The paper presents an innovative and improved method for doubled calibration of an eddy-current conductometer. Implementation of the proposed method makes it possible to achieve independence of the instrument indications on surface condition of the examined parts. The calibrating function is determined on-line when the measurements are in progress on the basis of parameters that are assigned to the contact probe coil and calculated with use of general mathematic equations. The calculated correction parameter can be considered as a measure of the surface roughness. Application of the modification as described in this paper can contribute to efficiency improvement of operational examinations for components made of non-ferrous materials during their lifetime.

**Keywords:** eddy currents, conductometry, non-destructive testing, surface roughness.

Zaproponowano oryginalną, ulepszoną metodę podwójnego skalowania konduktometru wi bırakowego. Realizując tę metodę można uzyskać niezależność wskazań przyrządu od stanu powierzchni badanych elementów. Funkcja skalująca konduktometr jest wyznaczana dynamicznie w czasie trwania pomiaru na podstawie parametrów przypisanych eksperymentalnie sondzie-cewce stykowej oraz uniwersalnych równań matematycznych. Obliczona poprawka może być traktowana jako miara chropowatości powierzchni. Stosując opisaną modyfikację można znacznie usprawnić badania eksploatacyjne elementów wykonanych z metali nieżelaznych.

**Słowa kluczowe:** prądy wirowe, konduktometria, badania nieniszczące, chropowatość powierzchni.

1. Introduction

An eddy-current conductometer is an instrument that can be used for non-destructive measurements of conductance (specific electrical conductivity) of materials that the examined part is made of. The measuring probe, made as a contact coil and supplied with alternate current, is approached to the surface under test. As the coil conducts electric current, it generates primary magnetic field, which, in turn, entails induction of eddy currents in a conductive material beneath the coil. According to the Lentz rule the secondary magnetic field produced by such eddy currents is a negative feedback, i.e. the direction of its induction vector is opposite to the one of the exciting magnetic field. Consequently, a resulting magnetic field is originated within the area of the coil probe that amends the coil impedance. The field component produced by eddy currents depends on the conductance parameter of the material the examined part is made of. Therefore the differential impedance of the probe shall also depend on the measured conductance. Eddy-current conductometers measure impedance components of the coil probe and use the measurement results to calculate the actual conductance. Conductometers, being instruments designed to measure absolute values of conductance, not only are meant to measure specific electrical conductivity of non-ferrous materials or electrolytes, but also can be used to determine purity of metals, monitor structures and homogeneity of non-ferrous alloys and for indirect tests of strength and hardness, estimation of phosphor content in copper, control of casting operations (polarization of copper), monitoring of separation processes, e.g. Cu-Cr, sorting and separation of metal scrap, detection of fatigue and thermal defects in aluminium alloys (operation tests of aircrafts) and many other applications.

Conductometers are calibrated against dedicated patterns, where the determinable parameter, i.e. conductance is assigned to physical variables that can be measured in a direct manner (coil resistance and inductance. But the measurement results are accurate and reliable only when structures and shapes of investigated workpieces are similar to reciprocal parameters of patterns that were already used for calibration. Unfortunately, examinations with use of eddy currents is associated with two detrimental effects that make the measurements more troublesome and difficult. Application limits for the eddy current method are determined by consequences of the following phenomena:

