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# THE DYNAMICS OF THE CUTTING PROCESS WITH DUFFING NONLINEARITY

# DYNAMIKA PROCESU SKRAWANIA Z NIELINIOWOŚCIĄ DUFFINGA*

*The paper presents the nonlinear one degree of freedom model of cutting process. To describe the dynamics the Duffing model with time delay part is used. The model is solved analytically by using the multiple time scale method. The stability lobe diagrams are determined numerically and analytically. The obtained results show that stability region depends on initial conditions of the system.*

***Keywords**: Duffing oscillator, nonlinear vibrations, time delay, stability.*

*W artykule przedstawiono jednowymiarowy nieliniowy model skrawania. Do opisu procesu przyjęto model Duffinga z opóźnieniem czasowym. Model rozwiązano analitycznie za pomocą metody wielu skal czasowych. Wykres stabilności otrzymano numerycznie i analitycznie. Wykazano, że obszary stabilności zależą od warunków początkowych układu.*

***Słowa kluczowe**: oscylator Duffinga, drgania nieliniowe, opóźnienie czasowe, stabilność.*

## 1. Introduction

Systems with time delays are of interest when modelling processes in engineering, finance, and others [3]. They belong to a class of systems in which the current state of the process is an effect of the former state, delayed in time. Sometimes, the time delay is introduced into the system for control purposes. The mathematical description of delay dynamical systems naturally involve the delay parameter in some specified way. A differential equation with delay (DDE) describing a dynamical system belongs to the class of retarded functional differential equations (sometimes the equations are called *retarded differential-difference equations - RDDE*) [4]. The Duffing's oscillator is the simplest model of the dynamical behaviour of many complex systems. The equation with added delay part can be used of a model of cutting process [1]. In the turning process, a cylindrical workpiece rotates with a constant angular velocity, and the tool generates a surface as material is removed. The cutting force, which is a strong function of the chip thickness, becomes strongly dependent on the tool's delayed position $x(t-\tau)$ as well as its current position $x(t)$. Thus, to represent such a phenomenon, delay differential equations have been widely used as models for regenerative machine tool vibration (*regenerative chatter*).

Several phenomena occur during machining which adversely affect the course of the machining process, as well as tool life and surface quality. The main reason for these adverse events is self-excited vibration caused by the regenerative effect. This effect is caused by the overlap in the preceding trace tool to the passage trace from the current transition tools. The research has been conducted to increase productivity in machining processes, predict and avoid regenerative-type chatter.

In this paper we present the one degree of freedom model of a cutting process which is described by the Duffing's equation with time delay. The analytical study with numerical examples of chatter investigation is performed. Based on numerical simulation, the stability lobe diagram are constructed and compared with numerical results. The method of multiple scales (MMS) is used to solve the problem analytically. This method was also applied to do research on similar Duffing's system with time delay and external excitation [10].

## 2. Model of cutting

Here, the classical Duffing oscillator is connected with a time delay element to model regenerative effect in the cutting process [5]. Then, the model of regenerative cutting with nonlinear stiffness is presented in (Fig. 1). Vibrations that occur during machining can be described by delay differential equations (DDE) with shifted argument in the form:

$$x''(t) + \delta x'(t) + \omega_0^2 x(t) + \gamma x(t)^3 = \alpha\left[-\mu x(t) + x(t-\tau)\right], \quad (1)$$

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** where: $\delta$ is damping coefficient, $\gamma$ is nonlinear stiffness, $\omega_o$ is natural frequency of the linear system. A cutting force is represented by the right side of the equation where, $\alpha$ is a specific cutting force factor, $\tau$ is time delay, $\mu$ is switching parameter which for the regenerative model of cutting is equal to one. The term with time delay represents a solution at previous state.

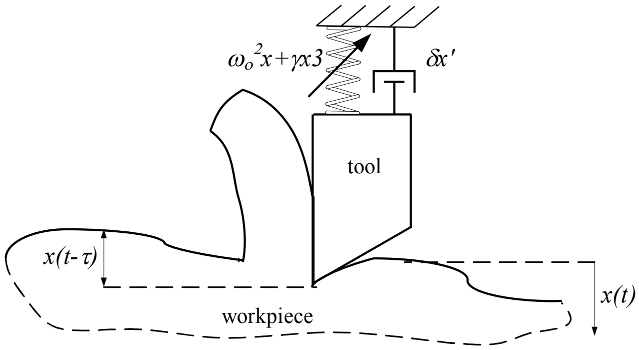


*Fig. 1. Model of regenerative cutting with Duffing's stiffness*

Chatter vibrations are the main problem occurring during cutting process [2, 6, 11], therefore this aspect is analysed here. In order to find stable cutting regions and the amplitude of chatter vibrations which exist in instability zones, an analytical and numerical solution of equation (1) and influence of the process parameters are determined in next sections.

