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HYBRYDOWY ALGORYTM WZAJEMNEJ ENTROPII DO OCENY NIEZAWODNOŚCI SYSTEMÓW TYPU KONFIGURACJA-REDUNDANCJA

A HYBRID CROSS-ENTROPY ALGORITHM FOR RELIABILITY ASSESSMENT OF CONFIGURATION-REDUNDANCY SYSTEMS

Stosowane w praktyce inżynieryjnej różnorakie redundancje zwiększają dostępność danego systemu zarazem powiększając jego złożoność, co czyni niepewnymi ocenę niezawodności i wykrywanie uszkodzeń komponentów systemu. Wobec powyższego, poddano badaniom system typu konfiguracja-redundancja oraz sformułowano jego funkcję niezawodności. Kiedy niedostępna jest wiedza na temat poprzednich uszkodzeń komponentów systemu, problem uszkodzeń systemu ma charakter problemu stochastycznego. Tymczasem, aby wyeliminować niepewność systemu, konieczne jest wykrycie uszkodzeń w serii komponentów. Zaproponowano model przewidywanej najkrótszej ścieżki oraz model wykrywania uszkodzeń mające służyć optymalizacji niezawodności. Metodę wzajemnej entropii wykorzystano jako algorytm heurystyczny do oceny niezawodności systemu i wykrywania uszkodzeń komponentów. Zastosowane stochastyczne podejście do generowania próbek umożliwia otrzymanie ważnych próbek. W celu poprawienia wydajności obliczeniowej, stworzono hybrydowy algorytm wzajemnej entropii, który łączy w sobie stochastyczne podejście do generowania próbek i metodę wzajemnej entropii. Wyniki numeryczne wskazują na potencjalną poprawę alokacji niezawodności złożonych systemów, która prowadziłaby do jak najlepszego działania wszystkich komponentów systemu.

Słowa kluczowe: ocena niezawodności, konfiguracja-redundancja, optymalizacja systemu, metoda wzajemnej entropi, generowanie próbek stochastycznych.

Engineering practices with various redundancies increase the availability of a system as well as complexity which bring the uncertainty of reliability estimation and failure detection of system components. Under such conditions, a configuration-redundancy system is studied and the reliability function of the system is formulated. When no prior failure of system components is available, failure problem of system is a stochastic shortest path problem. Meanwhile to eliminate the uncertainty of system, it is necessary to detect failures series of components. The expected shortest path model and failure detecting model are proposed for system reliability optimization. The Cross-Entropy (CE) method is applied as a heuristic algorithm to estimate the system reliability and detect the failure of components. A stochastic sample generating approach is designed to obtain some valid samples. In order to improve the efficiency of computing, a hybrid CE algorithm which combines the stochastic sample generating approach and the CE method is developed. Numerical results indicate potential improvements in reliability allocation of complex systems that would lead to the best performances of all system components.

Keywords: *reliability assessment, configuration-redundancy, system optimization, cross-entropy method; stochastic samples generating.*

1. Introduction

Physical and functional redundancies are the two basic redundancy allocations for reliability optimization. Physical redundancy allocation means the use of multiple independent hardware channels, such as adding redundancy components or using redundancy in the form of standby components and subsystems [2-3, 13, 17, 42, 43]. Functional redundancy has its initial meaning of use of mathematical relations to obtain the redundant measurements or functional compensation [1, 4, 10, 39] for control systems. In most cases the performances of a failure-dependent system are related to its configuration. An initial disturbance may cause some components to fail by exceeding their loading limit, and failures of these components always incur the failure of another component. It is recognized [37] that it is difficult to obtain the reliability of a complex system at the optimal level only based on single redundancy method. System configuration is the arrangement of elements or subsystems. Inspired by the idea of redundant measurement by the method of functional redundancy, system reliability can be changed by adding the redundancy functions to its configuration which is defined as configuration-redundancy [11]. Instead of directly adding redundancy elements to improve system reliability, the configuration-redundancy improves performance of a system by providing multiple functions of system elements essentially. The redundancy functions of components can correlate with each other and together form system redundancy of output functions. Therefore for maximizing the reliability of a system, the configuration-redundancy technology is concerned as a novel technology in modern engineering system design. Some complex systems consist of large numbers of components that can also have various redundancy functions, and these components functioning together form system configuration redundancies. Nevertheless for the reason that redundancy is essentially different from reliability, the redundancy is not completely capable of compensating for the interacting and varying failure modes of a complex system. One of the main obstacles facing current applications of redundancy allocation is the dataset dimensionality, which brings the possible inaccurate estimation of reliability indices.

Effective models and algorithms of reliability allocation make the applications of redundancy in practical engineering systems become possible. Diversified redundancy models have been developed for several decade years. It is obvious that to determine an optimum reliability redundancy allocation for a complex system, accurate models of the process which can completely describe the process mechanisms should be considered in advance [14]. For realizing the relationship of redundancy and reliability, active survivability methods [35, 38] are always associated with logic analyses to evaluate the uncertainty problems of redundancy. Among all different redundancy models, reliability of components or subsystems can be classified as binary-state or multi-state performances. The binary-state means that a system or its components can only take two possible states, working or failed. A multi-state system can perform its intended function at multi-state processes, from perfectly working to completely failed. Meanwhile, traditional reliability models are always based on the assumption that a system and its elements have the characteristic of independent failures. Independent failures imply that the failure of one component has no affect on the remaining components. In reality, the assumption is not necessarily true. Often in a multi-state system, the failure of a component affects in some way one or more of the remaining components of the system such as acting as a shock [24-27], or forcing the remaining components to be progressively more loaded as the failure proceeds [9]. These affections may cause performance delay or failure of other components which in turn would result in the system dynamic cascading failures.

