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Andrzej TOMPOROWSKI
Marek OPIELAK

STRUCTURAL FEATURES VERSUS MULTI-HOLE GRINDING EFFICIENCY

KONSTRUKCYJNE KSZTAŁTOWANIE WYDAJNOŚCI ROZDRABNIANIA WIELOOTWOROWEGO*

The purpose of this paper has been to describe the influence of design features and properties of disintegrated grain biomaterials on the dynamics of the process efficiency. It has been determined that the basis for improved functionality of the grinding machine is the analysis of the influence of the effective component of gravitational force and angle of repose of the grain biomass on the dynamics of changes in the efficiency of the five-disc, multi-hole RWT-5KZ grinder. Reasonable efficiency may be obtained by means of purposeful control of cross-sections and volumes of grinding holes, i.e. design features of multi-hole multi-disc unit. The actions will however bring the planned benefits only when a mathematical description is developed for the flow of the disintegrated grains (biomass grains) through the working space of the multi-disc grinder as a resultant variable of the design and operation of the working unit. The search for design solutions of the units that grind grain, leading to efficient processing justify the research into the improvement of the theory and design of grinding machines.

***Keywords**: grinding, biomaterials, efficiency.*

W pracy podjęto próbę opisu wpływu cech konstrukcyjnych oraz właściwości rozdrabnianych biomateriałów ziarnistych na dynamikę wydajności procesu. Uznano, że podstawą do poprawy funkcjonalności działania maszyny rozdrabniającej jest przeprowadzenie rozpoznania wpływu efektywnej składowej siły grawitacji i kąta usypu biomasy ziarnistej, na dynamikę zmian wydajności pięciotarczowego, wielootworowego rozdrabniacza RWT-5KZ. Racjonalna wydajność może być osiągnięta, między innymi, na drodze celowego sterowania przekrojami i objętościami otworów uczestniczących w rozdrabnianiu, czyli cechami konstrukcyjnymi wielotarczowego zespołu wielootworowego. Aby jednak działania te przyniosły planowane korzyści, konieczne staje się opracowanie opisu matematycznego przepływu rozdrabnianych ziaren zbóż (ziaren biomasy) przez przestrzeń roboczą rozdrabniacza wielotarczowego, jako zmiennej wynikowej konstrukcji i działania zespołu roboczego. Poszukiwania rozwiązań konstrukcyjnych zespołów rozdrabniających ziarna zbóż, prowadzące do wydajnych procesów przetwórczych, uzasadniają podjęcie badań nad doskonaleniem teorii i konstrukcji rozdrabniaczy.

***Słowa kluczowe**: rozdrabnianie, biomateriały, wydajność.*

1. Introduction

Innovations in, designing and testing of drive systems of grinders constitute a big challenge for engineers. Experience gained over a few decades shows that the creation of an integrated, multi-purpose system is not simple [1, 3, 4, 5]. Grinding has become a common process popular in industry and everyday life.

Functional model of grain material grinders must take into account the impact of design features of working elements on the operation and the basis for the physical interactions in material processing, according to the principles of mechanics. The study focused on the relationship between design features of the multi-disc motion/process unit and performance characteristics of the machine [3, 8].

Model unevenness, as the exemplification of performance characteristics of multi-edge grinders for biomass grains depends on power, torque (useful work) and the angular velocity (also angle of rotation, acceleration of the cutting edges of the process unit in time) and movement (transport) of the feed material in working space of the grinding machine.

As regards multi-edge grinding of grain biomass, design features of the discs (Fig. 1) must ensure performance of two basic motion functions: movement and comminution in the inter-hole space. The results of previously conducted experiments confirmed the validity of the theoretical assumptions made and correctness of mathematical relations developed [9, 10].

According to the literature, research was carried out into e.g. the effect of a rotational mechanical medium on the behaviour of pieced materials inside that medium. In 1994 Walton made an attempt to determine the so-called dynamic angle of repose, taking into account the values of the friction factor during the cutting of granulate and, as a consequence, its deflection [12]. In 2007 Ulrich S., Schroter M. and Swinney H. investigated the influence of the friction factor on the behaviour of the mixture comprising spherical particles of materials [11]. As yet, the literature is exclusive of any researches into the influence of the gravitational force effective component and the angle of repose of biomass grains on the dynamics of changes in the efficiency of multi-disc grinding.

The primary objective of this study was to describe phenomena, process variables, relations of the multi-disc grinding, and especially to answer the question: do the effective gravitational force and angle of repose influence the dynamics and efficiency of five-disc, multi-hole 5KZ RWT grinder for grain biomass?