## 3. Analytical solution

The system described by Eq. (1) is solved analytically with the help of the multiple time scale method [8, 9], we confine the study to the case of small damping and weak nonlinearity. We assume two scales (fast and slow) expansion of the solution. A fast scale $T_0$ and slow scale $T_1$ are described by eq. (2), then a solution in the first order approximation is sought in the form (3) and (4):

$$T_0 = t, T_1 = \varepsilon t\ , \quad (2)$$

$$x(t) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1)\ , \quad (3)$$

$$x(t-\tau) = x_\tau = x_{0\tau}(T_0,T_1) + \varepsilon x_{1\tau}(T_0,T_1)\ . \quad (4)$$

It is assumed that:

$$\omega_0^2 = \omega^2 + \varepsilon\sigma, \gamma = \varepsilon\tilde{\gamma}, \alpha = \varepsilon\tilde{\alpha}, \delta = \varepsilon\tilde{\delta}\ , \quad (5)$$

where: $\varepsilon$ is a formal small parameter [7]. Next, in order to facilitate notation, the tilde is omitted. By using the chain rule, the time derivative is transformed according to the expressions (6) and (7):

$$\frac{d}{dt} = \frac{\partial}{\partial T_0} + \varepsilon \frac{\partial}{\partial T_1}\ , \quad (6)$$

$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial T_0^2} + \varepsilon \frac{\partial^2}{\partial T_0 \partial T_1} + \varepsilon \frac{\partial^2}{\partial T_1 \partial T_0} + \ldots = \frac{\partial^2}{\partial T_0^2} + 2\varepsilon \frac{\partial^2}{\partial T_0 \partial T_1} + \ldots\ . \quad (7)$$

Substituting eqs.(2) – (7) into (1) we let:

$$\frac{\partial^2 x(t)}{\partial T_0^2} + 2\varepsilon \frac{\partial^2 x(t)}{\partial T_0 \partial T_1} + \varepsilon\delta \frac{\partial x(t)}{\partial T_0} + \left[\omega^2 + \varepsilon\sigma\right] x(t) + \varepsilon\gamma x(t)^3 = \varepsilon\alpha[-\mu x(t) + x(t-\tau)]\ . \quad (8)$$

Expanding derivatives of the equation (8) we obtain (12):

$$\frac{\partial x(t)}{\partial T_0} = \frac{\partial x_0}{\partial T_0} + \varepsilon \frac{\partial x_1}{\partial T_0}\ , \quad (9)$$

$$\frac{\partial^2 x(t)}{\partial T_0^2} = \frac{\partial^2 x_0}{\partial T_0^2} + \varepsilon \frac{\partial^2 x_1}{\partial T_0^2}\ , \quad (10)$$

$$\frac{\partial^2 x(t)}{\partial T_0 \partial T_1} = \frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \varepsilon \frac{\partial^2 x_1}{\partial T_0 \partial T_1}\ , \quad (11)$$

$$\frac{\partial^2 x_0}{\partial T_0^2} + \varepsilon \frac{\partial^2 x_1}{\partial T_0^2} + 2\varepsilon \frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \varepsilon\delta \frac{\partial x_0}{\partial T_0} + \omega^2 x_0 + \varepsilon\omega^2 x_1 + \varepsilon\sigma x_0 + \varepsilon\gamma x_0^3 = \varepsilon\alpha(-\mu x_0 + x_{0\tau})\ . \quad (12)$$

Equating coefficients of powers of $\varepsilon^0$ and $\varepsilon^1$, we obtain:

$$\varepsilon^0 \Rightarrow \frac{\partial^2 x_0}{\partial T_0^2} + \omega^2 x_0 = 0\ , \quad (13)$$

$$\varepsilon^1 \Rightarrow \frac{\partial^2 x_1}{\partial T_0^2} + 2\frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \delta \frac{\partial x_0}{\partial T_0} + \sigma x_0 + \omega^2 x_1 + \gamma x_0^3 + \mu\alpha x_0 - \alpha x_{0\tau} = 0\ . \quad (14)$$

It is convenient to express the solution of equation (13) in the complex form (15) and (16):

$$x_0(T_0,T_1) = A(T_1)e^{iT_0} + \overline{A}(T_1)e^{-iT_0}\ , \quad (15)$$

$$x_{0\tau}(T_0,T_1) = A(T_1)e^{i(T_0-\tau)} + \overline{A}(T_1)e^{-i(T_0-\tau)}\ , \quad (16)$$

where: $\overline{A}$ is the complex conjugate of $A$, which is an arbitrary complex function of $T_1$.