1. Surfaces of examined workpieces are not always flat, with frequent buckles and cavities. It is possible to place a spacing ring below the measuring coil to eliminate swiveling of the probe on the surface, but anyway it is impossible to approach the probe coil to the examined surface so that the distance between the probe and the surface would be always constant during the calibration process. Variations of the distance between the coil and the examined surface considerably affect instability of the coil impedance, even more than variation of the conductance to be determined.
The problem is usually resolved by introduction of the compensation mechanism that counterbalances variations of the distance between the coil and the surface. The conventional mechanism was initially proposed by F. Förster and is described in details in the paper of Dziczkowski [5]. The compensation method is based on a bridge circuit, where one branch of the bridge incorporates the measuring coil connected in series with a capacitor. The second branch represents also a serial resonance circuit that is made up of a reference coil and an adjustable capacitor. The measurement consists in unidimensional balancing of the bridge by appropriate variations of the capacitor value. Characteristic parameters of the resonance circuit vary in pace with alterations of coil resistance and inductance, whilst these parameters, in turn, depend simultaneously on conductance of the material under test and distance between the probe and the examined surface. By crafty tuning of the resonance circuits it is possible to achieve growth of indications for the measured conductance when the coil is moved away to a small distance. Farther retreating of coil lead to a rapid drop of indications coupled with the measurement result. Finally a range of certain distances between the coil probe and the surface is obtained where the result of measurements fits into limits of the assumed deviation. Consequently, measurement accuracy is sacrificed in favour of the possibility to take measurements for uneven workpieces. The authors strongly believe that it was the most important achievement in techniques of measurements carried out with use of eddy currents in the area of non-destructive tests. The amount of data processing is not a problem as modern measuring instruments are provided with microcontrollers with massive computation performance and capable to carry out calculations in the real time mode when the measurements are still in progress. Therefore it is necessary to seek for such computation algorithms, where variations in the distance between the measuring coil and the examined surface can be compensated without noticeable increase of errors in determination of conductance.

2. The magnetic field induced by eddy currents is directed opposite to the exciting field. Therefore a specific barrier is formed that prevents from penetration of the primary field deeply inwards the examined workpieces. Consequently, eddy currents are induced exclusively within the surface-adjacent layer of examined workpieces and properties of only that layer affect results of measurements. Penetration depth of eddy currents substantially depends on their frequency. Research studies are described, for instance, in the collective study that deals with modeling and detection of surface flaws [8]. This study, similarly to other literature references, uses the simplified definition for penetration depth of eddy currents that has been sourced from the eddy current heating technology. Somewhat more detailed description of that phenomenon can be found in the paper of Dziczkowski [2]. Since eddy currents flow only on the surface of examined workpiece, any surface roughness makes flow of these currents more difficult. If so, measurements of conductance with use of an eddy current instrument make it possible to identify surface roughness with apparent growth of conductance and apparent increase of distance between the coil and the workpiece surface. The measuring instrument was initially calibrated against polished specimen, thus one should expect considerable deviations of measurement results. The widely accepted approach, described in technical guidelines and literature references, assumes application of low frequencies as in such a case eddy currents are capable to penetrate much deeper and possible roughness only slightly disturb measurements. More detailed analysis leads also to the conclusion that frequency of eddy currents affects not only penetration depth but also sensitivity of the measuring instrument [4, 6].

For a specific test it is frequently more convenient to adjust the most suitable frequency that is best for the desired sensitivity. But such processes as selection of the best frequency, elimination of the impact from surface roughness and compensation of deviations are always inseparably interconnected.

Impedance of a measuring coil is always a complex parameter, therefore measuring of its two components enables independent determination of two parameters for the workpiece under test or makes it possible to find out one parameter and compensate effect of another one. The manner, how results of two measurements are used to determine a singe, discretely selected parameter depend on type of the measuring instrument. The instruments that can be possible used for measurements, i.e. flaw detectors and conductometers substantially differ from one another. The measurement process carried out with use of a conductometer is carried out as a sequence of operations aimed at determination of the absolute conductance value for the material the workpiece is made of. The final result should be independent on other factors that unfortunately also influence variations of components for the coil impedance. These factors include position the measuring coil against a workpiece under tests and condition of the workpiece surface. For investigations with use of flaw detectors the information about absolute value of the conductance for the material the workpieces is made of is irrelevant as the tests are focused on detection of any possible discontinuities within the structure under test [9] as for flaw detection it is necessary to find out all possible cracks or excessive surface roughness. Therefore it is the instrument that should be used to measure apparent variations of conductance as opposed to a conductometer that, by its nature, is insensitive to such apparent variations.

