1. Introduction

Fault or health trend prediction technique has become one of the effective ways to protect the safe operation of high reliable systems. However complex systems often show complex dynamic behaviors and uncertainty, which lead to hardly establishing their precise physical models. In this case, in order to obtain the satisfactory prediction results, time series analysis methods are often used to perform the prediction in practice [2, 12, 15, 19, 26]. Among the known non-linear time series prediction methods, the effectiveness of statistics theory based methods have been demonstrated, such as Artificial Neural Networks (ANN), Support Vector Regression (SVR), etc.

ANN has been applied in many fields due to its universal approximation property. However ANN suffers from local minimum traps, difficulty in determining the hidden layer size and learning rate, poor capacity for generalization, etc. [8, 10, 32] On the contrary, SVR overcomes the problems existing in ANN. SVR aims at the global optimum and exhibits better accuracy in non-linear and non-stationary time series data prediction due to its implementation of the structural risk minimization principle [10, 27, 28]. But complexity of SVR depends not only on the input space dimension, but also on the number of sample data. For large sample data, the quadratic programming (QP) problem is more complex, it will cost a lot of computing time. For this reason, LS-SVR was proposed by Suykens et al. [16, 23] In
LS-SVR, the inequality constrains are replaced by equality constrains. This way, solving a QP is converted into solving linear equations, and the calculation time is reduced significantly. Thus, LS-SVR attracts more attention in time series prediction [5, 6, 19, 20, 26, 33].

In many applications of fault or health condition prediction, one certain condition may be represented by one major variable and several relevant variables. In order to achieve satisfactory prediction, these auxiliary time series relating to the major time series are utilized to enrich the information and improve the prediction accuracy. In this case, how to fully present the information hiding in the multiple time series data becomes a key issue. The kernel function is used to map the input data to high dimensional feature space, so it influences the learning performance of LS-SVR, that means a appropriate kernel function can more fully present the information in time series data. However, LS-SVR with a single kernel function is not a good choice to all the data sets, especially for multiple time series data, although the kernel parameters can be optimally chosen to enhance the generalization capability.

Some researchers applied Multiple Kernel Learning (MKL) to solve the above problems [13, 29]. MKL provides a more flexible framework than single kernel. Under the framework, the information in time series data can be mined more adaptively and effectively, i.e., MKL explicitly learns the weights of basis kernels from different time series data sources, and the relationships among them are learned meanwhile. Moreover, MKL can avoid the difficulty of appropriate kernel function selection. Thus, multi-kernel LS-SVR has better prediction accuracy in practice [13,14].

However in order to obtain better prediction results, some problems which accord to the requirements of applications, should still be considered, such as fast and accurate prediction.

(1) In MKL framework, the time series data samples are generally learned by a linear convex combination of basis kernels. The reported methods of determining the combining weights of basis kernels, such as software packages [1] and joint optimization selection algorithm [9, 34], are always complex. They are generally unapt for applications.

(2) Although some researchers also used multiple relevant time series to perform prediction [6, 17, 31, 35, 36], different interrelated levels between major time series and auxiliary time series have different influences on prediction accuracy. It is necessary to determine the interrelated levels between them, and they represent the weight values of each time series for the prediction.

(3) The optimal prediction methods always assume that all the training time series data have same contribution to the prediction. According to the new information principle [3], the data near the current prediction point will affect the prediction much more. Thus, in order to achieve more accurate results, each sample data should be weighted according to their distance far from the current prediction point.

Thus a weighted prediction method with multi-kernel LS-SVR using multiple relevant time series is proposed in this paper. According to the application requirements, we apply three ways to achieve better prediction results. One is to compute correlative levels of multiple relevant time series to represent their different contributions to prediction results; Secondly, we propose a weight function to present the different influence of each history data on prediction; Finally, we establish a new multi-kernel LS-SVR based on time-distance-weighted factor of each time series, and in order to improve the application value of the proposed method, a simple approach of determining the combining weights of the multiple basis kernels is proposed to reduce the calculation time.

The rest of the paper is organized as follows: Section 2 gives a brief review of LS-SVR and multiple kernel learning (MKL) algorithm; Section 3 proposes the weighted prediction method which includes three computational approaches: (1) combination coefficients of multiple basis kernels, (2) correlative levels of the multiple time series, and (3) time-distance-weighted factors of each time series data; Section 4 shows simulation and application experiments; and the conclusions are drawn in Section 5.

