1. Introduction

The rear axle is an important part of the automobile transmission system; its working performance directly influences the whole system operation. According to statistics, 20% of car accidents are caused by rear axle failures, in which the most common fault is gear failure. If we could predict the residual life of the rear axle accurately during operation, it would not only prevent the fault occurrence, but also reduce unnecessary maintenance, thereby cutting down expenses and enhancing the equipment service life.

Extensive literature is available on gear damage prediction. Several studies of life prediction of gears based on the fracture mechanics have been carried out. Kramberger analyzed thin-rim gear fatigue life by using the finite element method and employed the continuum mechanics based approach for the prediction of the fatigue process initiation phase, where the basic fatigue parameters of the material are taken into account, and the remaining life of gear with an initial crack is evaluated using the linear-elastic fracture mechanics [5]. Khan made an effort to validate the competency of a standard gear useful lifetime estimation formula which is used for helical gear useful lifetime estimation under linear pitting fatigue conditions [4]. Several researchers approached gear life prediction by extracting effective characteristic parameters. Loutridis proposed energy-based features for gear fault diagnosis and prediction which are obtained when defective teeth are engaged [9]. Burstein developed a theory and a computing algorithm for machine residual service life prediction using a thermal diagnostic method which adopted the diagnostic parameter derived from the temperature change rate during running of a machine [1]. Statistic methods are often chosen as a traditional technique for gear fatigue life estimation. Naqamura developed a model to predict crack propagation and fatigue life of a carburized steel gear based on crack initiation on carbide precipitation and crack propagation life at tooth base by using the Monte Carlo simula-

**Keywords**: Damage, Prediction, Vibration Signals, RBF Neural Network.
tion method [11]. Loutridis introduced multiscale local statistic tools for gear failure prediction and established an empirical law that related variance at various scales to crack magnitude [10]. Topac applied a simulation based method and utilized a finite element method to analyze the pressure and fatigue of rear axle prototype. The initial position of a fatigue crack and the least load cycle before fatigue germination can be obtained [15]. Model-based analysis is a commonly used method and has been chosen in the present paper. Singh developed a two-stage cumulative damage model which divided fatigue life into two phases; a crack initiation phase and a crack propagation phase. The results show that the proposed method greatly improved life prediction capabilities and retained the simplicity of the S-N based approach with relatively simple material data and straightforward calculations [13]. Li proposed a model-based method to predict the remaining useful life of a gear with a fatigue crack which consisted of an embedded model to identify gear meshing stiffness, estimate crack size, simulate gear meshing dynamics and forecast the remaining life based on the estimated crack size and dynamic load [8]. Zhan developed a robust model-based technique for the detection and diagnosis of gear faults under varying load conditions using the gear motion residual life and a noise-adaptive Kalman filter-based auto-regressive (AR) model which is fitted to the gear motion residual signals. The percentage of outliers exceeding the three standard deviation limits of baseline AR model residuals is applied to evaluate the state of the gear [20], but the research depended on statistical methods which made prediction slow and could not meet the on-line prediction requirement of gear residual life. Simultaneously, another analysis method could be used, such as that introduced by Virtic where frequency spectra of simulated sound signals enabled an analysis of the error that could be used for calculating the remaining service life and determining the control cycle of maintenance [17,6,2,3]. Tanaka developed a method to diagnose gear conditions using a laser beam and estimated the condition of tooth surface such as initial or abnormal abrasion, pitting, and spalling by comparing the variations of laser reflections between initial and present conditions [16]. Lewicki proposed the effect of moving gear tooth load on crack propagation and studied two-dimensional analysis of an involute spur gear and three-dimensional analysis of a spiral-bevel pinion gear, and also investigated a modified theory for prediction of gear crack propagation paths based on the criteria by Erdogan and Sih [7].

