N)$ for m=2, are solutions of the following integrals:

$$B(t_N) = \int_0^t \alpha\left(t_N\right)dt = a_0(e^{\lambda\bar{C}_2 t_N} - 1) \tag{15}$$

$$A(t_N) = \int_0^t \beta\left(t_N\right)dt = \frac{1}{2}a_0^2\bar{C}_2\omega\left(e^{2\lambda\bar{C}_2 t_N} - 1\right) \tag{16}$$

where: $\bar{C}_2 = C_2 E[(\sigma^{max})^2\ ]$, $C_2 = CM_k^2\pi$, $\omega = \dfrac{E[(\sigma^{max})^4\,]}{\left(E[(\sigma^{max})^2\,]\right)^2}$.

With use the crack length density function (14) and formula (12) one can determine probability that: in range of the flying time (0, t_N), current crack length achieves limiting value:

$$Q(t_N, a_d) = \int_{a_d}^{\infty} u(a,t_N)da \tag{17}$$

Flying time density function for the period of time when fatigue crack growth achieves limiting value can be defined:

$$f(t) = \frac{\partial}{\partial t}\ Q(t;a_d) \tag{18}$$

Equation (18) can be also defined by the following form:

$$f(t) = \int_{a_d}^{\infty} \{\frac{\partial}{\partial t}\ u(a,t)\}\ da \tag{19}$$

In order to calculate integral (19), it's necessary to:

- determine derivative of $u(a,t)$ function with respect to time;
- find antiderivative of a function (19).

Determination of derivative of function (14) was realized in following way:

$$\frac{\partial}{\partial t}[u(a,t_N)] = \left(\frac{1}{\sqrt{2\pi[\frac{1}{2}a_0^2\bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N} - 1)]}}\right)'\ e^{-\frac{(a-a_0(e^{\lambda\bar{C}_2 t_N} - 1))^2}{2\frac{1}{2}a_0^2\bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N} - 1)]}} +$$

$$+\frac{1}{\sqrt{2\pi[\frac{1}{2}a_0^2\bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N} - 1)]}}\ \left(e^{-\frac{(a-a_0(e^{\lambda\bar{C}_2 t_N} - 1))^2}{2\frac{1}{2}a_0^2\bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N} - 1)]}}\right)' =$$

$$= \left(\frac{2(a-a_0(e^{\lambda\bar{C}_2 t_N} - 1)a_0\lambda\ \bar{C}_2\ e^{\lambda\bar{C}_2 t_N}}{a_0^2\bar{C}_2\omega(e^{2\lambda\ \bar{C}_2\ t_N} - 1)} + \right.$$

$$+\frac{2(a-a_0(e^{\lambda\ \bar{C}_2\ t_N} - 1))^2\lambda\ \bar{C}_2 e^{2\lambda\ \bar{C}_2\ t_N}}{a_0^2\bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N} - 1)^2} -$$

$$\left. -\frac{\bar{C}_2\lambda\ e^{2\lambda\ \bar{C}_2 t_N}}{(e^{2\lambda\bar{C}_2 t_N} - 1)}\ \right)\ u(a,t) \tag{20}$$

Determination of antiderivative of function was carried out with use of following expression:

$$w(a,t) = u(a,t)\cdot\theta(a,t) \tag{21}$$

where: $\theta(a,t)$ is wanted expression.

Derivative of function – $w(a,t)$ with respect to crack length should be equal expression (20). Derivative $w(a,t)$ function with respect to „a" has following form:

$$\frac{\partial w(a,t)}{\partial a} = u'(a,t)\cdot\theta(a,t) + u(a,t)\cdot\theta\ \ '(a,t)$$

Hence, derivative $\frac{du(a,t)}{da}$ determination allow as follows:

$$\frac{\partial w(a,t)}{\partial a}=u(a,t)\Big(-\frac{2(a-a_0(e^{\lambda\bar{C}_2 t_N}-1))}{a_0^2\ \bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N}-1)}\Big)\cdot\underbrace{(?)}_{\theta(a,t)}+u(a,t)\cdot\underbrace{(?)}_{\theta'(a,t)} \qquad (22)$$