2. Geometric form of biomass grains

Biomass grains with stabilized moisture parameters and grain dimensions were used as the feed material in the analysis of volumetric efficiency. An assumption was

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

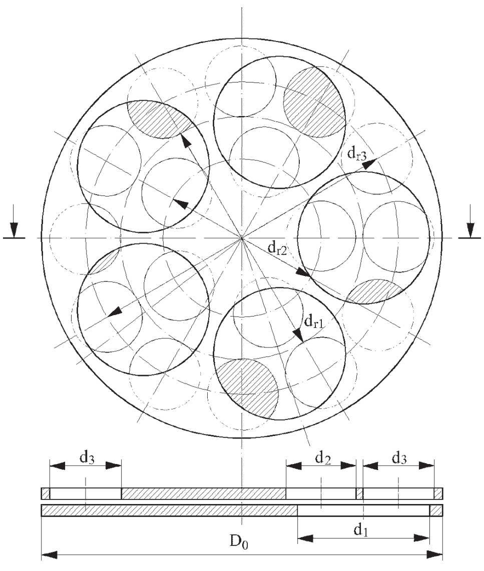

Fig. 1. *Design features, cross-sectional area and effective grinding surface for two grinding working discs between cutting edges [5]: D_0–outer disc diameter, d_{r1}– diameter of working holes mounting in the preceding disc, d_{r2}– mounting diameter of working holes inner row in the next disc, d_{r3}– mounting diameter of working holes outer row in the next disc, d_1- diameter of working hole in the preceding disc, d_2, d_3–diameters of working holes in the next disc.*

made, therefore, that for the purpose of this analysis an output model of the feed material is the material selected by dimensions with a repeatable constant size of an individual grain. Taking into consideration shapes of the basic grains (Fig.2), an assumption was made that biomass grains take the position in the holes of the working discs along their longer axis, perpendicularly to the inter-disc cutting plane.

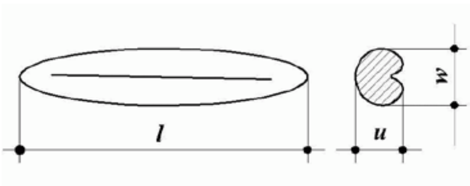

Fig. 2. Model of the geometric form of the basic grains: l, u, w – maximum dimensions of grain within three planes

Arrangement of the disintegrated biomass within the working space of the multi-hole grinder has been described with a probability distribution for grain length. Because in the holes of the same disc the material is described with the same grain-size state and is subject to the same cutting process in each hole, its state for the purpose of this analysis will be indexed with the cut number (*m*) and disc number (*n*):

$$\rho_n^m : [0, l_{\max}] \to [0,1], \quad \int_0^{l_{\max}} \rho_n^m dl = 1. \tag{1}$$

Since the desirable consequence of technological movement of the disintegrated material is its transfer to the next disc, the process results in the quasi-cutting at the disc edges. The initial state of the material in the first layer (disc) before the first quasi-cut is ρ_0^0 and constitutes an input state of the feed material in the working space of the analysed grinder. This state is hereinafter described by a function focused on value l_{max}. Efficiency-related state of the material during grinding changes as a result of two factors, mechanisms: grinding (quasi-cutting) and the removal of grains of the desired size from the inter-disc space of the grinding device operating unit.

3. Grinding efficiency model

When holes of two adjacent discs meet (Fig.1 and Fig.3) and their common cross-sectional area begins to increase (0, max), as a result of the movement of the feed material, the next grinding hole (in the next disc) is filled with some of the material from the preceding hole (in the preceding disc). In order for the cutting process to be efficient, the following conditions must be met:

- the hole of the preceding disc must be filled completely when it meets the next hole (when the common cross-sectional area of the discs appears),
- the hole of the next disc must be completely filled when the common cross-sectional area of the holes begins to decrease (the maximum common temporary cross-sectional area of the two grinding holes).

The two conditions must be met:

1. The volume of the feed material in the preceding hole must always be smaller than the space available in the next hole. Otherwise, undesirable idle movement of the feed material may occur, i.e. the material may move from the hole of the preceding disc to the hole of the next disc without being cut (quasi-cut) along the whole length of the device. Because some of the material as small particles is removed through the inter-disc space (inter-disc gap is a required design feature of this type of grinder), the volume of holes in the next disc should always be smaller than in the preceding disc. This is obtained through a reduced total cross-sectional area of holes or reduced thickness of the disc; there are however some limitations related to design, stress and load that the disc must transfer. Thickness *h* of disc *n*, for the purpose of the analysis has been identified with y_n, while $\widetilde{y}_n^{(k)}$ is used to identify height to which the material fills the hole in disc *n* before cut *k*.