Substituting equations and into equation and expanding derivatives we get:

$$\frac{\partial x_0}{\partial T_0} = A(T_1)ie^{iT_0} - \overline{A}(T_1)ie^{-iT_0}\ , \quad (17)$$

$$\frac{\partial^2 x_0}{\partial T_0 \partial T_1} = A'(T_1)ie^{iT_0} - \overline{A}'(T_1)ie^{-iT_0}\ , \quad (18)$$

and then the following equation is obtained:

$$\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+2\left[A'(T_1)ie^{iT_0}-\bar{A}'(T_1)ie^{-iT_0}\right]+\delta\left[A(T_1)ie^{iT_0}-\bar{A}(T_1)ie^{-iT_0}\right]+$$
$$\sigma\left[A(T_1)e^{iT_0}+\bar{A}(T_1)e^{-iT_0}\right]+\gamma\left[A(T_1)e^{iT_0}+\bar{A}(T_1)e^{-iT_0}\right]^3+\mu\alpha\left[A(T_1)e^{iT_0}+\bar{A}(T_1)e^{-iT_0}\right]-$$
$$\alpha\left[A(T_1)e^{i(T_0-\tau)}+\bar{A}(T_1)e^{-i(T_0-\tau)}\right]=0. \tag{19}$$

Ordering equation (19) we get its final form:

$$\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+\gamma A(T_1)^3 e^{3iT_0}+\gamma\bar{A}(T_1)^3 e^{-3iT_0}+$$
$$\left[-\alpha A(T_1)e^{-i\tau}+i\delta A(T_1)+\mu\alpha A(T_1)+\sigma A(T_1)+3\gamma A(T_1)^2\bar{A}(T_1)+2iA'(T_1)\right]e^{iT_0}+$$
$$\left[-\alpha\bar{A}(T_1)e^{-i\tau}-i\delta\bar{A}(T_1)+\mu\alpha\bar{A}(T_1)+\sigma\bar{A}(T_1)+3\gamma A(T_1)\bar{A}(T_1)^2-2i\bar{A}'(T_1)\right]e^{-iT_0}=0. \tag{20}$$

The secular term of equation (20) vanishes if and only if equations (21) are complied. This leads to the equations (22) and (23):

$$ST_1 e^{iT_0}=0,\ ST_2 e^{-iT_0}=0\,, \tag{21}$$

$$-\alpha A(T_1)e^{-i\tau}+i\delta A(T_1)+\mu\alpha A(T_1)+\sigma A(T_1)+3\gamma A(T_1)^2\bar{A}(T_1)+2iA'(T_1)=0\,, \tag{22}$$

$$-\alpha\bar{A}(T_1)e^{-i\tau}-i\delta\bar{A}(T_1)+\mu\alpha\bar{A}(T_1)+\sigma\bar{A}(T_1)+3\gamma A(T_1)\bar{A}(T_1)^2-2i\bar{A}'(T_1)=0\,. \tag{23}$$

where: $ST_1$ and $ST_2$ are secular generating terms.

Eliminating from equation the secular generating terms we have equation (24):

$$\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+\gamma A(T_1)^3 e^{3iT_0}+\gamma\bar{A}(T_1)^3 e^{-3iT_0}=0\,. \tag{24}$$