2. Properties of the desired mathematical model

The calibration process with use of standard patterns results in establishing the interrelationship between the parameters that can be measured in a direct way and the value to be detected. Each pattern that has been used for calibration is meant to find out a single point for the calibration scale, but between the calibration points the function relationship must be approximated. The more accurate the approximation function is, the less number of calibration patterns is necessary to find out the calibration scale that guarantees acceptable deviations within the assumed measurement range.

When an eddy current device is used to perform measurements for various frequencies of the exciting field or even this frequency is automatically adjusted in course of the test with mandatory compensation for variations in the distance between the probe (coil) and the examined surface, the calibration process is equivalent to setting up a function of four variables. Such a calibrating functions has the following arguments: frequency of eddy currents, distance between the probe and the
examined surface, measured differential resistance of the coil (real component of impedance) and the measured differential inductance of the coil (the parameter that depends on the imaginary component of impedance). The determined conductance value was unambiguously assigned to each set of four foregoing arguments.

The calibration process with use of standard patterns should be carried out for a set that is made up of a measuring instrument and a coil probe. After a probe is replaced with another one the calibration process must be repeated.

To set up the calibration function one must have a general but sufficiently accurate mathematical model that must enable to take account for all the foregoing provisions.

The available literature offers a number of mathematical models that can be suitable to carry out the foregoing operations. These models have been obtained after resolving the Maxwell equations. They differ between each other in terms of the resolving method and other approximations applied.

The most popular one is the Finite-Element Method (FEM). Quite a lot of computations is carried out with use of the Boundary Element Method (BEM) and the Finite Difference Time Domain (FDTD) methods. The foregoing methods are suitable for flaw detection technology to establish models of flaws and can be used to find out how a structure of any shape affects the parameters that can be measured directly.

Under assumption that the structure to be investigated is homogenous, sufficiently large and its shape can be naturally described within a cylindrical coordinate system, the available analytical methods seem to be more convenient. Complete systems of equations capable to describe the these phenomena were published by Dodd C. D., Deeds W. E and Luquire J. W. in early 70’s [1]. For the Dodd and Deed’s model the functions that are the result for the differential equations provided adopt the form that is suitable for practical applications. The left-hand side of figure 1 presents the model air-filled coil [12] with n turns encapsulated within a ring with the following dimensions: \( r_1 \) – internal radius of the winding, \( r_2 \) – external radius of the winding, \( z_1 \) – distance between the outermost part of the coil and the surface of the examined workpiece, \( z_2 \) – distance between the inner part of the coil and the surface of the examined workpiece.

It is assumed that a thick cylinder with the radius of \( b \) and made of a conductive material with the conductance of \( \sigma \) is positioned below the coil provided that the cylinder material is not a ferromagnetic stuff. The impedance variation of an air-filled coil with already known dimensions and approached to a conductive workpiece is expresses by means of the formula [12]:

\[
\Delta Z_f = \frac{j \omega 2 \pi \mu_0 \rho^2}{(r_2 - r_1)^3 (z_2 - z_1)^2} \sum_{i=1}^{n} \int_{r_1}^{r_2} \int_{z_1}^{z_2} \left( \exp(-q_i r_{q1}) \cdot \exp(-q_i r_{q2}) \right)^2 \frac{q_i}{q_i + p_i} \, dq_i \, dx
\]

where:

\[
\int_{r_1}^{r_2} \int_{z_1}^{z_2} \left( \exp(-q_i r_{q1}) \cdot \exp(-q_i r_{q2}) \right)^2 \frac{q_i}{q_i + p_i} \, dq_i \, dx = \int_{x_i}^{x_1} x f(x) \, dx
\]

The \( x \) stand for roots of the equation:

\[
J_n(x) = 0
\]

and \( q_i \) coefficients are expressed as:

\[
q_i = \frac{x_i}{b}
\]

It turn, \( p_i \) can be calculated from the relationship:

\[
p_i = \sqrt{q_i^2 + j \omega \mu_0 \sigma}
\]

The formula (1) makes it possible to calculate variations (differentials) of the resistance and inductance parameters for an air-filled coil with consideration to its geometrical parameters:

\[
r = \text{Re}(\Delta Z_f)
\]

\[
l = \frac{\text{Im}(\Delta Z_f)}{\sigma}
\]