Many everyday tasks in reliability research involve solving complicated combinational optimization problems. Throughout the researches made up to date, different parametric optimization algorithms have been considered and very complex mathematical formulas for reliability indices are formulated [2, 23, 40]. Yet in practices it is possible that these models still produce incorrect outputs for some ordinary inputs. It is suggested [2] that the traditional techniques of parametric optimization are insufficient to calculate all logical faults of a multidimensional redundancy model in the extreme case. It is well known that such a problem of system optimization is NP hard [5]. An increasingly popular approach is to tackle the problem via stochastic algorithms [22, 28, 36, 41], especially via some hybrid algorithms of simulation methods and certain heuristic algorithms [7, 12, 19]. The majority of recent works in the reliability research areas have been devoted to develop various heuristic algorithms solving complex system reliability problems. Kuo [20] provided a particular survey of the literature classification of Meta-heuristic methods, including Genetic algorithm (GA), Simulated Annealing algorithm (SA), Ant Colony Optimization (ACO) and Great Deluge Algorithm (GDA), etc, which have been successfully applied in optimal reliability design because of their robustness and efficiency.

Another adaptive heuristic algorithm called the Cross-Entropy (CE) method [8] has been proposed in recent years. The entropy concept plays a fundamental role in modern information fields which provides new tools for reliability optimization [15, 30]. The CE method derives its name from the cross-entropy distance, and was renowned by an adaptive algorithm for estimating probabilities of rare-events involves CE minimization [16, 32-33]. The power and generality of this new method was soon realized when used for solving the uncertainty problems of discrete events. One advantage of the CE method is over routing algorithms that to detect possible shortest routings of a system to failure, which can effectively simulate the excess capacity of a system when conditions are not uniform [34]. One important uncertainty inherent to the reliability of a system concerns the different conditional failure rates of system components whose failure distributions may be changed. In such cases the completion time of system performance can be considered as a random variable, and simulation for the reliability objective function is "corrupted" with an additive mass "noise". It is inspiring that the CE method can deal successfully with both deterministic and "noisy" problems which typically occur in the reliability optimization problems. The CE method deals these noisy problems with some adaptive algorithms which can be found in Refs.[6, 18, 29].

The rest of the paper is structured as follows. In section 2, a configuration-redundancy system model is illustrated and the structure function and reliability function of the system are formulated. In section 3, two problems about reliability evaluation and failure detection of the system are proposed. In section 4, the basic principles of the CE method are introduced and a stochastic samples generating approach is developed. Then, hybrid CE method which is used to solve reliability optimization problems of Configuration-Redundancy systems is designed. We illustrate the effectiveness of the hybrid CE algorithms via a number of numerical experiments in section 5. Finally, in section 6, we present our conclusions and future research.

2. Model Description and Reliability Function

2.1. Reliability Model of Configuration-Redundancy System

Modern reliability models always assume that a complex system be a multistate system. Active redundancy defines every element of a system is operating and the system can continue to operate despite the loss of one or more redundancy elements Levitin *et al.* [21] developed a joint active redundancy model which is generally used for optimizing the reliabilities of multistate systems. In this kind of models, a system that consists of n components connected in series is considered, and each subsystem can contain no more than B_{max} elements of various types connected in parallel. In view of the deterministic element arrangement of subsystems, we can regard the system as "static" redundancies.

This paper studies another kind of redundancy model, which is defined as configuration redundancy [11]. Similar to the definition of joint redundancy of multistate systems described above, a configuration redundancy model is illustrated in Fig.1. Consider an active redundancy system which consists of n components, $A_1,...,A_n$, connected in series and each component contains no more than $B_{\max}$ different functions connected in parallel. Let

$$\Omega = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m_1 1} & \cdots & a_{m_n n} \end{pmatrix} \qquad (1 \le m_i \le B_{\max})$$

be the set of functions of components of the system, where functions of the same first subscript with different second subscripts, such as $a_{11},...,a_{1n}$, are of the same characteristic corresponding to the system function demands. Assume any one out-of-order combination of n different functions a_{ij} $(1 \le i \le m_i,\ 1 \le j \le n)$ connected in series has the same output function. The reliabilities of functions of a component can differ according to their characteristics. Then, each component has N-1 dimension functional redundancy. Taking into the consideration of adaptive function arrangement of components, we define the system as "dynamic" redundancy.

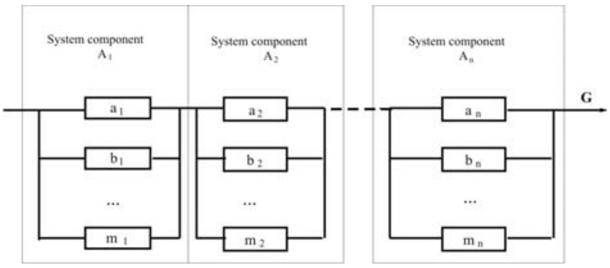

Fig.1. Structure of a configuration-redundancy system

It can be seen that the structure of a configuration-redundancy system is essentially different from the traditional redundancy models. The system only contains N components and each component has different redundancy functions rather than physical redundancies. The configuration-redundancy system model shows how all components form a system, and reflects the dynamic interrelationships of all the system functions. Instead of directly improving reliability of the system's elements, the redundancy model improves reliability of the system by providing multiple subsystem configurations. In such a redundancy allocation model, system can be considered to have different redundancy structure of permutation and combination, which is formed by parallelizing function of multiple configurations.