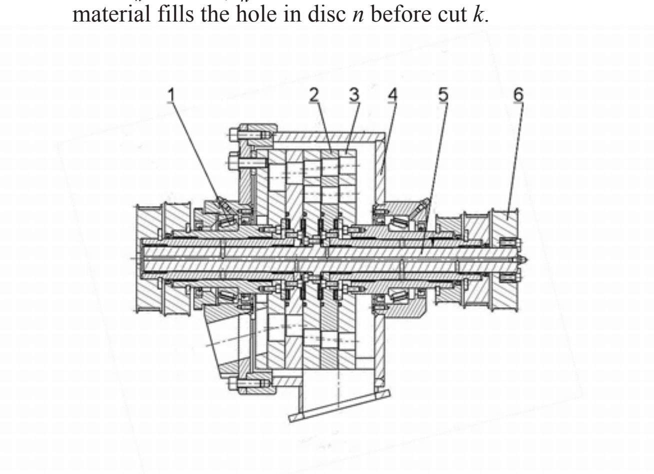

Fig. 3. The working unit of the five-disc, multi-hole RWT-5KZ grinder [4]: 1- bearing, 2- grinding disc, the so-called „preceding" disc, 3- grinding disc, the so-called "next" disc 4- body, 5- shaft, 6- pulley

2. Each point of the cross-sectional area of the holes in the next disc should at some time during the relative motion of the disc be located within the cross-sectional area of the holes in the preceding disc and when the cutting occurs, the so-called next hole of the given layer (of the next disc) is closed for the transfer to the next layer.

The analysis shows that cut *k* after disc ($n-1$) occurs earlier than cut *k* after *n* disc. With such numbering of cuts/quasi-cuts (the numbers begin with the first cut on each disc), the grain may, at each boundary between discs, be subject to a cut identified with the same number. Once a given hole is filled, the common cross-sectional area of the holes begins to decrease and cutting (quasi-cutting) occurs. An initial assumption was made that each single grain within the cross-sectional area of the grinding holes is subject to cutting. The orientation of grains in relation to the cutting plane is random with even distribution. Grains of any length will be disintegrated with the same probability into two smaller particles, with the total length equal to the length (size) before cutting. The division of the grain as a result of quasi-cutting always occurs in the material located in the preceding disc before filling. With efficient cutting, the distribution of length of grains that filled the empty space in disc (n+1) changes according to the following relation:

$$\tilde{\rho}_{n+1}^{m}(x) = A_{n,m}\rho_n^m = \left(1 - \frac{x}{y_{n+1} - \tilde{y}_{n+1}^m}\right)\rho_n^m(x) + \frac{1}{y_{n+1} - \tilde{y}_{n+1}^m}\int_x^{l_{\max}} \rho_n^m(l)\,dl, \tag{2}$$

and of those that remained in disc n:

$$\tilde{\rho}_n^{m+1}(x) = \tilde{B}_{n,m}\rho_n^m = \left(1 - \frac{x}{\tilde{y}_n^m}\right)\rho_n^m(x) + \frac{1}{\tilde{y}_n^m}\int_x^{l_{\max}} \rho_n(l)\,dl, \tag{3}$$

where: A, B – stochastic operators for m-th cutting, n-th disc.

For the purpose of simplification, an assumption has been made that after a cut, the distribution of disintegrated grains in hole spaces of disc (n+1) will be even (cut fraction and fraction already in the hole before the cut will mix) and therefore it will constitute the weighted average of ρ_{n+1}^k and ρ_n^k:

$$\rho_{n+1}^m(x) = \frac{\tilde{y}_{n+1}^m}{y_{n+1}}\rho_{n+1}^{m-1} + \frac{y_n - \tilde{y}_{n+1}^m}{y_{n+1}}A_{n,m}\rho_n^m(x). \tag{4}$$

4. Dynamics of efficient movement of grain material

Technological transport of the feed material within the working space of the analysed grinders is not even due to the nature and dynamics of the design and properties of the disintegrated biomass. This analysis begins when planes of two adjacent holes in adjacent working discs overlap and their common cross-sectional area caused by the difference in angular velocity begins to increase. The linear movement of the feed material is caused by the longitudinal force from the feeding screw and effective component of the gravitational force (inclined rotation axis of the disc package in the analysed grinder) (fig. 4). These forces cause ground biomass move between subsequent interconnecting holes in respective structural layers of the machine. Because the main axis of the working unit of the grinder (device) is not vertical, the direction of gravitational force is not perpendicular to the plane of the grinder rotational discs. Angular velocity of a disc results in an additional component of centrifugal acceleration. The sum of those two accelerations within the hole constitutes effective gravitational acceleration. Its direction depends on the phase of disc hole rotation.