A substantial inconvenience of the foregoing model that prevents from application of the same for design of calibration equipment is the infeasibility to take account for all dimensions of the coil. When additional factors are introduced into computation the foregoing equations become much more sophi-
sticitated, the computation time is inadmissibly prolonged and calculation of the calibration factors in the real time mode turns out to be infeasible hence the test is still in progress. Regardless to the fact that two coils with the same dimensions and the same number of turns were prepared, the spooling process for them during manufacturing is not controlled, therefore the probes designed with use of these coils demonstrate slightly different sensitivities. Consequently, a separate calibration procedure must be carried out for each of the coils.

The calibration method that is proposed by authors of this paper uses a simplified model that considers a coil as a 2D structure. It is assumed that all \( n \) turns are encapsulated by a circle with the radius of \( r_0 \) and positioned in parallel to the workpiece surface within the distance of \( h \). It is also assumed that the conductive workpiece is a half-space. The right-hand side of Fig. 1 presents a schematic diagram of a model 2D coil. A similar solution was applied in [11] with further modification in [3]. The primary reason for such simplification was the wish to develop a mathematical model that would be sufficiently fast for computations and would enable to find out the optimum eddy current frequency with the criterion of minimum errors (deviations). It is why application of the following generalized parameters proves to be convenient:

\[
\alpha = \frac{2h}{r_0} \tag{8}
\]

\[
\beta = r_0 \sqrt{\mu_0 \sigma} \tag{9}
\]

If so, variations (differential) of coil impedance is described by means of the equation below:

\[
\Delta Z = \omega \sigma r_0 \mu_0 \mu_n n^2 \cdot j \beta \left\{ \frac{\lambda - \sqrt{\lambda^2 + j}}{\lambda + \sqrt{\lambda^2 + j}} \right\} e^{-\alpha \beta \lambda} J_1^2 (\beta \lambda) d\lambda \tag{10}
\]

Upon extraction of the real and imaginary parts it was possible to find out variation (differential) of the coil resistance:

\[
r = R - R_0 = n^2 \omega \sigma r_0 \mu_0 \mu_n \phi (\alpha, \beta) \tag{11}
\]

where:

\[
\phi (\alpha, \beta) = \text{Re} \left\{ j \beta \left\{ \frac{\lambda - \sqrt{\lambda^2 + j}}{\lambda + \sqrt{\lambda^2 + j}} \right\} e^{-\alpha \beta \lambda} J_1^2 (\beta \lambda) d\lambda \right\} \tag{12}
\]

and also the coil inductance:

\[
l = L_0 - L = n^2 \pi \mu_0 \mu_n \chi (\alpha, \beta) \tag{13}
\]

where:

\[
\chi (\alpha, \beta) = -\text{Im} \left\{ j \beta \left\{ \frac{\lambda - \sqrt{\lambda^2 + j}}{\lambda + \sqrt{\lambda^2 + j}} \right\} e^{-\alpha \beta \lambda} J_1^2 (\beta \lambda) d\lambda \right\} \tag{14}
\]

Expressions (12) and (14) serve as formulas for generalized description of variations (differentials) exercised by the coil impedance due to presence of a conductive half-space.

4. Determination of the calibration function for conductometers

Implementation of the proposed methods needs to use standard patterns with already known conductance values, with polished surfaces and thickness values much exceeding the expected penetration depth of eddy currents. The contact coil is approached to a standard pattern and variations of the coil resistance \((r)\) and inductance \((l)\) are measured at a specific frequency of eddy currents. The system of equations, obtained after substitution of (8) and (9) to (11) and (13), enables to calculate values of \( r \) and \( l \) when the measured variations of \( r \) and \( l \) are already known. The calculated values of \( r \) and \( l \) are equivalent parameters of the coil, thus they are stored for further use – each time when the conductance parameter is to be found out. During each working cycle variations (differentials) of the coil resistance and inductance are measured and then, with use of an embedded controller, necessary numerical computations are performed. The computation consist in resolving of the equation system (11) and (13) with determination of the \( \alpha \) and \( \beta \) values. Then the equation (9) together with the modified equation (8):