Expression (22) should be equal to expression (20), and as a result of the comparison we can receive following form of θ(a,t) function:

$$\theta(a,t)=\Big(-a_0\lambda\bar{C}_2\ e^{\lambda\bar{C}_2 t_N}-\frac{(a-a_0(e^{\lambda\bar{C}_2 t_N}-1))\ \lambda\bar{C}_2\ e^{2\lambda\bar{C}_2 t_N}}{(e^{2\lambda\bar{C}_2 t_N}-1)}\Big) \qquad (23)$$

and derivative of $\theta(a,t)$ function with respect to „a":

Hence, expression describing antiderivative of function (19) has been described in the form:

$$w(a,t)=U(a,t)\ \cdot[-(a_0\lambda\ \bar{C}_2\ e^{\lambda\bar{C}_2 t_N}+$$
$$+\frac{(a-a_0(e^{\lambda\bar{C}_2 t_N}-1))\ \lambda\ \bar{C}_2\ e^{2\lambda\bar{C}_2 t_N}}{(e^{2\lambda\bar{C}_2 t_N}-1)})] \qquad (24)$$

Calculation of integral (19) gave opportunity to determine flying time density function for the period of time when fatigue crack growth achieves limiting value:

$$f(t_N;a_d)=w(a,t)\Big|_{a_d}^{\infty}=u(a_d,t_N)[(a_0\lambda\ \bar{C}_2\ e^{\lambda\bar{C}_2 t_N}+$$
$$+\frac{(a_d-a_0(e^{\lambda\bar{C}_2 t_N}-1))\ \lambda\ \bar{C}_2\ e^{2\lambda\bar{C}_2 t_N}}{(e^{2\lambda\bar{C}_2 t_N}-1)})] \qquad (25)$$

where:

$$u(a_d,t_N)=\frac{1}{\sqrt{2\pi(\frac{1}{2}a_0^2\ \bar{C}_2\omega(e^{2\lambda\bar{C}_2 t_N}-1)}}\ e^{-\frac{(a_d-a_0(e^{\lambda\bar{C}_2 t_N}-1))^2}{a_0^2\ \bar{C}_2\ \omega(e^{2\lambda\bar{C}_2 t_N}-1)}}$$

$a_d=\frac{K_c^2}{k\ \ M_k^{\ 2}\sigma_{kr}^2\pi}$ - crack length limiting value.

4. Determination of flying time distribution for the period of time when fatigue crack growth achieves limiting state for coefficient m≠2

Density function of crack length was given in the following form:

$$u(a,t)=\frac{1}{\sqrt{2\pi\ \ A(t)}}\ e^{-\frac{(a-B(t))^2}{2A(t)}} \qquad (26)$$

where: a - crack length, t - flying time of the aircraft, $B(t)$ - an average value of crack length for $m\neq 2$, in the form:

$$B(t)=[a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda CM_k^m\ \pi^{\frac{m}{2}}\ E[(\sigma^{max})^m]\ \ t]^{\frac{2}{2-m}}-a_o \qquad (27)$$

$A(t)$- a variance of crack length for $m\neq 2$ in the form:

$$A(t)=\frac{2}{2+m}\ CM_k^m\ \pi^{\frac{m}{2}}\frac{E[(\sigma^{max})^{2m}]}{E[(\sigma^{max})^m]}\cdot$$
$$\cdot\Big[(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda CM_k^m\ \pi^{\frac{m}{2}}\ E[(\sigma^{max})^m]\ \ t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}\Big] \qquad (28)$$

Notations:

C - material constant;

M_k - coefficient of the finiteness of the component's dimensions and position of the crack;

$E[(\sigma^{max})^{2m}]=P_1(\sigma_1^{max})^{2m}+P_2(\sigma_2^{max})^{2m}+...+P_L(\sigma_L^{max})^{2m}$;

$E[(\sigma^{max})^{m}]=P_1(\sigma_1^{max})^{m}+P_2(\sigma_2^{max})^{m}+...+P_L(\sigma_L^{max})^{m}$;

a_o - initial crack length;

λ - intensity of load cycles occurring $(\lambda=\frac{1}{\Delta\ \ t})$;

Δt - load cycle duration time.

Flying time density function for the period of time when fatigue crack growth achieves limiting value can be defined:

$$f(t)_{a_d}=\frac{\partial}{\partial t}\ Q(t;a_d) \qquad (29)$$

Equation (29) can be also defined by the following form:

$$f_{a_d}(t)=\int_{a_d}^{\infty}\{\frac{\partial}{\partial t}\ u(a,t)\}\ da \qquad (30)$$

Using previously way, in order to calculate integral (30), it's necessary to:

- determine derivative of u(a,t) function with respect to time;
- find antiderivative of a function (30).