When the common area of an overlapping pairs of holes reaches temporary maximum value and begins to approach zero the quasi-cutting process begins. For the purpose of further analysis, the following simplifications were made with respect to the design of working holes: cross-section of a hole is a convex, the volume of the convex is a cylinder (Cartesian product of cross-sectional area of the hole and thickness of the hole), the hole can connect only with one hole in the neighbouring discs at a time. The following structural and technical parameters were determined: thickness of n-th disc, y_n, minimum height (at the disc end) up to which biomaterial fills the hole in n-th disc following k-th cut, $\tilde{y}_n^{(k)}$ volume of grain biomaterial in the hole of n-th disc following m-th cut, V_n^m.

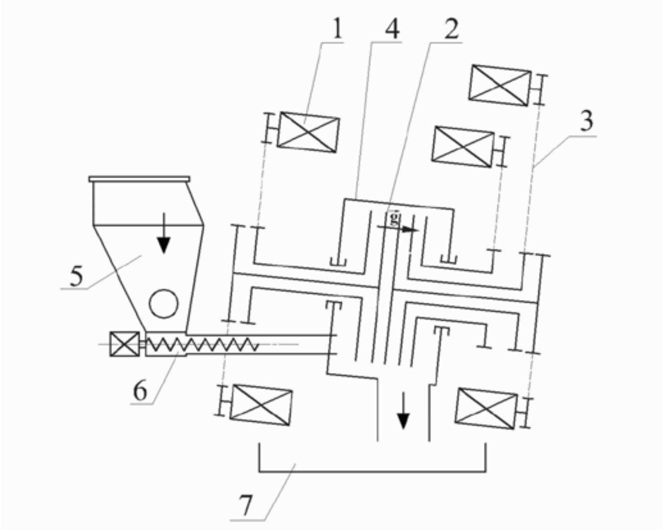

Fig. 4. Kinematical diagram of the multi-disc with inclined axis of working disc rotation for gravitational facilitation of feed movement: 1-electric motor, 2-grinding discs, 3-gears, 4-working space body, 5-container, 6-feeding screw, 7-finished product tank

At this stage, a zero angle of repose for ground medium was assumed (material behaves like a liquid) [7]. For this simplified assumption, the material surface in the working hole, at any time of its movement, is perpendicular to the direction of the effective gravitation vector within the hole area. In addition, an assumption was made that the variability of the effective gravitation within the hole area is omitted and the material surface is a flat surface on the side hole plane. When the common area of the upper and lower hole area is maximal, the common area starts to decrease. Generally, no cutting operation occurs already at this point as there is no need yet for the material to fill the entire common area. Therefore, such phase of the relative motion must be established where the entire common area of the holes cross-sections is filled with the material. The following simplifications are assumed:

1) Each hole is unambiguously assigned its cross-sectional point. This may be e.g. its geometrical centre. The relative motion phase for two holes can be described by providing an angular distance α between the centres of the holes. The ends of the ranges α are marked as α_p, α_k. The D, G, CW functions were defined which return, for a specified $\Delta\alpha$, the area of the following (lower) hole, preceding (upper) hole, and their common area. The symbol α_{max} is used to identify the value of the angular distance for which $|CW|$ function assumes a maximum value (if such maximum is achieved for more than one value α, the highest one of them is assumed as α_{max}). The average vector of the effective gravitation is marked with $\vec{g}_D, \vec{g}_G$ (in the lower and upper hole respectively). A simplified assumption was made that the centrifugal components $\vec{g}_D, \vec{g}_G$ are parallel to each other (they only differ in terms of value as a result of the difference in angular velocities of the two neighbouring discs). V^G, V^D, S^G, S^D stand respectively for the volumes of the preceding and next hole and the cross-sectional areas of the preceding and next hole.
2) For the vector $\vec{g} \in \mathbb{R}^3$ the plane $H_{\vec{g}}$ perpendicular to it was determined, tangent to the edge of the set $CW(\alpha)\times\{0\}$. There are two such planes – one positioned below the set $CW(\alpha)\times\{0\}$ and the second above it (existence of exactly two tangent planes results from the cross-sections common area convexity). The

upper plane was selected. It represents the surface of the material at the point when cutting is started.

3) The cylinder $G(\alpha) \times [0, y_n] \subset \mathbb{R}^3$ was divided with the plane $H_{\vec{g}G}$. The volume obtained over the plane was identified as (fig. 5):

$$V^G_{\vec{g}G}(\alpha) = \int_G min\{y_n, H_{\vec{g}G}(r)\} d^2r \quad (5)$$

This function attributes to the relative motion phase the volume which remains in the preceding hole at the time when the cutting starts.