\[
\alpha = \frac{2(h_0 + h)}{r_0} \tag{15}
\]

are used to calculate the parameters of \( \sigma \) and \( h \). Calculation of two parameters at a time is a natural solution used to compensates effect of the distance between a probe and a workpiece surface onto the result for conductance measurement. When surface of the examined workpiece is smooth and flat, the \( h \) value should be zeroed, as the probe was approached to the examined surface without a spacing ring. When the measured distance increases and amounts to \( h = h_0 \), it serves as the information about the surface roughness. The experience acquired during operational examinations suggests that two types of surface unevenness should be taken into account. The first type represents convex or concave surfaces that form the workpiece shape and can be observed down the distances comparable with horizontal dimensions of the coil. Such unevenness shall not be measured as apparent growth of the coil conductance. Application of the proposed method of measurements and computations makes it possible to compensate effects of such convex and concave unevenness due to compensation of the distance between the probe and the surface of the examined workpiece. In such a case the result of conductance measurements is burdened with merely an additional error that is caused by drop of the instrument sensitivity [6].

The second type of surface unevenness is rather associated with surface flaws than large-sized buckles. Such flaws include cracks, scratches, delamination, remnants after machining processes. In the theory of machinery design such faults are referred to as surface roughness [10]. Occurrence of surface roughness is pronounced by apparent growth of the coil conductance, whilst the proposed method for measurements, calculations and calibration is not sufficiently effective for that application.

The \( r_0 \) and \( h_0 \) parameters can be considered as establishing a correlation between each real coil and a certain dimensionless model coil. If so, the simplified mathematical model applied to that dimensionless coil can be used to determine the calibration factor for the specific instrument. If the measurements conditions only slightly differ from the actual circumstances for calibration with standard patterns, the error (deviations) for determination of conductance shall be insignificant and acceptable. However, the ensuing problem must be resolved, i.e. how to determine the number of calibration points and, consequently, the number of standard patterns that is necessary for calibration. One has to keep on mind that the overall objective is to assure
the maximum permissible calibration error is never exceeded over the entire measurement range.

5. Compensation of the roughness effect onto results of conductance measurements

The measurements with use of the proposed calibration method lead to determination of two parameters: the conductance \((\sigma)\) and distance \((h=hp)\). The \(hp\) value serves as the measure for apparent growth of the distance between the probe and surface of the examined workpiece. Such an apparent growth provides information that the roughness really occurs.

After completion of the first calibration cycle with use of polished standard samples the instruments should be recalibrated, but with use of standard samples with defined roughness. It was assumed that the average roughness profile of the examined workpieces is similar to the roughness profiles demonstrated by available standard patterns. Now it is enough to measure the apparent distance between the probe (coil) and the sample surface and repeat the measurements for several standard patterns with already known conductance values. One has to keep in mind that apparent variation (differential) of the mentioned distance is caused by the surface roughness. Upon taking the measurements it is necessary to store the associated pairs of figures: the apparent distance \(h\) and the apparent variation of conductance expresses by the differential of the \(\beta\) parameter. Such a differential can be expressed by means of the following formula:

\[
\Delta \beta_{zo} = \beta_h - \beta_z = h_0 \frac{2\pi f}{\mu_0} \left(\sqrt{\sigma_z} - \sqrt{\sigma_z}\right) \quad (16)
\]

where: \(\sigma\) - actual conductance of the applied standard pattern with defined roughness or actual conductance determined from measurements, \(\sigma\) - measured conductance, i.e. apparently reduced due to roughness of the workpiece surface, \(\beta\) - generalized parameters specified by the equation (9) for \(\sigma = \sigma_z\), \(\beta\) - generalized parameters specified by the equation (9) for \(\sigma = \sigma_z\).