Solving (24) for:

$$x_1(T_0,T_1)=B(T_1)e^{3iT_0}+\bar{B}(T_1)e^{-3iT_0}\,, \tag{25}$$

$$x_{1\tau}(T_0,T_1)=B(T_1)e^{3i(T_0-\tau)}+\bar{B}(T_1)e^{-3i(T_0-\tau)}\,, \tag{26}$$

where:

$$B(T_1)=-\frac{\gamma A(T_1)^3}{\omega^2-9}\,, \tag{27}$$

$$\bar{B}(T_1)=-\frac{\gamma\bar{A}(T_1)^3}{\omega^2-9}\,, \tag{28}$$

we obtain:

$$x_1(T_0,T_1)=-\frac{\gamma A(T_1)^3}{\omega^2-9}e^{3iT_0}-\frac{\gamma\bar{A}(T_1)^3}{\omega^2-9}e^{-3iT_0}\,, \tag{29}$$

$$x_{1\tau}(T_0,T_1)=-\frac{\gamma A(T_1)^3}{\omega^2-9}e^{3i(T_0-\tau)}-\frac{\gamma\bar{A}(T_1)^3}{\omega^2-9}e^{-3i(T_0-\tau)}\,. \tag{30}$$

Substitution into equations and the polar form of the complex amplitude:

$$A(T_1)=\frac{1}{2}a(T_1)e^{i\beta(T_1)}\,, \tag{31}$$

$$\bar{A}(T_1)=\frac{1}{2}a(T_1)e^{-i\beta(T_1)}\,, \tag{32}$$

$$A'(T_1)=\frac{1}{2}a'(T_1)e^{i\beta(T_1)}+\frac{1}{2}ia(T_1)\beta'(T_1)e^{i\beta(T_1)}\,, \tag{33}$$

$$\bar{A}'(T_1)=\frac{1}{2}a'(T_1)e^{-i\beta(T_1)}-\frac{1}{2}ia(T_1)\beta'(T_1)e^{-i\beta(T_1)}\,. \tag{34}$$

results in:

$$-\frac{1}{2}\alpha a(T_1)e^{-i\tau+i\beta(T_1)}+\frac{1}{2}i\delta a(T_1)e^{i\beta(T_1)}+\frac{1}{2}\mu\alpha a(T_1)e^{i\beta(T_1)}+\frac{1}{2}\sigma a(T_1)e^{i\beta(T_1)}+$$
$$\frac{3}{8}\gamma a(T_1)^3 e^{i\beta(T_1)}+2i\left[\frac{1}{2}a'(T_1)e^{i\beta(T_1)}+\frac{1}{2}ia(T_1)\beta'(T_1)e^{i\beta(T_1)}\right]=0, \tag{35}$$

$$-\frac{1}{2}\alpha a(T_1)e^{i\tau-i\beta(T_1)}-\frac{1}{2}i\delta a(T_1)e^{-i\beta(T_1)}+\frac{1}{2}\mu\alpha a(T_1)e^{-i\beta(T_1)}+\frac{1}{2}\sigma a(T_1)e^{-i\beta(T_1)}+$$
$$\frac{3}{8}\gamma a(T_1)^3 e^{-i\beta(T_1)}-2i\left[\frac{1}{2}a'(T_1)e^{-i\beta(T_1)}-\frac{1}{2}ia(T_1)\beta'(T_1)e^{-i\beta(T_1)}\right]=0. \tag{36}$$

After transformations (35) we obtain (37):

$$-\frac{1}{2}\alpha a(T_1)e^{-i\tau}+\frac{1}{2}i\delta a(T_1)+\frac{1}{2}\mu\alpha a(T_1)+\frac{1}{2}\sigma a(T_1)+\frac{3}{8}\gamma a(T_1)^3+$$
$$ia'(T_1)-a(T_1)\beta'(T_1)=0. \tag{37}$$

Then recalling:

$$e^{-i\tau}=\cos\tau-i\sin\tau\,. \tag{38}$$

The normal form is obtained:

$$-\frac{1}{2}\alpha a(T_1)\cos\tau+\frac{1}{2}i\alpha a(T_1)\sin\tau+\frac{1}{2}i\delta a(T_1)+\frac{1}{2}\mu\alpha a(T_1)+\frac{1}{2}\sigma a(T_1)+$$
$$\frac{3}{8}\gamma a(T_1)^3+ia'(T_1)-a(T_1)\beta'(T_1)=0. \tag{39}$$

Separating real and imaginary parts, the two, so called, modulation equations are found:

$$\frac{1}{2}\delta a(T_1)+\frac{1}{2}\alpha a(T_1)\sin\tau+a'(T_1)=0 \tag{40}$$

$$\frac{1}{2}\mu\alpha a(T_1)+\frac{1}{2}\sigma a(T_1)+\frac{3}{8}\gamma a(T_1)^3-\frac{1}{2}\alpha a(T_1)\cos\tau-a(T_1)\beta'(T_1)=0 \tag{41}$$