4) Similarly, the cylinder $D(\Delta\alpha)\times[-y_{n+1},0]\subset\mathbb{R}^3$ was divided by the plane $H_{\vec{g}}$. The volume obtained under the plane was marked as (fig. 5):

$$V^D_{\vec{g}D}(\alpha)\int^D min\{y_{n+1}, H_{\vec{g}D}(r)+y_{n+1}\} d^2 \quad (6)$$

This function attributes to the relative motion phase the volume which remains in the next hole at the time when the cutting starts.

5) The expression $(V^G_{\vec{g}G}+V^D_{\vec{g}D})(\alpha)$ for the range $\alpha \in (\alpha_{\max}, \alpha_k)$ is a decreasing function. This function being reversed on its image, the phase of the motion where the cutting starts is obtained for selected volume of material in both grinding holes.

$$\alpha_c(V) = \left(V^G_{\vec{g}G}+V^D_{\vec{g}D}\right)^{-1}(V) \quad (7)$$

This function was extended to include the set $[0, V^G + V^D]$, putting α_k on the left and α_{max} on the right hand side compared with the original function domain.

6) The common area when the cutting for selected volume starts (fig. 5):

$$S_c = \left|\left(Cw\cdot\left(V^G_{\vec{g}}+V^D_{\vec{g}}\right)^{-1}\right)\right| : \left|O, V^G+V^D\right| \to \mathbb{R} \quad (8)$$

This is an increasing function – the highest the amount of the feed material in both working holes, the quicker the cutting is started, involving thus a larger common area of cross-sections.

The resulting function can be treated as the function with the four variables – material volume in both holes, direction of the effective gravitation radial component and the value of the effective gravitation radial components in both holes S_c: $[0, V^G + V^D] \times S^1 \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$. These functions must be determined numerically and treated as the encoding of the influence of the holes shape on the quasi-cutting process for various directions of effective gravitation vectors.

It was assumed that an angle of repose is determined for given material and it does not depend on the distribution of the grain length and that the material is moved only in the direction of the forced displacement (movement direction of the preceding hole in relation to the next hole). This means that the material is inclined down the plane by the angle of repose in a direction corresponding to the direction of the material forced displacement. Because the direction of the relative motion of the holes is always perpendicular to the direction of the centrifugal acceleration, it means that the effective gravitation vector is modified by turning it around the centrifugal component by the angle of repose.

In the cylindrical coordinate system with basis vectors $\{e_r, e_\varphi, e_z\}$, the effective gravitation vectors has the coordinates $(\omega^2 r, 0, g)$. After it is turned around the radial component by the angle γ, the effective gravitation vector will have the components $(\omega^2 r, \pm g\sin\gamma, g\cos\gamma)$. The mark ”–” corresponds to the situation where the preceding disc

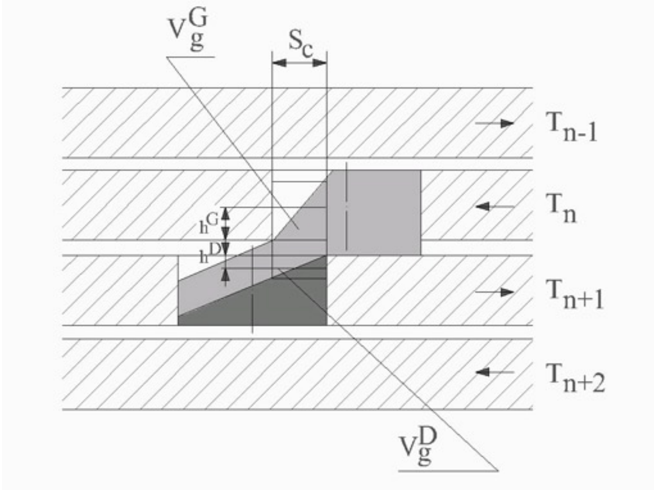

Fig. 5. Diagram presenting two adjacent working holes of the quasi-cutting unit filled with biomass grains. Tn-1 do Tn+2 - next grinding discs, hG – material column height before the cutting plane, hD – material column height behind the cutting plane, VgG- calculated volume of material before the cutting plane, VgD- calculation volume of material behind the cutting plane, SC – cross-sectional area of common pair of quasi-cutting holes

rotates in relation to the next disc with a positive angular velocity and the mark ”+” represent the reverse situation.