Determination of derivative of function (26) was realized in following way:

$$\frac{\partial}{\partial t}[u(a,t)]=\underbrace{\left(\frac{1}{\sqrt{2\pi\ \ A(t)}}\right)'}_{D}\ e^{-\frac{(a-B(t))^2}{2A(t)}}+$$
$$+\left(\frac{1}{\sqrt{2\pi\ A(t)}}\right)\cdot\underbrace{\left(e^{-\frac{(a-B(t))^2}{2A(t)}}\right)'}_{F} \qquad (31)$$

$$D=\frac{\lambda CM_k^m\ \pi^{\frac{m}{2}}\ E[(\sigma^{max})^m]\ (a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda CM_k^m\ \pi^{\frac{m}{2}}\ E[(\sigma^{max})^m]\ \ t)^{\frac{2m}{2-m}}}{2\frac{2}{2+m}\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda CM_k^m\ \pi^{\frac{m}{2}}\ E[(\sigma^{max})^m]\ \ t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}\ ]\ \sqrt{2\pi\ \ A(t)}} \qquad (32)$$

Notation: $\_\_=CM_k^m\pi^{\frac{m}{2}}E[(\sigma^{max})^m]$ (33)

Hence: $$D=-\left(\frac{\lambda\_\_(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\ \_\_\lambda t)^{\frac{2+m}{2-m}}}{2\frac{2}{2+m}\ \ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\ \_\_\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}\ ]\ \sqrt{2\pi\ \ A(t)}}\right) \qquad (34)$$

Calculation F:

$$F=\left(e^{-\frac{(a-B(t))^2}{2A(t)}}\right)'=$$
$$=e^{-\frac{(a-B(t))^2}{2A(t)}}\underbrace{\left(-\frac{[(a-B(t))^2]'2A(t)-(a-B(t))^2\cdot 2A(t)'}{4(A(t))^2}\right)}_{G} \qquad (35)$$

Notations:

$$\omega=\frac{E[(\sigma^{max})^{2m}]}{(E[(\sigma^{max})^m])^2} \qquad (36)$$

$$A(t)=\frac{2}{2+m}\ \_\_\omega\ \ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\ \_\_\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}\ ] \qquad (37)$$

$$B(t)=\left\{[a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t]^{\frac{2}{2-m}}-a_o\right\} \quad (38)$$

Calculation G:

$$G=\frac{(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{m}{2-m}}\cdot(a-B(t))\ (\frac{2-m}{2})}{\frac{2}{2+m}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]}+$$

$$+\frac{(a-B(t))^2\ 2\ ^2\omega\ \lambda(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2\ \frac{4}{(2+m)^2}\ ^2\omega^2\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]^2} \quad (39)$$

Hence, taking equations (34) and (39) into account, equation (31) takes the form:

$$\frac{\partial}{\partial t}[u(a,t)]=-\frac{\lambda(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]\ (\frac{2}{2+m})}\ u(a,t)+$$

$$+\frac{(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{m}{2-m}}\cdot(a-B(t))\ (\frac{2-m}{2})}{\frac{2}{2+m}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]}\ u(a,t)+$$

$$+\frac{(a-B(t))^2\ \lambda(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2\ \frac{4}{(2+m)^2}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]^2}\ u(a,t) \quad (40)$$

Antiderivative function in (30) expression, takes the form:

$$w(a,t)=u(a,t)\cdot\theta(a,t) \quad (41)$$

where: $\theta(a,t)$ is wanted expression.