Transforming, we obtain the modulation equations in the form (42) and (43):

$$a'(T_1)=-\frac{1}{2}\delta a(T_1)-\frac{1}{2}\alpha a(T_1)\sin\tau\,, \tag{42}$$

$$\beta'(T_1) = \frac{3}{8}\gamma a(T_1)^2 + \frac{1}{2}\mu\alpha + \frac{1}{2}\sigma - \frac{1}{2}\alpha\cos\tau \; . \qquad (43)$$

In a case of a steady state solutions $a'$=0 and $\beta'$=0 then the chatter frequency ($\omega$) and amplitude ($a$) is given as follows:

$$\omega = \sqrt{\omega_0 + \alpha\mu - \alpha\cos\tau} \qquad (44)$$

$$a_1 = \frac{2\sqrt{3}}{3}\sqrt{\frac{\sqrt{\left(\alpha^2-\delta^2\right)} - \alpha\cos\tau}{\gamma}} \qquad (45)$$

$$a_2 = \frac{2\sqrt{3}}{3}\sqrt{\frac{-\sqrt{\left(\alpha^2-\delta^2\right)} + \alpha\cos\tau}{\gamma}} \qquad (46)$$

Chatter frequency ($\omega$) given by equation (44) depends only on delay parameters $\alpha$, $\tau$ and natural frequency of linear system $\omega_o$. Interestingly, the parameter of nonlinearity ($\gamma$) and vibrations amplitude do not influence the frequency. The amplitude of steady state chatter vibrations exist when:

$$\alpha^2 - \delta^2 > 0 \qquad (47)$$

Assuming, that system parameters are always positive, only for $\alpha > \delta$ the periodic solution appears. Then the critical value of $\alpha$ can be introduced $\alpha_{cr}$=$\delta$. On the other hand, the amplitude of vibrations is equals zero when the system parameters fulfil the condition:

$$\sqrt{\left(\alpha^2-\delta^2\right)} - \alpha\cos\tau = 0 \qquad (48)$$

The amplitudes $a_1$ and $a_2$ represented by equation  and  are displayed in Figs. 2 and 3 as maps where colour means value of the vibrations amplitude.

The second solution exists only in narrow regions where the first solution does not exist. That means that when the condition (47) is fulfilled always periodic solution appears in the autonomous delayed Duffing's system regardless time delay. Whereas the condition (48) is satisfied (amplitude equals zero) exactly on the border of the lobs shown in Fig. 2.

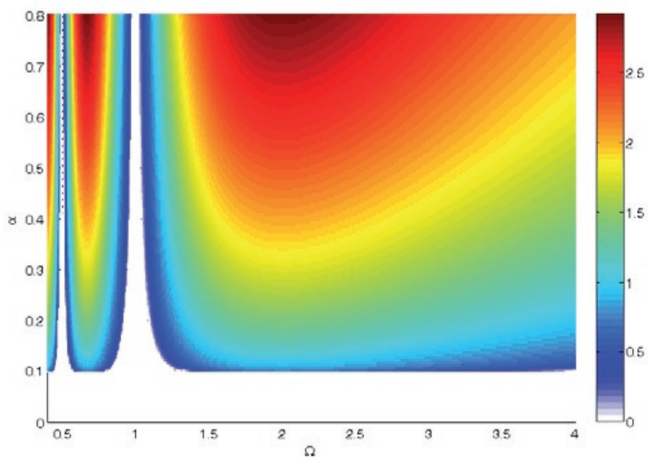


*Fig. 2. Analytical calculated amplitude $a_1$ of steady state solution represented by equation of (45) versus Ω and α*

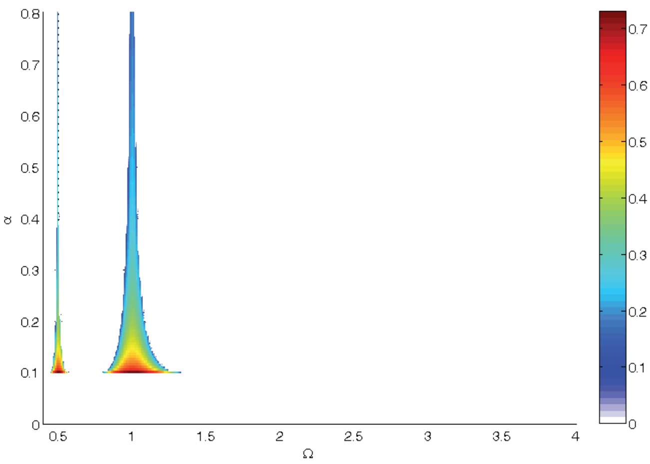


*Fig. 3. Analytical calculated amplitude $a_2$ of steady state solution represented by equation (46) versus Ω and α*

In the next section numerical simulations are done to show when the solution presented in this section can appear.