2. A brief review of related work

2.1. Least squares support vector regression

LS-SVR has many advantages, such as simpler algorithm, faster operation speed, etc. It is widely applied in regression. The goal of LS-SVR is to estimate a function that is as “close” as possible to the target values for every data point, and at same time, is as “flat” as possible for good generalization. The regression principle of LS-SVR can be expressed as follows.

Consider a training data set of \( n \) data points \( \{x_i,y_i\}_{i=1}^{n} \) with input data \( x_i \in \mathbb{R}^d \), and \( y_i \in \mathbb{R} \) is the corresponding output or target value.

LS-SVR is to construct the regression function with the following form:

\[
 f(x) = y = w^T \phi(x) + b
\]

where \( \phi(x) \) is used to non-linearly map the input data to the high dimensional feature space, \( w \) is the weight vector and \( b \) is the bias term.

According to the structural risk minimization principle [27,28], the function regression problem can be represented as a constraint optimization problem as follows

\[
 \begin{align*}
 \min_{w,b} & \quad J(w,b) = \frac{1}{2} \|w\|^2 + \frac{c}{2} \sum_{i=1}^{n} \epsilon_i^2 \\
 \text{s.t.} & \quad y_i = w^T \phi(x_i) + b + \epsilon_i
\end{align*}
\]

(2)

where \( i = 1,2,...,n \), \( J(w,b) \) is the cost function, \( c \) is a positive real constant (regularization parameter) and \( \epsilon_i \in \mathbb{R} \) is an error variable.

In order to solve the above constraint optimization problem, the Lagrangian function is constructed by transforming constraint optimization problems into unconstraint ones

\[
 L(w,b,\alpha,\epsilon) = \frac{1}{2} \|w\|^2 + \frac{1}{2} \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i (w^T \phi(x_i) + b + \epsilon_i - y_i)
\]

(3)

where \( \alpha_i \) is the \( i \)-th Lagrange multiplier. It is obvious that the optimal solution of Eq.(2) satisfies the Karush-Kuhn-Tucker (KKT) conditions. The optimal conditions are expressed as follows

\[
 \begin{align*}
 \frac{\partial L}{\partial w} &= w - \sum_{i=1}^{n} \alpha_i \phi(x_i) = 0 \Rightarrow w = \sum_{i=1}^{n} \alpha_i \phi(x_i) \\
 \frac{\partial L}{\partial b} &= -\sum_{i=1}^{n} \alpha_i = 0 \Rightarrow \sum_{i=1}^{n} \alpha_i = 0 \\
 \frac{\partial L}{\partial \epsilon_i} &= w^T \phi(x_i) + b + \epsilon_i - y_i = 0 \Rightarrow y_i = w^T \phi(x_i) + b + \epsilon_i \\
 \frac{\partial L}{\partial \alpha_i} &= \epsilon_i - \alpha_i = 0 \Rightarrow \epsilon_i = \frac{1}{c} \alpha_i
\end{align*}
\]

(4)

After eliminating \( w \) and \( \epsilon_i \) from Eq.(4), we could obtain the solution by the following linear equations
to obtain the combining weights $\mu_j$ and the parameters of LS-SVR simultaneously. But all the solution methods are complex in practice. Thus, we proposed a simple method to fix this problem in this paper.

3. Proposed weighted multiple time series prediction method

In this section, we propose a new scheme to obtain better prediction performance. Firstly, we use multiple kernel functions consisting of several basis kernels to show the information more effectively in the high dimensional mapping feature space. A simple approximate approach is presented to compute the combining weights with less calculation complexity. Then we propose the weighted prediction method. In the method, we calculate the correlation coefficient of each time series as weight factor, which present the influence factor on prediction accuracy with each time series, and based on the distance of each time series data far from the current prediction point, we weight the effects of the history data on prediction via a modified weight function.