Cutting is always done in the material which was present in the preceding disc and within the area $S = S_C\left(V^m_n + V^m_{n+1}\right)$ before feeding. The height of the material column over the cutting plane is (fig. 5):

$$h^G = \int_{cw(\alpha_c)} min\{y_n, H_{\vec{g}G}(r)\} d^2r \quad (9)$$

and under the cutting plane:

$$h^D = \int_{cw(\alpha_c)} min\{y_{n+1}, -H_{\vec{g}D}(r)\} d^2r \quad (10)$$

The probability that a particle from within a length range of $(x, x + dx)$ will be present in the material below the cutting plane following cutting is:

$$\rho^m_n dx - \rho^m_n \frac{x}{h^D} + \int_{l_{\min}}^{l_{\max}} \frac{l}{h_D}\frac{dx}{l}\rho(l)dl \quad (11)$$

The first component corresponds to the probability before the cutting. The probability that a particle with x size reaches the cutting area must be deducted from this component. It is expressed by the occurrence probability product with regard to the occurrence of a particle with x size and the relation x/h^D (particle end must be within the segment $(0, x) \subset (0, h^D)$). The third factor are the probabilities, integrated from x to l_{max}, that there is a particle with a length of $l(\rho(l)dl)$ and that it will reach the cutting plane l/h^D, and that a particle from within a range of $(x, x + dx)$ (dx/l) will be obtained after cutting.

Based on the above, the distribution of the grain length in the material filling an empty space within the disc $(n + 1)$-th changes in the following way:

$$\tilde{\rho}^m_{n+1}(x) = A_{n,m}\rho^m_n = \left(1-\frac{x}{h^D}\right)\rho^m_n(x) + \frac{1}{h^D}\int_x^{l_{\max}} \rho^m_n(l)dl \quad (12)$$

while in the material remaining in the n-th disc (analogous reasoning):

$$\tilde{\rho}^{m+1}_n(x) = \tilde{B}_{n,m}\rho^m_n = \left(1-\frac{x}{h^D}\right)\rho^m_n(x) + \frac{1}{h^G}\int_x^{l_{\max}} \rho_n(l)dl \quad (13)$$

It must be remembered that the column of the material subject to cutting is not on the whole the entirety of the material which has been moved to the lower hole. Its volume is $S_c \cdot h^D$, while that of the entire material moved from the preceding hole to the next one is:

$$V_{\vec{g}}^{D}\left(\alpha_c\right)-V_n^m$$

This means that the second and third component in (13) should be multiplied by their volumetric ratio:

$$\tilde{\rho}_n^{m+1}\left(x\right)=\tilde{B}_{n,m}\rho_n^m=\left(1-\frac{Sc\cdot x}{V_{\vec{g}}^{D}\left(\alpha_c\right)-V_{n+1}^m}\right)\rho_n^m\left(x\right)+\frac{Sc}{V_{\vec{g}}^{D}\left(\alpha_c\right)-V_{n+1}^m}\int_x^{l_{\max}}\rho_n\left(l\right)dl \tag{14}$$

It was assumed for simplification purposes that after cutting the distribution of the grain length in the disc (n+1)-th will be even (cut fraction and fraction already in the hole before the cut will mix) and therefore it will constitute the weighted average of $\rho_{n+1}^{(m-1)}$ and $\tilde{\rho}_{n+1}^{m-1}$:

$$\rho_{n+1}^m\left(x\right)=\frac{V_{n+1}^m}{V^D}\rho_{n+1}^{m-1}+\frac{V^D-V_{n+1}^m}{V^D}B_{n,m}\rho_n^m(x) \tag{15}$$

The filling of the quasi-cutting unit and thus the cutting efficiency depends on the value of the function V^D, V^G and S_c, which depend on the direction of effective gravitation and the sum of material volume in both holes before cutting $(V_{n+1}^m+V_n^m)$.

After grains are cut, two layers of the feed material move in relation to each other according to the rotational direction of the neighbouring discs and the gradient of mutual velocities. Material particles are removed from the preceding hole (they are subjected to the effect of the gravitational force effective component with a direction perpendicular to the inter-disc gap), but they are not removed from the next hole.

After cutting, length distribution will be:

$$\tilde{\rho}_n^{m+1}\left(x\right)=B_{n,m}\rho_{n,m}=\begin{cases}\tilde{\rho}_n^{m+1}\left(x\right)\left(\int_{l_{\min}}^{l_{\max}}\tilde{\rho}_n^{m+1}\left(x\right)\right)^{-1} & x\rangle l_{\max} \\ 0 & x\langle l_{\min}\end{cases} \tag{16}$$

The level of material for m-th cut (before m+1 cut) in n-th gap, $\tilde{y}_n^{m+1}$ is:

$$\tilde{y}_n^{m+1}=\left(y_n-y_{n+1}+\tilde{y}_{n+1}^k\right)\left(1-\frac{\int_0^{l_{\min}}\tilde{B}_{n,m}\rho_n^m\left(x\right)xdx}{\int_0^{l_{\max}}\tilde{B}_{n,m}\rho_n^m\left(x\right)xdx}\right) \tag{17}$$