2.2. Reliability Function of Configuration-Redundancy System

2.2.1. Structure Function

The *structure function* of a system expresses the state of the system in term of the states of all components. Let $A = (a_i,\ i = 1,...,n)$ be the set of all components of a system. Use x_i to represent the state of component i and define variables $\mathrm{x} = (x_1,...,x_n)$ represent the states of all components of the system. Assume the state of each component A_i is a binary variable: x_i=1 if the component works and 0 otherwise. Then the state of the system, s say, is a binary variable as well, s = 1 if the system works and 0 otherwise. We assume that there exists a function $\phi : \{1,0\}^n \to \{1,0\}$ such that

$$s=\phi(\mathrm{x}) \tag{1}$$

This function is defined as the structure function. A *series* system is defined as a system which only functions when *all* components are operational. Its structure function is given by

$$\phi(\mathrm{x}) = \min\{x_1, x_2, ..., x_n\} = x_1 x_2 \cdots x_n = \prod_{i=1}^{n} x_i \tag{2}$$

A system that functions if and only if at least one component is operational is called a *parallel* system. Its structure function is

$$\phi(\mathrm{x}) = \max\{x_1, x_2, ..., x_m\} = 1-(1-x_1)(1-x_2)\cdots(1-x_m) = \coprod_{i=1}^{m} x_i \tag{3}$$

The structure function of a configuration-redundancy system can be developed by analyzing its series-parallel configuration. The output function of a configuration-redundancy system can be comprised by a corresponding list of sub functions in the design stage. For each component of the system, different components can be designed into containing all or part of the list of sub functions. Let $\Omega = (A_i,\ i$=1,...,$n)$ be the set of all components of the system. Different functions of A_i may drop into different states in a different order, which makes the system drop into different states, denoted by $S = (s_i,\ i$=1,2,...,$k)$. States in which the system can perform the expected output function are valid, and states can not perform the expected output functions are invalid. Define binary variables $x_i \in (x_1, ..., x_n)$, which represents the states of component A_i: x_i= 1 if the i-th component works, and 0 otherwise. Meanwhile, let vectors $x_{ij} = (x_{i1}, ..., x_{im_i})$ represents the function states of component i : x_{ij}= 1 if the j-th function of the i-th component is valid for the expected output functions of system and 0 otherwise. Then the vectors X=$\{x_{ij}\}$, $(1 \le i \le m_i,\ 1 \le j \le n)$ define the entire system structure. Let $\Phi = (\phi_1,...,\phi_q)$ be the set of all structure functions of all series configurations of the system, where a series configuration ϕ_i contains n functions from each component and all functions are linked in series. Note that the system states are determined by all the construction functions, $\phi_1,...,\phi_q$, of the system. For any valid state s_i, there is in Φ at least a structure function ϕ_i which can perform the expected output function of system independently. Then, the relationship of the number k and q is

$$k = \mathrm{C}_\mathrm{q}^1 + \mathrm{C}_\mathrm{q}^2 + \cdots + \mathrm{C}_\mathrm{q}^\mathrm{q} = 2^q - 1$$

The *method of paths and cuts* can be used to establish the relationship of structure function and components states. Here, we assume that the structure function is monotone by defining $\mathrm{x}<\mathrm{y} \Rightarrow \phi(\mathrm{x})<\phi(\mathrm{y})$ for all vectors **x** and **y**, where x < y means that $\mathrm{x}_i \le \mathrm{y}_i$ for all i and $\mathrm{x}_i<\mathrm{y}_i$ for at least one i. A *minimal path vector* is a vector **x** such that ϕ(x)=1 and ϕ(y) = 0 for all y < x. A *minimal cut vector* is a vector **x** such that ϕ(x) = 0 and ϕ(y) = 1 for all y > x. The *minimal path set* corresponding to the *minimal path vector* **x** is the set of indices i for which x_i= 1. The *minimal cut set* suggest that the *minimal cut vector* **x** is the set of indices i for which x_i= 0.

The structure function can be determined by the minimal path and cut sets. Namely, let $P_1,\ldots, P_p$ be the minimal path sets and $K_1,\ldots,K_k$ be the minimal cut sets of the system with structure function Φ, then,

$$\Phi(\mathrm{x}) = \coprod_{j=1}^{p} \prod_{i\in p_j} x_i \tag{4}$$

and

$$\Phi(\mathrm{x}) = \prod_{j=1}^{k} \coprod_{i\in K_j} x_i \tag{5}$$

The equation (4) suggests that the system works if and only if there is at least one minimal paths set with all components working. Similarly, the equation (5) means that for each of the cut sets the system works if and only if at least one component is working.

