1. Introduction

During recent years, more and more attention has been paid to the acquisition of knowledge of the factors that cause critical situations in the motor vehicle traffic. To analyse the related issues, results of the testing of vehicles and vehicle models are taken as a basis. Double-deck buses constitute a separate category of motor vehicles; they are in growing demand because of their high passenger transportation capability. The number of double-deckers used in intercity traffic is rapidly increasing. Therefore, research becomes obviously needed to get to know, *inter alia*, the characteristic features of behaviour of such vehicles in critical traffic conditions. However, the possibilities of testing double-deckers in the conditions of high rollover risk are limited by low availability and high cost of such buses. In consequence, it is very difficult to find reliable data for modelling the pre-accident situations of such vehicles and for exploring the double-deck bus rollover process.

Among the factors that cause critical situations in the operation of motor vehicles, those worth mentioning at first are the rapidly performed manoeuvres in curvilinear motion, including the avoidance of an obstacle having suddenly sprung up [18]. The critical situations in curvilinear motion are usually analysed from the point of view of improving the simulation models of driver assistance systems for passenger cars, sport utility vehicles (SUV), and light transport vehicles (LTV), with a predefined vehicle trajectory [2, 5, 7, 15]. The processes of rollover of motor trucks and special vehicles are also examined by analysing the behaviour of scaled-down remotely controlled mobile physical models of such vehicles [14, 19]. The works concerning the analyses of accident hazards and improvement of bus passenger protection systems [6, 10, 11, 16] may be considered as a separate group. On the other hand, there is a lack of reports of exploring the processes where hazards are created during the curvilinear motion of double-deck buses and where the effects of such processes may provide recommendations for defining the acceptable
conditions of operation of such vehicles and for training bus drivers in respect of minimizing the risk of rollover of such vehicles in road traffic. From this point of view, a factor of significant importance is also the very low position of driver’s seat in double-deckers (below the floor of the lower deck), in result of which the driver cannot detect easily enough the developing symptoms of the imminent rollover risk.

Statistical data on accidents confirm that the most severe accidents (i.e. the events with the highest numbers of casualties) are those where bus rollover occurred [10]. A characteristic feature of such accidents is the fact that most of them took place without a collision with other vehicles or road obstacles [6]. For bus rollover accidents, the ratio of the number of deaths to the number of accidents is 1.5 as high as that for all the other types of bus accidents [6]; for the number of severely injured people, this ratio is 1.8.

The main purpose of the analysis presented in the subsequent part of this paper was to determine the values of the factors that characterize the critical situations of double-deckers in the road traffic. The risks that reveal themselves in critical situations were analysed. The paper includes calculation results, which make it possible to formulate recommendations important for the training of double-decker drivers and to define the acceptable conditions of operation of specific vehicle. The conclusions of this analysis are important for defining the limitations in the conditions of vehicle operation [14, 16, 18]. In the calculations carried out, a critical value of the speed is sought that might warn the driver about a pre-accident situation. The recording of the quantities thus selected will facilitate the analysis of road accidents [13]. The rollover of a motor vehicle is counted among the most complex (difficult for analysing) road accidents.

2. The risk of bus rollover

In practice, an indicator referred to as SSF (Static Stability Factor) [7] is used as the starting point for determining the risk of vehicle rollover. This indicator is calculated from vehicle design parameters:

\[
SSF = \frac{b_x}{2h_3}
\]

where \(b_x\) is the vehicle wheel track and \(h_3\) is the height of the centre of mass.

The calculated values of this indicator have been given in Table 1. The lower this indicator value, the higher the risk of vehicle rollover [18]. Hence, the rollover risk is very high for double-deckers, which is confirmed by statistical data on road accidents [1, 10, 11]. The estimated relation between the vehicle rollover risk and the SSF value is illustrated in Fig. 1. The rollover risk represents the ratio of the number of vehicle rollover accidents to the total number of accidents with vehicles of the category under consideration.