To obtain distribution across the entire space of the hole before (m+1)-th cut (after re-feeding), the weighted average must be determined:

$$\rho_n^{m+1}\left(x\right)=\frac{y_n-\tilde{y}_n^{m+1}}{y_n}A_{n-1,m}\rho_{n-1}^m+\frac{\tilde{y}_n^{m+1}}{y_n}B_{n,m}\rho_n^m \tag{18}$$

The operator $B_{n,m}$ is no longer a linear operator like $A_{n,m}$ because it depends on the level of the material left in the *n-th* after *m-th* cut: $\tilde{y}_n^{m+1}$ and is the function of the probability distribution in the material (which determines what part of the material is removed from the machine during the cutting). So that $B_{n,m}$ can be treated as linear operators, the values y_n^m should be treated at every stage of the procedure as predetermined and iterative, which take into consideration the results of tests and experiments.

Determination of the cross-sectional area of the common part as the cross-section of grinding must be based on efficient mathematical procedures using the essence of the integral calculus or analytical geometry. Based on comprehensive analyses of cross-sections and multi-disc grinding resistances, it can be stated that the method for calculation of the grinding cross-section common area, in the analyses of the multi-disc seed grinding energy efficiency, depends directly on computing capabilities [8].

During the modelling of the grinding surface and the common surface, integration of the momentary grinding cross-section was used at the first stage (fig.1 and fig.6):

$$S_C=\int_{x_1}^{x_2}\left\{b_2+\left[R_2^2-\left(x-a_2\right)^2\right]^{1/2}\right\}dx-\int_{x_1}^{x_2}\left\{b_1-\left[R_1^2-\left(x-a_1\right)^2\right]^{1/2}\right\}dx \tag{19}$$

where:

a_1, a_2, b_1, b_2 - holes coordinates: C_1 i C_2,
R_1, R_2 – holes radius

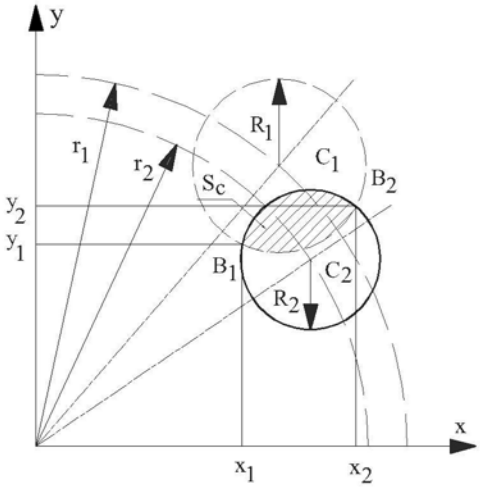

Fig. 6. Common area of two holes, grinding cross-section between the edges of neighbouring grinding holes: C1, C2 - working holes centres, R_1, R_2 – working holes radiuses, r1, r2 – working holes positioning radiuses, x1, x2, y1, y2 - coordinates for B1 and B2 quasi-cutting pair circles intersection points, SC –quasi-cutting hole pair common area

5. Summary

To sum up, it must be remembered while identifying the mathematical model of the grinder structure solution that the mathematical model describes the actual grinder only in an approximate way. There may be a number of various discrepancies between the model and the grinder which most frequently occur for the following reasons:

1. For derivation of the mathematical dependencies, classic mechanics laws are usually used which were formulated in reference to stiff non-deformable bodies. Moreover, various heat, friction and wear related phenomena as well as other phenomena are omitted as a rule which accompany the analysed mechanical processes.
2. The discrete model being identified has a finite number of degrees of freedom, while the mechanical grinder is a system with an infinite number of degrees of freedom.
3. The mechanical multi-drive grinder, within the full range of blades dynamic work and two overlapping holes, is mostly a nonlinear system. However, within some ranges of the tool common area (pairs of holes), the grinder behaves like a linear mechanical system, and therefore the assumption of the linear model is fully justified in reference to these ranges.

Furthermore, it must be added that the conditions in which the identification of the solution model is carried out may be materially different from the actual operation conditions of the grinder. This refers mainly to the nature of signals (determined or random) and the points of forcing signals. The mathematical model is in such cases defined under conditions differing from the actual operating conditions of the grinder, which may result in significant features of the mechanical system being omitted. If, for example, the ranges of the actual forcing signals are much wider than the ranges of forcing signals conducted at the time of a planned experiment, certain nonlinearities can be found important for the operation of the innovative system.

Omission of certain qualitative features of forcing signals may result in the distortion of the model actual dynamic properties. Bearing that in mind, one should, during innovative implementation, inspect carefully dynamic properties of the grinder and such analysis should demonstrate that the properties of the grinder correspond to its mathematical model.