The traditional methods for predicting gear damage life are generally by means of statistical tools and do not consider the influence of load and rotational speed on the vibration signals and therefore do not meet the demand of on-line monitoring and predicting. However, the rear axle gear is a key part of the automobile transmission system with non-uniform velocity and variable load. The vibration signal has a large fluctuation range because of fluctuating speeds and loads as well as external disturbances when the rear axle gear is running. If the characteristic parameters can be extracted from vibration signals directly, higher predictive precision can be obtained.

In this study, a coupled analysis method for the prediction of damage in rear axle gears mounted in a fatigue test system is carried out. This paper adopts a characteristic parameter tracking method, which predicts the parameter values according to the historical values of the characteristic parameters and diagnoses the possibilities of equipment faults so as to make prediction of gear damage life and provide the necessary information to support normal operation maintenance of rear axle gears. This study investigates the use of RBF neural networks combined with recursive preprocessing as a new predictive method. That is, the recursive preprocessing method is used to reduce the effects of instantaneous load and speed fluctuation on the extracted features of gear damage from vibration signals. At the same time the RBF neural network is characterized by self-adaptive and fast convergence. Simultaneously, a comparison is made between the traditional methods and the proposed analysis method.

This paper is organized as follows: the vibration characteristics of rear axle gears and the preprocessing method of vibration signals are described in Section 2. The characteristics and the topological structures of RBF neural networks are discussed in Section 3. A new method of damage prediction for rear axle gears on the basis of vibration signal preprocessing coupled with a neural network is described in Section 4. In Section 5, simulated waveforms have been used to verify the feasibility of the proposed method and obtain good predictive accuracy. Simultaneously, the experimental fatigue test system is described and damage prediction for rear axle gears using vibration signal preprocessing coupled with RBF neural networks is investigated. Concluding remarks are presented in Section 6.

2. Vibration characteristics of the rear axle gears and vibration signal preprocessing methods

Many parameters could be used for gear condition monitoring, such as root-mean-square (RMS) value, crest factor, kurtosis factor, FM4 and NA4, etc. [18]. The kurtosis factor is sensitive to the failures generating shock pulse signatures, especially the early stage failures. However, this indicator is not stable enough during advanced stages of deterioration. The stability of the RMS value is good, but is not sensitive to early stage failures. RMS values increase with the deterioration of faults [14]. Both the RMS indicator and the kurtosis factor are selected as the condition assessment variables in this paper.

The RMS of the vibration signal is defined as:

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} s_i^2}$$  \hspace{1cm} (1)

where $s_i$ is the $i$-th member of dataset $s$. The mathematical definition of kurtosis is given by Eq. (2):

$$\text{Kurtosis} = \frac{N \sum (s_i - \bar{s})^4}{\left( \sum (s_i - \bar{s})^2 \right)^2}$$ \hspace{1cm} (2)

where $N$ is the number of points in dataset $s$ and $\bar{s}$ is the mean of the dataset $s$.

The original RMS value and kurtosis factor sequences of the rear axle gear are shown in Fig. 1. Diagram a) of Fig.1 shows the relationships of the RMS value with time and the kurtosis factor versus time every 15 minutes when the rear axle gear is in normal operation. The peak-peak value is used to reflect the fluctuation ranges of characteristic parameters and is remarked as FP. The FP of the RMS value and the kurtosis factor are 5.442 m/s² and 0.5837 respectively; meanwhile, many obvious spikes appear in these curves. The relationships every 2
minutes when the rear axle gear experiences a small crack fault and progresses to breakdown are shown in b) and c) of Fig. 1, respectively. Totally 246 time series are used here and the FP of the RMS value and the kurtosis factor are 10.818 m/s² and 4.0975 respectively. At the same time obvious spikes appear in the two curves.