**Table 1. SSF values**

<table>
<thead>
<tr>
<th>Motor vehicle category</th>
<th>SSF value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger cars</td>
<td>1.2±1.55</td>
</tr>
<tr>
<td>SUV and LTV</td>
<td>1.0±1.2</td>
</tr>
<tr>
<td>Conventional buses</td>
<td>0.85±1.0</td>
</tr>
<tr>
<td>Double-deck buses</td>
<td>0.6±0.75</td>
</tr>
</tbody>
</table>

The risk was calculated with taking into account only the accidents where no other vehicles were involved. In Fig. 1, the heavy lines have been plotted on the grounds of analyses of road accidents with passenger cars (dashed line) and SUVs and LTVs taken together (solid line) [1, 7]. In paper [7], the results of analysis of the accidents where SUVs and LTVs were involved have been analytically represented in the form of a regression curve, plotted with the use of a logistic regression model, which makes it possible to calculate the SSF-based prediction of a rollover for vehicle during a road accident (referred to as “rollorate,” to be distinguished from the real-world rollover risk):

\[
Rollorate = \frac{1}{1 + e^{(c_1+c_2 \log(SSF-0.9))}}
\]

where \(c_1 = 2.7546\) and \(c_2 = 1.1814\).

The regression curve dependent on the SSF value (heavy solid red line) has also been shown in Fig. 1, where two fine lines have been added to represent the results of authors’ own extrapolation calculations made with the use of the data given in publications [1] and [7]. These curves have been characterized in Table 2.

**Table 2. Regression equations and estimates of the vehicle rollover risk indicator (RW)**

<table>
<thead>
<tr>
<th>Regression equation for the risk indicator</th>
<th>Coefficient of determination (R^2)</th>
<th>Source of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW = 1.045SSF^2-3.312SSF+2.702</td>
<td>0.998</td>
<td>[1], dashed line</td>
</tr>
<tr>
<td>RW = 14.989SSF^4-8.81.179SSF+164.875SSF^2-1.149 .18SSF+50.999</td>
<td>0.999</td>
<td>[7], solid line</td>
</tr>
</tbody>
</table>

When extrapolating from these data to the area of the SSF values that correspond to double-deck buses, one may draw a conclusion that the double-decker rollover risk is almost 3 times as high as the rollover risk for passenger cars. This clearly highlights the degree of rollover risk faced by the double-decker buses and justifies the necessity of investigation into this problem.

3. Preparation of a static model

Predominantly, vehicle rollover is caused by the excessive lateral forces (usually inertia forces) that occur when complex road manoeuvres are performed. The research works on this problem may be based on analytical calculations and computer simulations and they are chiefly aimed at:

- Determining the critical value of vehicle speed;
- Obtaining information about the vehicle operation factors that might indicate the emerging hazard to the driver even before the critical value of the bus body tilt angle is reached.
To make the calculations, a set of reliable data must be prepared that would characterize important design features of double-deck buses. However, the necessary data are difficult to be obtained. In this situation, a static model of the bus under consideration was built for the necessary values of the parameters describing the bus design to be determined [17]. The model was used for determining the coordinates of the centre of vehicle mass and the central moments of inertia of the bus body with and without passengers (and their luggage) with the best possible accuracy. The static model was built on the grounds of an analysis of the construction of double-deck buses. It consisted of a number of solids whose geometry and mass distribution corresponded to those of the major bus components. As the starting point, the available technical specifications of the Skyliner L, TD 925, and B9TL 6x2 buses were used [4, 21]. The general construction of the static model has been shown in Fig. 2.

The main results of the calculations made on the grounds of the model thus built have been given in Table 3.

4. Model of vibrations of the suspension system

A separate stage of the preliminary calculations was dedicated to the determining of the stiffness and damping coefficient values for the suspension system of individual vehicle axles. The calculations were carried out with the use of a model of vertical and angular lateral vibrations of a solid representing the vehicle body seated on three axles, where tandem axles 2 and 3 being close to each other were replaced with a single equivalent axle situated at a distance of L from the front axle of the bus (Fig. 3).