2.2.2. Reliability Function Formulation

The *reliability* of a component is defined as the probability that the component will perform a required function under stated conditions for a stated period of time. The reliability of the configuration-redundancy system is

$$\mathrm{P}[\textit{system works}]=\mathrm{P}[\Phi(\mathrm{x})]=\mathrm{E}[\Phi(\mathrm{x})]$$

where **X** is the component vector $(X_1,...,X_n)$. The function $r(p)=\prod[\phi(\mathrm{X})=1]$ is the *reliability function* of the system. We use a vector $p = (p_1,...,p_n)$ to gather the reliabilities of the components, where p_i contains m_i functions of component i. Then, the function matrix of components of the system is

$$p=\begin{pmatrix} p_{11} & \cdots & p_{n1} \\ \vdots & \ddots & \vdots \\ p_{1m_1} & \cdots & p_{nm_i} \end{pmatrix}$$

and the reliabilities of the functions. Assuming the component states are independent, the system reliability can be expressed in terms of the vector $p=(p_1,...,p_n)$. When the random variables X_i are independent, the reliability function of series and parallel systems can be expressed by the following formulations:

$$r(\mathrm{p}) = \prod_{i=1}^{n} p_i \tag{6}$$

and

$$r(\mathrm{p}) = \coprod_{i=1}^{n} p_i \tag{7}$$

respectively. Based on the system structure function $\Phi(X)$, the reliability function of configuration-redundancy systems is given by

$$\begin{aligned} r(p) &= E\left[\Phi(\mathrm{X})\right] = \sum_{\mathrm{x}\in \mathrm{S}} \phi(\mathrm{x}) P[\mathrm{X}=\mathrm{x}] \\ &= \sum_{\mathrm{x}\in \mathrm{S}} \phi(\mathrm{x}) \prod_{i=1}^{n} \left[p_i^{x_i} (1-p_i)^{1-x_i} \right] \end{aligned} \tag{8}$$

The formulation above can be used as a general method to evaluate the reliability of configuration-redundancy systems. Note that the system structure function $\phi(\mathrm{x})$ involves the valid combination analysis of component functions and the random variables of X_i are not necessarily independent, i.e., the accurate value of $p_i^{x_i}$ is difficult to obtain. When a configuration-redundancy system contains a large number of components, the exact value of the system reliability is not feasible to be obtained directly from the formulation. To optimize a configuration-redundancy system, it is reasonable to expect that the reliability of each component is the same. According to the maximum entropy principle when all subsystem failure events have the same failure probability in a series system, the system reliability amounts to the maximum and the entropy of system reliability equals to *log m*.

3. Reliability Evaluation of Configuration-Redundancy System

Due to the complexity, we design two stochastic simulation models to evaluate and detect uncertain reliability problems of configuration-redundancy system. The first model is to search for the expected shortest path which can provide the information for further system reliability optimization. The second model deals with the failures time series of components and functions to eliminate the uncertainties of system failures.

3.1. Expected shortest path model

For a complex system there are many ways in which component failures interact, which result in that random variables X_i of component states are usually not subject to the independent failures. The uncertainty of failure interactions would result in the system dynamic cascading failures. In such situations the decision-makers expect to find the path with the minimum expected time of system performance, which help them make decisions for reliability optimization of system. Use w_{ij} to denote the length of x_{ij}, and use $T(\mathrm{x},w)$ to represent the total length of a random failure path of system. Thus, the reliability evaluation problem of the configuration-redundancy system is transferred to be a stochastic path problem. That is

$$min\ \left\{ T(\mathrm{x}_i, w) = \sum_{j=1}^{m_i} w_{ij} x_{ij},\ (1 \le i \le m_i) \right\} \tag{9}$$

In order to depict the stochastic shortest path problem, a directed weighted graph $\Im = [\mathbb{N}, \ell]$ is considered as an auxiliary tool. The graph $\Im$ consists of a finite set of nodes $\mathbb{N} = \{1, 2, ...m_i\}$ and a set of routes ℓ, in which the lengths of the routes are assumed to be stochastic. Each route is denoted by an ordered pair (i, j), where $(i, j) \in \ell$. It is assumed that there is only one route ℓ from i to j. Moreover, the nodes in a acyclic directed graph $\Im = [\mathbb{N}, \ell]$ can be renumbered so that $i<j$ for all $(i, j) \in \ell$. Use $x = \{x_{ij} \mid (i, j) \in \mathbb{N}\}$ to represent the journey for i to j, where $x_{ij}=1$ means that the vector (i,j) is in the route, $x_{ij}= 0$ means that the vector (i,j) is not in the route.

It has been proved that $x = \{x_{ij} \mid (i, j) \in \mathbb{N}\}$ is a route from nodes 1 to n in a directed acyclic graph if and only if

$$\sum_{(i,j)\in\mathbb{N}} x_{ij} - \sum_{(j,i)\in\mathbb{N}} x_{ji} = \begin{cases} 1, & i=1, \\ 0, & 2 \le i \le n-1, \\ -1 & \mathrm{i}=n, \end{cases} \tag{10}$$

where $x_{ij}=1$ or 0 for any $(i, j) \in \ell$.