Fig. 1. Parameters before preprocessing: a) Parameters (normal operation), b) RMS value (fault), c) Kurtosis factor (fault)

Fig. 1 shows that a number of fluctuations and spikes of the RMS and the kurtosis factor, which influence the predictive precision seriously, occur in the curves when the rear axle gear is in normal operation and when it has small crack faults. In order to enhance the accuracy of prediction, a recursive preprocessing method is investigated in the present study. The recurrence tracing formula is as follows:

\[ \mu_{sn} = \frac{n-1}{n} \mu_{s(n-1)} + \frac{1}{n} x_n \]  

where \( \mu_{sn} \) is the recurrence value, \( x_n \) is the present value, and \( n \) is the number of times.

After recurrence tracing preprocessing of the parameter values in b) and c) of Fig. 1, the preprocessing results can be depicted respectively in a) and b) of Fig. 2.

Fig. 2. Parameters after preprocessing: a) RMS value, b) Kurtosis factor

Comparing the curves of the parameter sequences in diagrams b) and c) of Fig. 1, and a) and b) of Fig. 2 respectively, the results show that after preprocessing the two parameters of RMS value and kurtosis factor remain consistent in tendency with the original parameter values, meanwhile the kurtosis factor holds fast tracking response whereas the RMS value is not so fast as the kurtosis factor.

The fluctuation comparisons of the RMS value and the kurtosis factor are shown in Table 1.
3. Topological structure of RBF neural networks

RBF neural network is a kind of tectonic feed forward network based on function approximation theory, having a simple architecture of three layers known as input, hidden and output layers as shown in Fig. 3 [19]. The inputs of the network are fed to the hidden layer by a nonlinear mapping from the input space to a new space. The hidden layer utilizes activation functions like Gaussian functions known as radial basis functions, whereas the output layers are linearly combined with a set of weights.

![Fig. 3. Structure of typical RBF model](image)

In Fig. 3, $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ is the input vector of the network, $W \in \mathbb{R}^{m \times n}$ is the output weight matrix, $b_0, b_1, \ldots, b_m$ are the output unit biases, $y = [y_1, \ldots, y_n]^T$ is the output of the network, $\phi(*)$ is the $i$-th activation function of the hidden node, $c_i$ is the centroid value of data values, and $||*||$ denotes Euclidean norm.

The basis functions of the hidden nodes create local responses to the input signals. The commonly used radial basis function is the Gaussian function:

$$R(x) = \exp\left[-\frac{||x-c_i||^2}{2\sigma_i^2}\right], i = 1,2,\ldots,m$$  \hspace{1cm} (4)

The output of the network is:

$$y_m = \sum_{i=1}^{m} w_i \phi(||x-c_i||)$$  \hspace{1cm} (5)

It has been theoretically proven [12] that radial-basis-function networks having one hidden layer are capable of making universal approximations (under certain norm cases) to any arbitrary given function in a wide range if the number of hidden layer nodes, the centroid value, and the weight between hidden and output layer are appropriately regulated, which is the theoretical basis for its nonlinear mapping ability and its wide applications.

4. Algorithm of damage prediction of the rear axle gear using vibration signal preprocessing coupled with RBF neural network

The basic structure of the rear axle gear damage prediction process is shown in Fig. 4. This approach involves two parts: vibration signal preprocessing and RBF neural network prediction. The preprocessing part includes two sections: the first is the feature extraction analysis model which is used to extract a feature set $F_m$, the second is the recursive preprocessing method which is applied to decrease the fluctuation of $F_m$ and obtain the more stable feature set $P_m$. In addition, the RBF neural network prediction module is developed to predict the life of a rear axle gear.

The feature extraction module can process the signals $x$ measured from the rear axle gear and extract characteristic parameters $F_m$ reflecting the condition of the gear. Subsequently $F_m$ is used as the input of the parameter preprocessing module, thereby obtaining a more stable sequence $P_m$ which is easy to gain high accuracy for prediction. Once the feature set $P_m$ is obtained and could be developed to make a time series $x_1, x_2, \ldots, x_n$ for gear life prediction. Taking $x, x_1, x_2, \ldots, x_n$ sequence values to predict $x_{m+1}$ value at time $m+1$.