The results of these calculations (Table 5) were used as input data for the computations of the bus model motion at the avoidance of an obstacle having suddenly sprung up.

The fulfillment in practice of the ride comfort criterion is evident from e.g. the fact that the frequencies of free vertical and angular lateral vibrations of the bus body were brought to values within the range from 1 to 1.6 Hz. The description of detailed calculations within this scope has been omitted here.

5. Analytical calculations

In consideration of the unavailability of results of research on the behaviour of double-deck buses in critical situations in curvilinear motion, analytical calculations were carried out, where the previously determined characteristics of the static model were used. The objective was to estimate, for the vehicle moving along a road bend, the limit speed values where the exceeding of such a limit would result in the following critical situations:

- side-slip of driving axle wheels;
- wheel lift-off on one vehicle side;
- vehicle body tilt to the state of unstable equilibrium (Fig. 5b).

At analyses of this type, the instant of wheel lift-off is in most cases taken as the beginning of the process of vehicle rollover on the side. The system of the forces applied to the static model representing the bus driven along a road bend has been shown in Fig. 4.

The small impact of tyre deflection on the body tilt was ignored. The tilting bus body rotates around the axis of tilt, the trace of which (R) has been marked on the sketch in Fig. 4.

Based on the equation of equilibrium of the moments of forces acting on the bus body relative to tilt axis R, we have:

\[ F_{y} \cdot h_{y} + Q_{y} \cdot b = M_{spr} \]  \hspace{1cm} (3)

where \( M_{spr} \) = moment of the suspension spring forces, calculated from the following:

The arrangement of solids and seats in the static model [17]

Table 3. Values of the design parameters of the double-decker model with and without passengers (and their luggage) [17]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>14.00 m</td>
</tr>
<tr>
<td>Wheelbase, 1-2</td>
<td>6.90 m</td>
</tr>
<tr>
<td>Width</td>
<td>2.50 m</td>
</tr>
<tr>
<td>Bus without passengers, mass 17 275 kg:</td>
<td></td>
</tr>
<tr>
<td>Location of the centre of mass (cf. Fig. 3)</td>
<td></td>
</tr>
<tr>
<td>( X_{0} )</td>
<td>5.28 m</td>
</tr>
<tr>
<td>( Y_{0} )</td>
<td>0</td>
</tr>
<tr>
<td>( Z_{0} )</td>
<td>1.45 m</td>
</tr>
<tr>
<td>Moment of inertia of the bus body</td>
<td></td>
</tr>
<tr>
<td>( I_{x} )</td>
<td>35 000 kg-m²</td>
</tr>
<tr>
<td>( I_{y} )</td>
<td>298 000 kg-m²</td>
</tr>
<tr>
<td>( I_{z} )</td>
<td>288 000 kg-m²</td>
</tr>
<tr>
<td>Bus fully loaded with passengers, mass 25 000 kg</td>
<td></td>
</tr>
<tr>
<td>( X_{0} )</td>
<td>5.25 m</td>
</tr>
<tr>
<td>( Y_{0} )</td>
<td>0</td>
</tr>
<tr>
<td>( Z_{0} )</td>
<td>1.73 m</td>
</tr>
<tr>
<td>Moment of inertia of the bus body</td>
<td></td>
</tr>
<tr>
<td>( I_{x} )</td>
<td>49 000 kg-m²</td>
</tr>
<tr>
<td>( I_{y} )</td>
<td>411 000 kg-m²</td>
</tr>
<tr>
<td>( I_{z} )</td>
<td>396 000 kg-m²</td>
</tr>
</tbody>
</table>

Fig. 2. The arrangement of solids and seats in the static model [17]