3.1. Combination coefficients of multiple basis kernels

In this paper, we apply the new kernel with a linear combination of basis kernels, shown as Eq.(9). In order to reduce the computing complexity, we propose a simple method to determine combining weights, i.e., the combining weights of basis kernels are determined according to the root mean squared error (RMSE) of each LS-SVR with each single basis kernel. This way, smaller RMSE value will get bigger weight value. The RMSE of multiple time series prediction is defined as follows

$$\sigma_{\text{RMSE}} = \sqrt{\frac{1}{MN} \sum_{i=1}^{N} \sum_{k=1}^{M} (y(i,k) - \hat{y}(i,k))^2}$$

where $M$ is the number of the relevant parameters, $N$ is the number of original training sample data, and $y(i,k)$ and $\hat{y}(i,k)$ are the prediction value and actual value respectively. The linear combining weights $\mu_j$ can be computed as follows

$$\mu_j = \frac{\sum_{r=1}^{m} \sigma_r - \sigma_j}{(m-1) \sum_{r=1}^{m} \sigma_r}$$

where $\sigma_j$ is the prediction RMSE of the $j$-th kernel, $\sum_{r=1}^{m} \sigma_r$ is the sum RMSE of all basis kernels, and $\sum_{r=1}^{m} \sigma_r - \sigma_j$ presents the contribution of the $j$-th kernel.

Obviously, the proposed method of calculating combination coefficients has less complexity comparing with the methods described in section 2.

3.2. Weight factors of multiple time series

3.2.1. Weight factors of major and auxiliary time series

Multiple relevant time series are used to enrich the information in data. However, each time series has different effects on prediction. Therefore, some researchers also applied joint optimization selection algorithms with general-purpose optimization software packages[14]. Moreover, some researchers also applied joint optimization selection algorithms to obtain the combining weights $\mu_j$ and the parameters of LS-SVR simultaneously. But all the solution methods are complex in practice. Thus, we proposed a simple method to fix this problem in this paper.

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the system’s fault or health condition. In this paper, we select the time series, which mainly represents the system fault or health state, as major time series, the others are auxiliary time series. Then the correlation coefficients between the major time series and the auxiliary time series will be computed, and they will be utilized to improve the prediction accuracy.

The purpose of correlation analysis is to measure and interpret the strength of linear or non-linear relationship between two continuous variables [11, 22]. We select the commonly used correlation coefficients, Pearson correlation coefficient [4, 7], to assess the strength of the relationships of multiple relevant time series. The Pearson correlation coefficient computing formula is shown as follows

\[ R = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} \]  

(12)

where \( R \) is the correlation coefficient between bivariate data \( x_i \) and \( y_i \) values \( (i = 1, 2, ..., n) \), \( \bar{x} \) and \( \bar{y} \) are the mean values of the \( x_i \) and \( y_i \) respectively. The Pearson correlation coefficient may be computed by means of a computer-based statistics program “Microsoft Excel” using the option “Correlation” under the option “Data Analysis Tools”. Moreover it can also be calculated by Matlab.

3.2.2. Time-distance-weighted factors of each time series

The time series data closing to the current prediction point have greater relevance to current prediction, on the contrary, less relevance. Lorenz function's corresponding differential equations are shown as follows

\[ \frac{\partial x}{\partial t} = -a y x + z \]  
\[ \frac{\partial y}{\partial t} = b x y - x z - c y \]  
\[ \frac{\partial z}{\partial t} = x y - c z \]

(15)

Let \( a = 8/3 \), \( b = 10 \), \( c = 28 \), range of initialization as \([1,1,1]\), and simulation step as 0.1 with Fourth-order Runge-Kutta method. We collect 800 data of the three time series of \( x \) (major time series), \( y \) and \( z \) (auxiliary time series) respectively. We select the first 400 data of \( x \), \( y \) and \( z \) time series as training data and the last 400 time series as testing data. In addition, we apply C-C method[21] to generate training sample sets because Lorenz time series is chaotic time series. The prediction efficiency depends on the RMSE, training time (TrTime) and prediction time (PrTime). One Gaussian RBF

\[ \kappa(x, y) = \exp \left( \frac{|x - y|^2}{2\sigma^2} \right) \]

and one Linear kernel function \( \kappa(x, y) = xx' \) are adopted as basis kernel functions. All the parameters will be jointly optimized by traditional gridding search method with range of \([0.1, 1000]\)

In Experiment I, we use variable \( x \) time series alone to do prediction with tradition multi-kernel LS-SVR reported in Ref.[35] and Ref.[36]. This experiment compares the following two methods: one
is obtains the combining weights via optimization software packages (called Method A); the other one is the proposed simple approximate approach in section 3.1 (called Method B). The results are shown in Figure 1 and Table 1.

From Figure 1 and Table 1, we can see that although the prediction accuracy of the new simple computing method (Method B) is adequately reduced comparing with Method A, it also has a good results. Method B can greatly reduce the total computing time, especially training time. The results also indicate that the proposed approximate method is an effective method.