The main steps are as follows:
1) Firstly, divide $x, x_1, x_2, \ldots, x_n$ into $k$ training modes in terms of the requirement of the network input, the first $m - 1$ points are used as the input of network, only the last one as the designated value.
2) Train the network based on the training modes, so as to obtain the connection weights.
3) Obtain the predictive value $x_{m+1}$ when the $x_m^1, x_m^2, \ldots, x_m^k$ sequence values are used as the input of the neural network by using the connection weights after training the net. Adopting a recursive prediction algorithm when multi-step prediction is explored, namely using $x_{m+1}, x_{m+2}, \ldots, x_{m+k-1}$, to predict $x_{m+k}$, and so on.

In order to evaluate the effectiveness of prediction, the square root of the mean squared error of the $k$-th step can be calculated as the performance error ($k = 1,2,\ldots,10$), that is:

$$E_k = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - x_{i,k})^2}$$ \hspace{1cm} (6)

where $x_{i,k}$ is the predictive value of the $k$-th step. This parameter can be used to assess the performance of the network.

5. Simulation and experimental results

5.1. Simulation results

In order to verify the applicability of the new method, a signal of $y = 0.5x + \sin(15x)$ was sampled with the sampling frequency of 10Hz and several time sequence values were ob-

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**Tab.1. Fluctuation comparison of vibration parameters**

<table>
<thead>
<tr>
<th>Vibration parameter</th>
<th>FP before preprocessing</th>
<th>FP after preprocessing</th>
<th>Amount of FP decreased</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS value</td>
<td>10.818</td>
<td>1.309</td>
<td>8.26</td>
</tr>
<tr>
<td>Kurtosis factor</td>
<td>4.0975</td>
<td>0.1662</td>
<td>24.65</td>
</tr>
</tbody>
</table>
maintained to test the method. The simulation results are shown in Fig. 5.

Here, in Fig. 5, diagram a) is the original time sequence of the simulation values with the FP of 3.43, showing an increasing trend and obvious spikes. Diagram b) is the time series after recursive preprocessing with the FP of 1.7979, having the same trend as a). However, the time series becomes more stable and the spikes decrease significantly. Establishing the net by using the time series in b) based on the proposed method, the error curve of the net training process is shown in c). After

Fig. 4. Basic structure of damage prediction system

Fig. 5. Results of analyzing the simulation signals: a) Original waveform, b) Waveform after recursive preprocessing, c) Error curve of the training process, d) Prediction result
10 epochs training the goal of the net can be fulfilled, the net training meets the requirements and then the establishing of the net is finished. At the same time, the performance error $E_k$ is 0.0022 by using the already established net model for prediction and the prediction result is shown in diagram d). The zoomed-in part shows that the predictive values are very close to the simulation values, which tells us that the proposed method can obtain high predictive precision and fast convergence for fluctuating signals.

5.2. Experimental results

The proposed method can be applied to the signals collected from the rear axle gear fatigue test system in order to verify the validity of rear axle gear damage prediction. The rear axle gear measuring system, including a dynamometer, the transmission, a speed increaser, signal conditioning instrument, vibration acquisition unit and so on, is shown in Fig. 6.

![Fig. 6. The rear axle experimental system: 1) 570 kW dynamometer, 2) transmission, 3) torque and speed sensor, 4) rear axle, 5) large speed increaser, 6) speed increaser, 7) 150 kW dynamometer machine, 8) signal conditioning instrument, 9) DeWe-tron vibration acquisition unit, 10) indicator, 11) acceleration transducer 1, 12) acceleration transducer 2, 13) acceleration transducer 3]

The geometric parameters of the gear experimental fatigue test system shown in Fig. 6 are as follows: the number of pinion teeth $z_1 = 6$, the number of gear teeth $z_2 = 38$, transmission ratio is 6.3, rotational speed is 200 revolutions per minute (rpm), and the maximum load is 60 kW. The fluctuation range of the input rotational speed is 195.2~198.8 rpm and the load fluctuation is 55.145~61.258 kW in actual working condition.