Derivative of function w(a,t) with respect to crack length should be equal expression (40). Derivative $w(a,t)$ function with respect to „a" has following form:

$$\frac{\partial w(a,t)}{\partial a}=u'(a,t)\ \theta(a,t)+u(a,t)\ \theta'(a,t) \quad (42)$$

Derivative of $u(a,t)$ function:

$$\frac{\partial u(a,t)}{\partial a}=u(a,t)\left(-\frac{(a-B(t))}{A(t)}\right) \quad (43)$$

Taking (43) into account eq (42) takes the form:

$$\frac{\partial w(a,t)}{\partial a}=u(a,t)\left(-\frac{(a-B(t))}{A(t)}\right)\underbrace{(?)}_{\theta(a,t)}+u(a,t)\cdot\underbrace{(?)}_{\theta'(a,t)} \quad (44)$$

Equation (44) was compared to (40):

$$\left\{\frac{(a-B(t))(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{m}{2-m}}(\frac{2-m}{2})}{\frac{2}{2+m}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]}+\right.$$

$$+\frac{(a-B(t))^2\ \lambda(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}\ \omega\ 2}{2\ \frac{4}{(2+m)^2}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]^2\ \omega\ 2}-$$

$$\left.-\frac{\lambda(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}\ \omega}{2\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]\ \omega\ (\frac{2}{2+m})}\right\}u(a,t)=$$

$$=u(a,t)\left(-\frac{(a-B(t))}{A(t)}\right)\underbrace{(?)}_{\theta(a,t)}+u(a,t)\theta'(a,t) \quad (45)$$

Using equation (45), following equation was obtained:

$$u(a,t)\left\{\frac{(a-B(t))(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{m}{2-m}}(\frac{2-m}{2})}{\frac{2}{2+m}\ \omega\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]}+\right.$$

$$\left.\frac{(a-B(t))^2\ \lambda\omega\ ^2(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2\ \frac{4}{(2+m)^2}\ \omega^2\ 2\ [(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]^2}\right\}=$$

$$=u(a,t)\left(-\left(\frac{(a-B(t))}{A(t)}\right)\right)\underbrace{(?)}_{\theta(a,t)} \quad (46)$$

$$u(a,t)\left\{-\frac{\lambda\ ^2\ \omega\ (a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2(\frac{2}{2+m})\ \omega[(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}]}\right\}=$$

$$=u(a,t)\ \theta'(a,t) \quad (47)$$

Using equation (46), $\theta(a,t)$ function was determined:

$$\theta(a,t)\left[-(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{m}{2-m}}(\frac{2-m}{2})-\right.$$

$$\left.-\frac{(a-B(t))\lambda\omega\ ^2(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda\ t)^{\frac{2m}{2-m}}}{2A(t)}\right] \quad (48)$$

Derivative of (48) function with respect to „*a*" takes the form:

$$\frac{\partial\theta(a,t)}{\partial a} = -\frac{\lambda\omega^{2}\left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda t\right)^{\frac{2m}{2-m}}}{2A(t)} \qquad (49)$$

Received derivative gave the proof of equation (47) correctness.

Equations (46), (47), (48) i (49) have following notations: *B(t)* - given by equation (38), *A(t)* - given by equation (37), ω - given by equation (36), $\hat{C}$- given by equation (33).

Hence, expression described antiderivative of function (30) has following form:

$$w(a,t) = u(a,t)\left[-\left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda t\right)^{\frac{m}{2-m}}\left(\frac{2-m}{2}\right) - \frac{(a-B(t))\lambda\omega^{2}\left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda t\right)^{\frac{2m}{2-m}}}{2A(t)} \right] \qquad (50)$$

Calculation of the integral (30) gave opportunity to determine flying time density function for the period of time when fatigue crack growth achieves limiting value:

$$f(t;a_d) = w(a,t)\Big|_{a_d}^{\infty} =$$

$$= u(a_d,t)\left[\left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda t\right)^{\frac{m}{2-m}}\left(\frac{2-m}{2}\right) + \frac{(a+B(t))\lambda\omega^{2}\left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda t\right)^{\frac{2m}{2-m}}}{2(A(t))} \right] \qquad (51)$$

where: $u(a_d,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(a_d - B(t))^2}{2A(t)}}$

Equation (51) determines fatigue life of selected structural elements in using condition at a given spectrum of load and for Paris formula with coefficient m≠2.

5. Conclusion

Determined, in this paper, flying time density function for the period of time when fatigue crack growth achieves limiting value is very important for aircraft's structure durability research. Presented in second item equation can be expressed as a dependent on calendar time and combined with number of aircraft's flights.

The main advantage of the method is fact that takes into consideration random value of aircraft's structure loading. Using of this method can contribute to decrease of durability calculation time. Value of used constants and others required values can be estimate with aid of maintenance data and using method of moment or reliability function.

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