For a given expected time of system performance to failure, say γ, we expect the length of the shortest path of system will exceed γ. Then the stochastic shortest path problem of the configuration-redundancy system is to estimate

$$l = P(T(x_i, w) \geq \gamma) = EI_{\{T(x_i,w)\geq\gamma\}} \quad (11)$$

s.t.

$$\begin{cases} \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{m_i} x_{ki} = 1, & i = 1\ ; \\ \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{m_i} x_{ki} = 0, & i = 2, 3, ..., m_i\text{-}1; \\ \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{m_i} x_{ki} = -1, & i = m_i\ ; \\ x_{ij} = 0, 1, & i = 1, 2, ..., m_i,\ j = 1, 2, ..., n. \end{cases}$$

3.2. Failures Detection Model

Detecting the failure series of a system is necessary to eliminate the uncertainty of system failures. Suppose that we do not know the real failure time series X of a configuration redundancy system, we may introduce auxiliary vectors Y=$\{y_{ij}\}$,$(1 \leq i \leq n$, $1 \leq j \leq m_i)$ to reconstruct the failure vector X, where $i = (1,...,n)$ and $j = (1,...,m_i)$ corresponds to the component failures and function failures, respectively. To identify the failure time series of system, we use random simulation, $S(y \mid X)$, to represent the number of failure matches between the vectors of y and X. Use an algorithm to solve the maximum problem of $S(y \mid X)$, which can confirm the failure series of the system components and functions. The problem is to solve

$$max \left\{ S(y \mid X) = \sum_{i=1}^{n} n \times m_i - \sum_{j=1}^{n} \sum_{i=1}^{m_i} \left| y_{ij} - x_{ij} \right| \right\} \quad (12)$$

s.t.

$$\begin{cases} \sum_{j=1}^{n} y_{ij} - \sum_{k=1}^{m_i} y_{ki} = 1, & i = 1\ ; \\ \sum_{j=1}^{n} y_{ij} - \sum_{k=1}^{m_i} y_{ki} = 0, & i = 2, 3, ..., m_i\text{-}1; \\ \sum_{j=1}^{n} y_{ij} - \sum_{k=1}^{m_i} y_{ki} = -1, & i = m_i\ ; \\ x_{ij}, y_{ij} = 0, 1, & i = 1, 2, ..., m_i,\ j = 1, 2, ..., n. \end{cases}$$

4. A Hybrid Algorithm

The minimization and the maximization problems of Eq.11 and Eq.12 formulate a typical stochastic path problem and a combinatorial optimization problem, respectively. For a multi-dimensional system, an exhaustive examination of all possible solutions is unrealistic. A heuristic searching algorithm is needed to solve the problem. We now focus on the recently developed heuristic family of the CE method. It has been proved [31] that the CE method has the theoretical global convergence property for most practical stochastic problems.

4.1. Stochastic Samples Generating Approach

To solve the problems above, we need to generate a start random sample. Using the start sample path we can construct prearrange samples for heuristic algorithm updating the parameters at each of iterations. Firstly, we develop a stochastic samples generating algorithm for simulating the expected shortest path model. For convenience, ξ is presented in another way as $\xi = (\xi_1, ..., \xi_{m_i})$, where m_i is the number of the random variables of functions of component A_i. Then, we calculate the following function:

$$U_1 : x \rightarrow E\left[T(x, \xi)\right]$$

Here we design a stochastic generating as follows:

Step 1. Set $U_1(x) = 0$.

Step 2. Generate $\xi = (\xi_1,...,\xi_n)$ from the distribution functions.

Step 3. $U_1(x) \leftarrow U_1(x) + T(x,\xi)$.

Step 4. Repeat the second and the third steps N times.

Step 5. $U_1(x) = U_1(x)/N$.

Secondly for the failures detecting problem, let y represent a random paths which is

$$y = \begin{pmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{m_1 1} & \cdots & y_{m_i n} \end{pmatrix}$$

where y_{ij} are independent Bernoulli random variables. We calculate the value

$$U_2 : y \rightarrow E[S(y \mid X)]$$

Then a stochastic simulation sample can be developed as follows:

Step 1. Set $U_2(y) = 0$, $t = 1$.

Step 2. Generate $y = \begin{pmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{m_1 1} & \cdots & y_{m_i n} \end{pmatrix}$ from the Bernoulli vectors with success probability vectors $p_{t\text{-}1}$.

Step 3. $U_2(y) \leftarrow U_2(y) + S(y)$.

Step 4. Repeat the second and the third steps N times.

Step 5. $U_2(y) = U_2(y)/N$.

4.2. The CE Method

A tutorial on the Cross-Entropy method is presented in Ref.[8]. The usual CE procedure is to construct a sequence of reference vectors and converges to the degenerate probability that corresponds to the optimal vector. In the field of combination optimization problems, the CE method can be readily applied by first translating the underlying optimization problem into an associated estimation problem which typically involves Rare-Event estimation. The processes of the CE method for Rare-Event estimation referred to Rubinstein's book [31], is as follows:

Suppose we wish to search for a general minimization problem, such as $min\ S(x)=\sum_{i=1}^{n} c_i x_i$, over all states $\boldsymbol{x}$ in some set X. Let us denote the minimum of S(X) by γ^*, that is

$$S(x^*)=\gamma^*=\min_{x\in X} S(x) \tag{13}$$

where x* is the optimal vector. To proceed, we first randomize x by defining a family of pdfs $\{f(\bullet;u),u\in V\}$ on the set X. Next, we associate with the estimation of

$$l(\gamma)=P_u(S(X)\geq\gamma)=E_u I_{\{S(X)\geq\gamma\}} \tag{14}$$

A viable method to estimate l of Eq. (14) is to use a stochastic simulation method. Draw a random sample $X_1,...,X_N$ from the distribution of X, and use

$$\frac{1}{N}\sum_{i=1}^{N} I_{\{S(X)\geq\gamma\}} \tag{15}$$

as the unbiased estimation of l. However, for large γ the probability of l is very small, a crude stochastic simulation method require a very large N to obtain a small relative error. The CE method uses an adaptive likelihood ratio to solve the problem. It can be seen that there are in fact two possible estimation problems associated with Eq. (14). For a given γ we can estimate l, or for a given l we can estimate γ alternatively. For some large fixed γ and a small relative error (RE), a better way to perform the simulation is to use importance sampling as

$$l=\int I_{\{S(X)\geq\gamma\}}\frac{f(x)}{g(x)}g(x)dx=E_g I_{\{S(X)\geq\gamma\}}\frac{f(X)}{g(X)} \tag{16}$$

where the subscript g means that the expectation is taken with respect to g. We define

$$W(X_i)=\frac{f(x_i)}{g(x_i)} \tag{17}$$

as the likelihood ratio.

The CE method solves the problem efficiently by making adaptive changes to the probability density function f(X) and g(X). The changes are aimed to the minimum of their cross-entropy for obtaining the maximum likelihood ratio

$$W(X;u,v)=\frac{f(X;u)}{f(X;v)} \tag{18}$$

by defining any reference parameter v. Thus the optimal solution of (16) can be rewritten as

$$v^*=\arg\max_{v} E_w I_{\{S(X)\geq\gamma\}} W(X;u,w)\ln f\ (X;v) \tag{19}$$

However the accurate value of v* always can not be obtained directly, we may estimated v* by solving the following stochastic counterpart, that is

$$\max_{v} D(v)=\max_{v}\frac{1}{N}\sum_{i=1}^{N} I_{\{S(X_i)\geq\gamma\}} W(X_i;u,w)\ln f(X_i;v) \tag{20}$$

where X_1, … , X_N is a random sample form $f(\bullet;w)$. When the function D in (18) is convex and differentiable with respect to v, v^* can be solved by the following iterations of simulated cross-entropy program,

$$v^{(k)}=\arg\max_{v}\frac{1}{N}\sum_{i=1}^{N} I_{\{S(X_i)\geq\gamma\}} W(X_i;u,v^{(k-1)})\ln f(X_i;v) \tag{21}$$

Thus the program creates a sequence $f(\bullet;v)$ of pdfs that are "steered" in the direction of the theoretically optimal density $f(\bullet;v^*)$, which corresponds the degenerate density at an optimal point. In the adaptive process, the CE method simultaneity generates an adaptive updating of γ_t and v_t sequence, $\{\gamma_t,v_t\}$. The sequence converges quickly to a small neighborhood of the optimal $\{\gamma^*,v^*\}$.

4.3. Hybrid CE Algorithms

A start random sample is necessary for the CE method creating the iterative sequences. We integrate stochastic sample generating approaches and the CE method to design hybrid simulated algorithms. The kind of hybrid algorithms contain two stages described as follows:

Stage 1: Generate the stochastic paths according to pdfs of the random variables for the problems. Rearrange the sequence of reference vectors for adaptive updating of the process 2.

Stage 2: Using the CE method to search for the optimal parameter vector.

4.3.1. Failure Series Detection Algorithm

Use a Markov chain to represent random path y. The Markov chain transfers from node 1 to the end node after $\gamma^*=\sum_{i=1}^{n} m_i$ steps. Let

$$p=\begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m_1 1} & \cdots & p_{m_i n}\end{pmatrix}$$

denotes the one-step transition matrix of the Markov chain. Once the P is converged to the optimal parameter vector, say p^*, the real failure time series vector X is determined. The main algorithm for updating p_i is as follows.

Algorithm 1

Step 1. Start with some p_0. Set t =1 (iteration counter).

Step 2. Use the stochastic samples generating approach to generate random samples with success probability vector p_{t-1}. Calculate the performance $S(y_i)$ for all i, and order them from smallest to biggest, $S_{(1)}\leq\cdots\leq S_{(N)}$. Let γ_t be the sample (1-ρ) -quantile of performances, γ_t=$S_{[(1-\rho)N]}$, provided this is less than γ^*. Otherwise, put γ_t=γ^*.

Step 3. Use the same sample to calculate p_t=($p_{t,1}$,...,$p_{t,n}$), for j=1, …, n, via

$$p_{t,j}=\frac{\sum_{i=1}^{N} I_{\{S(y_i)\geq\gamma_t\}} I_{(y_{ij}=1)}}{\sum_{i=1}^{N} I_{\{S(y_i)\geq\gamma_t\}}}$$

where y_i=(y_{i1},...,y_{in}).

Step 4. If the stopping criterion is met, then stop. Otherwise set $t=t+1$ and reiterate from step 2.