The signals from point 11 (acceleration transducer 1 in Fig. 6) at the nearest position of the bearing house, where the vibrations are most obvious, were collected to be analyzed. After establishing the net models by using RBF and BP neural networks respectively based on the kurtosis sequence after preprocessing shown in Fig. 2 b), the error curves of the training process of the net model are shown in Fig. 7.

![Fig. 7. Error curves of the training process: a) RBF neural network, b) BP neural network]

Fig. 7 shows that when setting the same net error goal, the RBF neural network arrives at the error goal and meets the requirement after 197 training epochs; whereas the BP neural network could not reach the error goal even after 5000 training epochs. Meanwhile according to the training curve of the BP neural network, it is obvious that the convergence rate of the RBF neural network is better than that of the BP neural network.

Making predictions for rear axle gear damage life by using the already established net model, Table 2 shows the prediction performance error $E_k$ by using the RBF neural network, the BP neural network, and the ARMA model, respectively.

<table>
<thead>
<tr>
<th>Prediction model</th>
<th>$k=1$</th>
<th>$k=2$</th>
<th>$k=5$</th>
<th>$k=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBF neural network</td>
<td>0.0006</td>
<td>0.0012</td>
<td>0.0058</td>
<td>0.0074</td>
</tr>
<tr>
<td>BP neural network</td>
<td>0.0009</td>
<td>0.0025</td>
<td>0.0082</td>
<td>0.012</td>
</tr>
<tr>
<td>ARMA model</td>
<td>0.0143</td>
<td>0.0259</td>
<td>0.0303</td>
<td>0.0529</td>
</tr>
</tbody>
</table>

The results in Table 2 show that, the performance error $E_k$ of the RBF neural network model is smaller than both the BP neural network and the ARMA model under the same prediction steps. It could be summarized that the RBF neural network is superior to the BP neural network and the ARMA model for the prediction of rear axle gear damage. Simultaneously Table 2 shows that the performance error $E_k$ corresponding to the same prediction model increases gradually with the increasing
of predicting steps. We can obtain higher precision for multi-step prediction of rear axle gear damage by using the proposed method.

In order to verify the applicability for on-line prediction application of rear axle gear based on the proposed method, we emphasize the multi-step prediction of rear axle gear damage. Fig. 8 is a comparison of the RMS value and the kurtosis factor between predicting values and measured values of 10 prediction steps based on the RBF neural network, the BP neural network and the ARMA model respectively.

Fig. 8 shows that the prediction results of the RBF neural network and the BP neural network have a trend similar to the measured values. Simultaneously according to Table 2, the RBF neural network has the highest predicting accuracy while the ARMA model has the worst. All in all, it is feasible to make multi-step predictions for the rear axle gear damage and it can be used to achieve on-line monitoring of the rear axle gear. The kurtosis factor sequence exhibits a drastically increasing trend in diagram b) of Fig. 8, showing the fault of rear axle gear is aggravating.

The current condition of the gear is as shown in Fig. 9.

Fig. 9. Fracture photo of rear axle gear

Fig. 9 shows that obvious fracture faults have happened, which indicates that the proposed method could be used to predict damage of the rear axle gear.

6. Conclusions

Aiming at the transmission system of the rear axle gear with non-uniform velocity and variable load, this paper proposes a new damage prediction method of the rear axle gear based on the RBF neural network coupled with recursive preprocessing in order to improve the accuracy of gear residual life prediction. The method could be used to address the nonlinear problem of rear axle vibration signals. Simulated and experimental results have shown that the proposed method could be used not only for on-line monitoring of rear axle gear deterioration, but also for fault prediction based on normal state values with high accuracy, providing a reliable tool for condition monitoring and fault diagnosis of rotating machines.

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7. References


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