Next, the following function can be developed by interpolation and stored as a calibration curve:

\[
\Delta \beta_{zo} = f(h_p) \quad (17)
\]

The measurements comprise determination of the distance value \(h\) and the value for the generalized parameter \(\beta=\beta_z\) that is burdened by errors due to the surface roughness. The \(\beta\) value correspond to the incorrectly measured value of conductance \(\beta\). Therefore the equation (17) is used to find out the correction factor \(\Delta \beta_{zo}\) for the generalized parameter. Finally, the desired value of \(\sigma\) can be calculated by means of the equation (16).

6. Roughness measure

Application of the double calibration process makes it possible to find out the correction factor for measurements of conductance. Effectiveness of the proposed method has proved to be really good when roughness profile of standard patterns is identical with reciprocal profiles of examined workpieces. It was, for instance, the case, where the roughness standard patterns were sampled from the same manufacturing line where conductance measurements were carried out. The examination enabled to find out that the apparent variation of the distance \(h\) between the probe and the examined workpiece, initially me-

assured for compensation purposes, can serve as an eddy current measure of roughness. Although that measure is not equivalent with the parameter commonly used for technological evaluation, but a correlation between these two parameters can be found [7]. The examples of results for measurements with use of Ra and Rz methods as a comparison against the eddy current technique are summarized in table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td>Ra [(\mu m])</td>
</tr>
<tr>
<td>347</td>
</tr>
<tr>
<td>156</td>
</tr>
<tr>
<td>96</td>
</tr>
<tr>
<td>35</td>
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7. Verification of the calibration process

Implementation of the foregoing calibration method that consists in calculation of equivalent parameters \(r_0\) and \(h_0\) makes it possible to find out the calibration function within the vicinity of the point defined by parameters of the standard patterns and the applied frequency. The experiments revealed that bias of the calibration point, i.e. alteration of frequency or conductance, failed to entail any changes determined by means of the method involving \(r_0\) and \(h_0\) parameters. The recorded alterations of these parameters were not higher than the estimated errors (deviations) that could have resulted from properties of the electronic measurement path. Therefore it is enough to calibrate the instrument only in one point, with use of a single standard pattern. To verify this hypothesis some additional computations were carried out.

Application of the aforementioned TREE method enabled to calculate variations (differentials) of resistance and inductance for specific, already assumed dimensions. Then the system of equations (11) and (13) was resolved with consideration to (8) and (9). Upon the variations of resistance and inductance were known, it was possible to calculate generalized parameters \(r_0\) and \(h_0\). It turned out that the value of the \(r_0\) parameter is independent on eddy current frequency, coil conductance and the distance between the coil and surface of the examined work-

---

*Fig. 2. Effect of the distance between a 3D coil and the surface of the examined workpiece onto the corresponding distance of the equivalent model coil*
piece. Also the value of \( h \) proved to be unbounded with frequency and conductance values. To check relationship between \( h \) and the distance between the coil and surface of the examined workpiece some further computations were performed. It was assumed that dimensions \( r_1 \) and \( r_2 \) are constant alike to the value of the differential \( z_2 - z_1 \). The value of \( z_1 \) was made variable and the equivalent parameter of \( h \) was computed. Obtained results are presented on figure 2. It was observed that the relationship between the values \( h \) and \( z_1 \) represents a linear function with deviations from linearity even smaller that the expected errors resulting from inaccuracy of numerical calculations.

8. Conclusions

When the calibration method proposed in this study is applied to calibration of conductometers it is possible to effectively determine conductance in the real time mode and to simultaneously compensate effects of variations in the probe (coil) distance from surface of the examined workpiece. Implementation of the double calibration method enables efficient compensation of the effect exercised by surface roughness of the examined workpieces provided that the roughness profile of standard patterns already used for calibration is similar to the corresponding roughness profile of examined workpieces.

The determined compensating factor intended to eliminate effect of surface unevenness is actually the measure of roughness. Owing to equivalent parameters of the coil determined during the calibration process it is possible to substitute each real coil with a model one, not only for the calibration procedure but also for further numerical calculations. It particular, it is possible to use a simple mathematical model to find out the optimum frequency of eddy current, the best suitable for any specific application.

9. References


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