Table 4. Starting-point data used to calculate the characteristics of the bus suspension system; symbol "W0" indicates the set of nominal data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus without passengers, mass 17 275 kg:</td>
<td></td>
</tr>
<tr>
<td>Static deflection of the suspension system</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Bus with passengers</td>
<td></td>
</tr>
<tr>
<td>Static deflection of the suspension system</td>
<td>0.19 m</td>
</tr>
</tbody>
</table>
where:

- $k_i, b_{si}$ – see Figs. 3 and 4;
- $k_{si}$ – stiffness coefficient of the anti-roll bar of the $i$th axle;
- $k_{\phi} = f(\phi)$ – angular stiffness of the suspension system at a body tilt angle of $\phi$.

At an assumption that:

$$s_{pr} \cong h_{prz} \phi,$$

the following was obtained after transformations:

$$\phi \cong - \frac{F_h k_Q}{h_{prz} Q_p},$$

The process of vehicle rollover under the impact of centrifugal force $F_Q$ is initiated by the fact that the values of normal reactions $Z_i$″ approach zero (Fig. 4). It follows from the equation of equilibrium of moments calculated in relation to point A in Fig. 4 that

$$F_h Q_s K_{prz} \phi = \frac{Q_g v R}{h_{prz} Q_p} = 0,$$

where

$$M_{spr} = \left[ 2 \sum_i \left( k_i \frac{h_{si}^2}{4} + k_{si} \right) \right] \phi = k_{\phi} \phi$$

The substitution of (5) and (6) to the above equation produces:

$$\frac{Q v^2}{g} h_{prz} \phi - Q k_{prz} \phi = 0,$$

where $F_Q = \frac{Q v^2}{g} R$.

Based on this, the maximum value of the vehicle speed $v \to v_{MAX}$ was determined, such that the exceeding of this limit would result in starting the process of vehicle rollover on the road bend:

$$v_{MAX} = \sqrt{\frac{b_K R}{2 h_{prz} (k_{\phi} - Q_{prz}) + 2 Q_{prz}^2}},$$

At the next step, the process of increase in the side tilt angle, leading to vehicle rollover (Fig. 5), was considered. In particular, the speed $v_{kr}$ (higher than $v_{MAX}$) was calculated, at which the vehicle would reach the position of unstable equilibrium (Fig. 5b).

The above equations were used for analytical calculations, the results of which make it possible to evaluate the functional characteristics of, and the risks to be encountered by, the vehicle in curvilinear motion. The calculations were started with determining the vehicle speed at which a side-slip of wheels of the driving axles (equivalent driving axle) would occur [16]:

$$v_M = \sqrt{\beta_N g R},$$

Table 5. Calculated characteristics of the bus suspension system in the “W0” state

<table>
<thead>
<tr>
<th>Suspension characteristic</th>
<th>Unit</th>
<th>Bus without passengers</th>
<th>Bus fully loaded with passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension stiffness coefficient, calculated for the position of static equilibrium, for the suspension of axles Nos. 1, 2, and 3, respectively</td>
<td>kN/m</td>
<td>339; 395; 395</td>
<td>387; 452; 452</td>
</tr>
<tr>
<td>Dimensionless critical damping ratio, gamma</td>
<td></td>
<td>0.44</td>
<td>0.36</td>
</tr>
<tr>
<td>Frequency of free vertical vibration of the bus body</td>
<td>rad/s</td>
<td>8.09</td>
<td>7.00</td>
</tr>
<tr>
<td>Frequency of angular lateral vibration of the bus body</td>
<td>rad/s</td>
<td>6.58</td>
<td>5.73</td>
</tr>
</tbody>
</table>

Fig. 4. Side tilt of the bus body supported by a spring suspension system under the influence of a side force applied to the centre of mass of the bus body.

Fig. 5. Vehicle positions during the rollover process: a – initial position; b – position of unstable equilibrium.