4.3.2. Computing of the Shortest Path

By checking the system state if a configuration system is functioning at time t=1, we can change the distributions of random variables for the shortest path estimating problem. The aim is to translate an estimating problem of independent Bernoulli random variables into an estimating problem involving dependent exponential random variables. Ref.[31] gave a detail on how to translate the distributions of random variables. In other words, if the distributions of functions of a component x_i belong to a natural exponential family, the probability density of x_i is

$$f(x_i, u) = \exp\left(-\sum_{j=1}^{m_i} \frac{x_{ij}}{u_{ij}}\right)\prod_{j=1}^{m_i}\frac{1}{u_{ij}}$$

and the likelihood of Eq. (18) is changed into

$$W(x; u, v) = \frac{f(x; u)}{f(x; v)} = \exp\left(-\sum_{j=1}^{m} x_j\left(\frac{1}{u_j} - \frac{1}{v_j}\right)\right)\prod_{j=1}^{m}\frac{v_j}{u_j} \qquad (22)$$

Then, the updating program of Eq. (21) always become the following form,

$$v_{t,j} = \frac{\sum_{i=1}^{N} I_{\{S(X)\geq\gamma_t\}} W(X_i; u, v_{t-1}) X_{ij}}{\sum_{i=1}^{N} I_{\{S(X)\geq\gamma_t\}} W(X_i; u, v_{t-1})} \qquad (23)$$

However, the CE method is difficult to compute such problems [34] if the probability of the shortest path is too small, say smaller than 10^{-5}. It is important to note that the CE method is efficient when the probability of the shortest path is not too small. A two-level algorithm can be used to overcome the difficulty. In the first level of iteration in the algorithm, γ_t is updated, and in the second level v_t is updated. When the distributions of $f(x; v)$ belong to a natural exponential family that is parameterized by the mean, the general algorithm for computing the shortest path is as follows.

Algorithm 2

Step 1. Define v_0=u. Set t =1.

Step 2. Use the stochastic samples generating approach to generate a random sample according to the pdf $f(\bullet; v_{t-1})$. Calculate the performance $S(X_i)$ for all i, and order them from the smallest to the biggest, $S_{(1)} \leq \cdots \leq S_{(N)}$. Let γ_t be the sample (1-ρ) -quantile of performances, $\gamma_t = S_{[(1-\rho)N]}$, provided this is less than γ^*. Otherwise , put $\gamma_t = \gamma^*$.

Step 3. Use the same sample to calculate, for j=1, …, n

$$v_{t,j} = \frac{\sum_{i=1}^{N} I_{\{S(X)\geq\gamma_t\}} W(X_i; u, v_{t-1}) X_{ij}}{\sum_{i=1}^{N} I_{\{S(X)\geq\gamma_t\}} W(X_i; u, v_{t-1})}$$

Step 4. If $\gamma_t = \gamma^*$, then proceed to step 5; Otherwise set t = t+1 and reiterate from step 2.

Step 5. Let T be the final iteration. Generate a sample $X_1, \ldots, X_{N1}$ according to the pdf $f(\bullet; v_t)$ and estimate l via the important sampling estimator

$$l = \frac{1}{N_1}\sum_{i=1}^{N_1} I_{\{S(X)\geq\gamma_t\}} W(X_i; u, v_t)$$

5. Numerical Results

5.1. Example 1

As a example, assume the case where

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & / \\ / & x_{22} & / & x_{24} \\ x_{31} & x_{32} & x_{33} & / \\ x_{41} & / & / & x_{44} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & / \\ / & 0 & / & 0 \\ 1 & 0 & 1 & / \\ 0 & / & / & 1 \end{pmatrix}$$

Using the initial parameter vector

$$p_0 = \begin{pmatrix} 0.5 & 0.5 & 0.5 & / \\ / & 0.5 & / & 0.5 \\ 0.5 & 0.5 & 0.5 & / \\ 0.5 & / & / & 0.5 \end{pmatrix}$$

and taking N=100, ρ=0.1, the algorithm 1 quickly yields the result given in Table 1. We notice that p_t and γ_t converge very quickly to the optimal parameter vector $p^*=y$ and optimal performance γ^*=10 respectively. The table was computed in less than half 1 second.

Tab. 1. The evolution of the sequence $\{\gamma_t = p_t\}$.

t	γ_t	p_t									
0		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
1	7	0.60	0.4	0.00	1.00	0.00	0.20	0.00	0.40	0.40	0.80
2	9	0.80	0.80	0.00	1.00	0.00	0.00	0.00	0.80	0.40	1.00
3	10	1.00	1.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	1.00
4	10	1.00	1.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	1.00

5.2. Example 2

To illustrate the effectiveness of the hybrid CE algorithm for the shortest path estimating problem, a 6-node graph of Fig. 2 with random “weights” $X1, \ldots, X12$ is considered.

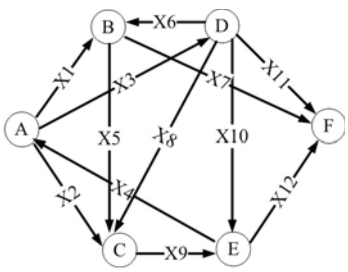

Fig. 2. Shortest path from A to F

Let S(X) be the total length of the shortest path from node A to node F. Suppose the weights are independent and exponentially distributed random variables with means $u_1,\dots,u_{12}$. The *nominal* parameter vector u is given in Table 2. Using the means of weights, we can calculate the unreliability q_i of each link.