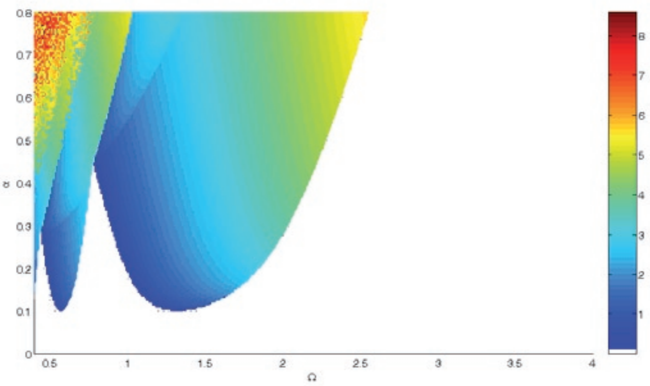


*Fig. 4. Colour map of amplitude versus Ω and α for initial condition x(0)=0.5*

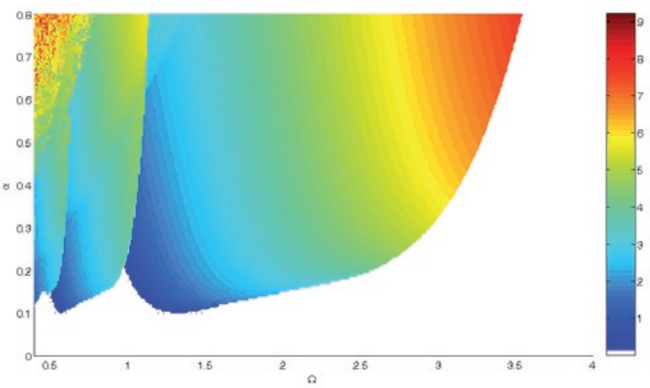


*Fig. 5. Colour map of amplitude versus Ω and α for initial condition x(0)=3.5*

## 4. Numerical results

Since, as it has been shown in the previous section, chatter vibrations can exist always when $\alpha$ crosses $\alpha_{cr}=\delta$. The most interesting, from practical point of view, are the initial conditions which favours getting high amplitude vibrations. Therefore, the numerical simulations are performed on the basis of DDE in Matlab-Simulink with the help of 4$^{th}$ order Rungge-Kutta procedure with variable integration time. The system parameters are fixed as follows: $\gamma$=0.25, $\delta$=0.1, $\omega_o$=1, $\mu$=1. The value of vibrations amplitude is presented in Figs. 4 and 5 as colour map on the of two parameters plot $\Omega=2\pi/\tau$ and $\alpha$.

The amplitudes of chatter vibrations are very sensitive on initial conditions because the region of unstable cutting is much wider for the initial condition x(0)=3.5 than x(0)=0.5. Moreover, vibrations amplitudes are bigger as well. Only $\alpha<\delta$ guarantees the cutting process without chatter vibrations regardless initial conditions.

## 5. Discussion and Final Conclusions

Since chatter vibrations are the main problem in cutting process therefore looking for regions of stable technological parameters is a primary goal. The linear model of regenerative cutting is well known and the derivation of its analytical solution does not introduce difficulties. But in the nonlinear case the system can have more than one periodic solution and also quasi-periodic, sub-harmonic or even chaotic ones. That depends, of course, on the system parameters and additionally on initial conditions. The analytical solutions shown graphically in Fig. 2 and 3 represent only steady state periodic solutions. Interestingly, for the analysed system there are no stable lobes, characteristic for the linear regenerative model. In the nonlinear model for any time delay chatter vibrations exist if delay amplitude $\alpha<\alpha_{cr}$. The numerical analysis is a complement of analytical research. Numerical investigations allow finding initial conditions regions which do not generate chatter vibrations. These regions are very important from practical point of view because the safe set of parameters ($\Omega$, $\alpha$) can be found providing the system stays in the proper initial conditions domains.

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