Now, the value of $v_{kr}$ was determined from the above after transformations as follows:

$$v_{kr} = \sqrt{\frac{2 R (r_{kR} - h_{kR}) + v_{MAX}^2}{b_K}}.$$
where \( R \) is the road bend radius, \( \gamma_i \) is the coefficient of extra load on the \( i \)th bus axle resulting from the driving force, and \( \gamma_2 \) is the specific driving force [16].

Results of \( v_{\text{MAX}} \) calculation according to equation (9) have been presented in Fig. 6. The determined \( v_{\text{MAX}} \) speed values indicate the beginning of the wheel lift-off process on the bus side where the wheel load is reduced in result of the action of the lateral force. These results have been compared with results of calculation of the vehicle speed \( v_{\text{MAX}} \) at which a side-slip of wheels of the driving axles would occur. The speed causing the side-slip was determined for two values of the coefficient of adhesion of bus tyres to the road surface, i.e. \( \mu = 0.5 \) and \( \mu = 0.8 \) (denoted by “VM 0.5” and “VM 0.8” in Fig. 6).

An important feature of the behaviour of double-deck buses can be clearly seen here: during motion along a road bend, the bus speed ranges where the side-slip and rollover processes begin coincide with each other. This is a meaningful difference in the behaviour of double-deckers in comparison with that of conventional buses, for which these speed ranges differ from each other (cf. Figs. 6 and 7). This difference in bus behaviour in a critical situation has a significant impact on the safety of ride, especially where the same drivers use different buses.

A comparison between the calculation results, a part of which has been presented in Fig. 6, indicates the following:

- The side-slip initiation speed \( v_{\text{MAX}} \) on road surfaces with the coefficient of adhesion of 0.5–0.6 is lower than the rollover initiation speed \( v_{\text{MAX}} \), which may “inform” the driver about the emerging hazard and warns against a critical situation in the movement of double-deck buses without and with passengers;
- The side-slip initiation speed on road surfaces with high values of the coefficient of adhesion (0.7–0.8) is not markedly lower than the rollover initiation speed \( v_{\text{MAX}} \), which will result in an extremely dangerous road situation when significant lateral forces occur.

\[ v_{\text{MAX}} = \frac{M_{\text{eff}}}{\gamma_2} \]

where \( M_{\text{eff}} = \text{weight distribution} \times \text{factors} \times \text{coefficient of adhesion} \). The speed at which the vehicle reaches the position of unstable equilibrium (Fig. 5b), calculated from equation (11), has been treated in the subsequent part of this paper as the critical speed for the motion along a road bend.

In Fig. 6, heavy lines AP1 and AP2 represent changes in the values of the critical speed of a double-deck bus vs. road bend radius. These values are definitely lower (by about 10 m/s) than those calculated for conventional buses (heavy lines AK1 and AK2).

The critical tilt angle was calculated on the grounds of an analysis of the tilting of the static bus model. The numerical values have been brought together in Table 7.

### Table 6. Characteristic values of bus speeds \( v_{\text{MAX}} \) and \( v_{\text{kr}} \) [m/s] on a road bend

<table>
<thead>
<tr>
<th>Description</th>
<th>Value for R = 100 m</th>
<th>Value for R = 200 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional bus, wheel lift-off speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• without passengers</td>
<td>30.9</td>
<td>28.4</td>
</tr>
<tr>
<td>• fully loaded with passengers</td>
<td>30.9</td>
<td>28.4</td>
</tr>
<tr>
<td>Double-deck bus, wheel lift-off speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• without passengers</td>
<td>26.9</td>
<td>24.9</td>
</tr>
<tr>
<td>• fully loaded with passengers</td>
<td>26.9</td>
<td>24.9</td>
</tr>
<tr>
<td>Conventional bus, critical rollover speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• without passengers</td>
<td>36.8</td>
<td>34.1</td>
</tr>
<tr>
<td>• fully loaded with passengers</td>
<td>36.8</td>
<td>34.1</td>
</tr>
<tr>
<td>Double-deck bus, critical rollover speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• without passengers</td>
<td>32.4</td>
<td>30.0</td>
</tr>
<tr>
<td>• fully loaded with passengers</td>
<td>32.4</td>
<td>30.0</td>
</tr>
</tbody>
</table>