In Experiment II, the variables $y$ and $z$ time series are utilized to enrich the information of variable $x$ time series. We use same multi-kernel LS-SVR model to compare the following methods: Method C doesn’t consider the different contributions of each time series and their history data on prediction; Method D applies the proposed approach proposed in section 3.2. Here, we select same kernel functions and optimization method as Experiment I. The results are reported in Table 2, Figure 2 and Table 3.

Figure 2 and Table 3 show that the weighted time series prediction method can improve the prediction accuracy efficiently, and the computing time is not large increase. These are due to that the proposed method takes the different influence factors on prediction accuracy with each auxiliary time series and their history data into account. The other reason is that almost all the middle values at the calculation process of weight factors are already computed and stored in the process of setting up the prediction method.

Table 1. Prediction Results of Method A and Method B

<table>
<thead>
<tr>
<th></th>
<th>TrTime/s</th>
<th>PrTime/s</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>4.1501</td>
<td>0.2406</td>
<td>2.2258</td>
</tr>
<tr>
<td>Method B</td>
<td>3.0480</td>
<td>0.2456</td>
<td>2.8927</td>
</tr>
</tbody>
</table>

4.2. Application experiment and results analysis

We apply the proposed method in a prediction application of one complex avionics system. Four relevant variables time series are collected. They are shown in Figure 3 after preprocessing (omit dimension).

We take the first 15 data of each time series as training samples and look at any continuous 6 as a sample, i.e., the data points from 1 to 15 in the time series are taken as the 10 initial training sample data. The first sample data set consists of points 1 through 6, with the first 5 as the input sample vector and the 6th point as the output. The second sample data set consists of points 2 through 7, with the points 2 through 6 as the input sample vector and the 7th point as the output. This way we have 10 training data out of the first 15 data points.

Table 2. Correlation Coefficient of Time Series

<table>
<thead>
<tr>
<th></th>
<th>$r_{XY}$</th>
<th>$r_{XZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Coefficient</td>
<td>-0.0581</td>
<td>-0.0348</td>
</tr>
</tbody>
</table>

Table 3. Prediction Results of Method C and Method D

<table>
<thead>
<tr>
<th></th>
<th>TrTime/s</th>
<th>PrTime/s</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method C</td>
<td>8.0479</td>
<td>0.8016</td>
<td>2.1765</td>
</tr>
<tr>
<td>Method D</td>
<td>9.1241</td>
<td>0.8203</td>
<td>2.0252</td>
</tr>
</tbody>
</table>
All the parameters are set same as simulation Experiments I and II. The contrast prediction experiment applies Method A and Method E (described in Section 3). The prediction results of the major time series (see Figure 3) are shown in Figure 4 and Figure 5.

In order to show the results clearly, we report them in Table 4 and Table 5.

Table 4. Correlation Coefficient of the Time Series

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>( r_1 )</th>
<th>( r_2 )</th>
<th>( r_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Coefficient</td>
<td>0.2791</td>
<td>0.8514</td>
<td>0.6065</td>
</tr>
</tbody>
</table>

Table 5. Prediction Results of Method A and Method E

<table>
<thead>
<tr>
<th>Method</th>
<th>TrTime/s</th>
<th>PrTime/s</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>1.4322</td>
<td>0.0808</td>
<td>0.6771</td>
</tr>
<tr>
<td>Method E</td>
<td>1.2927</td>
<td>0.0122</td>
<td>0.5789</td>
</tr>
</tbody>
</table>

From Figure 4, Figure 5 and Table 5, we can see that the proposed method has better prediction results in prediction accuracy and computing time. The results also indicate the proposed method is a good approach, and it can adapt the application better.

5. Conclusions

In this study, we aim at the requirements of applications and analyze the drawbacks of multiple time series prediction by LS-SVR, and then we propose a novel weighted multiple time series prediction method based on multi-kernel LS-SVR. In the new method, we determine the combining weights of each basis kernels by calculating the root mean squared error (RMSE) of prediction using each basis kernel, compute the different contributions to prediction results via correlation analysis between the major time series and auxiliary time series, and make the each historical data with different weight factor based on their distance far from the current prediction point via a modified weight function. The results of simulation and application experiments show that the proposed prediction scheme is an effective approach. It can satisfy the application requirements and may be more valuable in practice.

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