Tab. 2. "Weight" and unreliability of each link

X(i)	u_i	q_i	*X(i)*	u_i	q_i	*X(i)*	u_i	q_i
X1	1.0	2.72e0	X5	0.2	6.74e-3	X9	0.5	1.35e-1
X2	0.1	4.54e-5	X6	0.3	3.57e-2	X10	0.3	3.57e-2
X3	0.8	2.87e-1	X7	0.4	8.21e-2	X11	0.7	2.40e-1
X4	0.1	4.54e-5	X8	0.6	1.89e-1	X12	0.9	3.29e-1

Define the *relative error* as the index of estimating error, which is

$$\mathrm{RE} = \frac{\sqrt{\mathrm{Var}(l)}}{l^*}$$

where l^* is the actual value and l is the estimate value of the shortest path. For convenience, we select the mean value as the actual value. Assume the minimum path is greater than $\gamma=3$. Table 3 displays the estimated optimal parameter of the hybrid CE algorithm, using N = 1000 and $\rho=0.1$. This table was computed in less than 1 second. Using the estimated optimal parameter vector of v_5 described in Table 3 and with $N_1=10^5$, the final step of the hybrid CE algorithm gave an estimate of 5.95e-4 with a relative error of 3%. The simulation time was less than 2 seconds.

In order to express the efficiency of this hybrid CE algorithm, we compute example 2 with different ρ, N and N_1. It follows from Table 4 that the relative error does not exceed 4%.When we select $0.01\le\rho\le0.1$, $5\times10^2\le N\le5\times10^3$ and $10^5\le N_1\le5\times10^5$, all of the simulation time are less than 10 seconds. The results suggest that the hybrid CE algorithm is robust to the parameters selecting and effective to find the shortest path of the system.

Tab. 3. The evolution of the sequence $\{\gamma_t, v_t\}$.

t	γ_t	v_t											
0		0.1000	0.1000	0.8000	0.1000	0.2000	0.3000	0.4000	0.6000	0.5000	0.3000	0.7000	0.9000
1	1.2604	1.6754	0.1201	1.2416	0.0872	0.2099	0.2994	0.5457	0.5857	0.5779	0.3153	0.9670	1.4889
2	1.8835	2.2494	0.1025	1.5802	0.0959	0.2033	0.3113	0.6759	0.8147	0.8009	0.2540	1.2388	2.0165
3	2.4575	2.8894	0.1369	2.0033	0.0878	0.2049	0.2958	0.6737	0.6341	0.7397	0.3258	1.3578	2.5364
4	2.8746	3.0282	0.1146	2.1026	0.0959	0.2801	0.1945	0.5490	0.5023	0.6593	0.2053	1.6474	3.0674
5	3.0000	3.2373	0.0932	2.2060	0.0695	0.2140	0.2265	0.5319	0.5393	0.7264	0.2507	1.6919	3.1274

Tab. 4. Comparison solutions of the shortest path.

	ρ	N	N_1	Simulation time	Path lengths	Relative Error
1	0.01	1000	100000	1.3750	0.00059344	0.030877
2	0.01	1000	100000	1.4070	0.00061493	0.031567
3	0.05	1000	100000	1.3750	0.00062222	0.036199
4	0.05	1000	100000	1.4380	0.00060167	0.025946
5	0.10	1000	100000	1.3910	0.000604	0.037932
6	0.10	1000	200000	2.7350	0.00058653	0.019577
7	0.10	1000	200000	2.7970	0.00057352	0.034571
8	0.10	1000	500000	6.9840	0.00060106	0.012799
9	0.10	500	500000	7.0310	0.00057756	0.019987
10	0.10	5000	200000	3.2030	0.00061284	0.017413

6. Conclusion and Future Research

In this paper, a configuration-redundancy system is studied and reliability function of system is formulated. To obtain the desired demands, redundancy technologies have been widely employed in traditional engineering practices. Redundancy improves system performances by compensating its possible component and subsystem failures. However, systems with various physical redundancies have brought the high cost of system and the complexities to system structure. Studying on reliability evaluation of configuration-redundancy systems will provide novel views for system design. For the reason that the failure of a component affects one or more of the remaining components in a system, reliability estimation of a complex configuration-redundancy system is heavily dependent on the accurate analysis of the mutual effects among possible function failure events of system components.

One important uncertainty inherent to the model of configuration-redundancy used in the system optimization problems concerns the stochastic states of system component failures. Likelihood ratio estimators obey the laws of probability theory when no more prior probability is available. Conventionally this requires repeated evaluations of the failure probability for different values of the stochastic parameters, which is a direct but computationally expensive task. Reliability-based design sensitivity analysis involves studying the dependence of the failure probability on design parameters. The basic principle of the CE method are formulated and used as a versatile tool to detecting the possible failure time series. The implementation and performance of the CE method for solving the stochastic system failure problem are discussed. The results obtained are promising and show that the algorithm is less sensitive to the variations of technique parameters and offers an effective alternative for solving the system robust design with uncertainty.

Several issues must be resolved in future research to enhance the capacity of system reliability. First, based on the reliability research of configuration-redundancy systems it is feasible to evaluate integrated disposition of physical redundancy and function redundancy in a system. Second, the simulation algorithm used in this paper deals with only the relative rankings of system reliability. An improvement of the performance of the CE algorithm could be achieved by examining the failures of practical complex systems. Finally we will concern the non-convex optimization problem of complex systems, and the efficiency of our model and algorithm will be given much attention to solve such complex reliability allocation problems.

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