6. Conclusions

Phenomena, processes and relations in multi-disc grinding, despite their complexity, are among those that can be described formally. The answers to the questions about process factors (activities and methods), structural features (methods, equipment and systems), operating conditions and their influence on the dynamics and efficiency of biomass grain grinding, based on the example of a multi-disc grinder, was possible if assuming infinitely quick propagation of quasi-cutting stresses within the scope of:

- probability distribution for a grain length,
- probability distribution for particles in the stream leaving the machine through the gap between the discs.

At m-th cut, the output stream of particles leaving the machine through the inter-disc gap between n-th and (n+1)-th disc is described by a probability distribution:

$$s_n^m(x) = \begin{cases} \int_x^{l_{\max}} \rho_n(l)\,dl \left(\int_0^{l_{\min}} \int_x^{l_{\max}} \rho_n(l)\,dl dx \right)^{-1} & l \langle l_{\min} \\ 0, & l \rangle l_{\min} \end{cases} \quad (20)$$

and its volume equals:

$$V = \left(y_n - y_{n+1} + \tilde{y}_{n+1}^k \right) \frac{\int_0^{l_{\min}} \tilde{B}_{n,m} \rho_n^m(x)\,x dx}{\int_0^{l_{\max}} \tilde{B}_{n,m} \rho_n^m(x)\,x dx} \quad (21)$$

The effective component of gravitational force and the angle of repose influence the dynamics and efficiency of the five-disc multi-hole grain biomass RWT-5KZ grinder. The significance of the influence is visible in their relation to basic parameters of the multi-disc grinding process, e.g. surface filling (crosswise) of the carrying and grinding space (operating space), actual volume and weight (mass efficiency) of the multi-edge grain grinder hole space filling. It was described with the variability of the operating space structural features, brought to the volume V^G, V^D, and the cross-sectional area S^G, S^D of the co-working pairs of the quasi-cutting holes, kinematical parameters of motion and the influence of the vector of effective components of gravitational force ($\vec{g}_G, \vec{g}_G$) caused by the inclination of the main axis of the operating discs package.

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References

1. Chwarścianek F. The construction all increase of crumbling effectiveness. The Archive of Mechanical Engineering, 2007, Vol. LIV, No 4: 391÷408.
2. Detyna J. Analysis of nonequilibrium stases in the sieve separation process. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2011, 1(49): 78-85.
3. Flizikowski J. The Construction of the Food Grinders. Wydawnictwo Uczelniane Akademii Techniczno-Rolniczej w Bydgoszczy 2005.
4. Flizikowski J. Doskonalenie badań i rozwoju rozdrabniaczy. Inżynieria i Aparatura Chemiczna, 2006, 1-2: 38-39.
5. Flizikowski J, Bieliński M. Multidisc Grinder Especially for Grains. Patent.RP-144 566.
6. Flizikowski J, Lis A. Optymalizacja rozdrabniacza wielotarczowego. Inżynieria i Aparatura Chemiczna 2007, 46(38), Nr 1: 50-52.
7. Khazaei J, Ghanbari S. New method for simultaneously measuring the angles of repose and frictional properties of wheat grains. International Agrophysics, 2010, 24: 275-286.
8. Macko M, Czerniak J. The evolutionary method for optimising disk design of multi-edge grinders. Journal of Theoretical and Applied Mechanics 8211, 08/2011.
9. Razavi S. M. A, Farahmandfar R. Effect of hulling and milling on the physical properties of rice grains. International Agrophysics, 2008, 22: 353-359.
10. Tomporowski A. Studium efektywności napędu i rozwiązań innowacyjnych konstrukcji wielotarczowych rozdrabniaczy ziaren biomasy. Societsa Scientiarum Lublineneis, 2011.
11. Ulrich S, Schroter M, Swinney H. Influence of friction on granular segregation. Physical Review, E, 76, 042301, 2007.
12. Walton O. Effects of interparticle friction and particle shape on dynamic angles of repose via particle-dynamics simulation. Proc. Conf. Mechanics and Statistical Physics of Particulate Materials, June 8-10, La Jolla, CA, USA, 1994.

Dr inż. Andrzej TOMPOROWSKI
Department of Mechanical Engineering
University of Technology and Life Sciences in Bydgoszcz
ul. Prof. S. Kaliskiego 7, 85-789 Bydgoszcz, Poland
E-mail: a.tomporowski@utp.edu.pl

Prof. dr hab. inż. Marek OPIELAK
Lublin University of Technology
ul. Nadbystrzycka 38, 20-618 Lublin, Poland
E-mail: m.opielak@pollub.pl