### Table 7. Critical tilt angle values for the buses under consideration

<table>
<thead>
<tr>
<th>Bus type</th>
<th>Height of the centre of mass [m]</th>
<th>Critical tilt angle [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>1.1±1.3</td>
<td>38±45</td>
</tr>
<tr>
<td>Double-deck</td>
<td>1.4±1.7</td>
<td>31±38</td>
</tr>
</tbody>
</table>

6. Road situation. Modelling of the driver

In result of growing traffic intensity and drive speeds, the manoeuvre of avoiding an obstacle that has suddenly sprung up becomes increasingly difficult for being safely performed. This problem related to the traffic of passenger cars has been investigated in the work described in paper [5], where it has been shown that the manoeuvre of avoiding an obstacle may be more advantageous than braking. During the manoeuvre of avoiding an obstacle, the vehicle follows a trajectory consisting of curvilinear sections with varying radii of curvature. For the manoeuvre to be performed, a driver model was applied which automatically changed the turning angle of the steered wheels.
The driver model adopted was designed to follow a present bus trajectory. The experiments and measurements carried out within the work described in [3] made it possible to estimate the range of changes in the parameters characterizing the driver’s actions during the manoeuvre of avoiding an obstacle. For this purpose, a PID driver model, i.e. a driver model with a proportional–integral–derivative controller, was adopted, where the maximum turning angles of the steering wheel and the steered axle wheels were up to 500 deg and 25 deg, respectively, and they could be achieved within a time of 1 s.

The output of the driver model was applied as an input to the bus model.

7. Model of bus motion dynamics

The work was focused on vehicle movement on a flat horizontal section of a two-lane road, with no cross slope, 7 m wide, and with hardened road shoulders. The plan shown in Fig. 8 illustrates the following road situation:

- An obstacle has suddenly sprung up on the bus lane;
- At the instant of the obstacle being noticed by the driver, the distance between the bus and the obstacle is shorter than the minimum achievable bus stopping distance;
- The bus may only enter the adjacent lane for a while shorter than 3–4 s.

The road situation plan presented in Fig. 8 shows an outline of the trajectory of the bus centre of mass, where A = 30 m. This trajectory is represented by the red line in Fig. 11. The reference frames adopted when building the model of bus motion dynamics have been shown in Fig. 9.

In the model, the bus superstructure was treated as a rigid body with six degrees of freedom, where the movements of each wheel (along the axes parallel to the vertical axis of the bus body) in result of the spring action of the suspension system were taken into account separately. Another six degrees of freedom of the model are related to the rotation of each of the bus wheels taken separately.

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The equations of motion of the bus body solid have the form as follows:

\[
\begin{align*}
\dot{m}x &= \sum_{i=1}^{n} F_{xi} \\
\dot{m}y &= \sum_{i=1}^{n} F_{yi} \\
\dot{m}z &= \sum_{i=1}^{n} F_{zi} \\
I_x \dot{\alpha}_x + I_x \dot{\omega}_x \dot{\omega}_x - I_x \dot{\omega}_x \dot{\omega}_x &= \sum_{j=1}^{k} M_{xj}' \\
I_y \dot{\alpha}_y + I_y \dot{\omega}_y \dot{\omega}_y - I_y \dot{\omega}_y \dot{\omega}_y &= \sum_{j=1}^{k} M_{yj}' \\
I_z \dot{\alpha}_z + I_z \dot{\omega}_z \dot{\omega}_z - I_z \dot{\omega}_z \dot{\omega}_z &= \sum_{j=1}^{k} M_{zj}'
\end{align*}
\]

where:

- the symbol “prime” (‘) has the meaning that the physical quantity involved has been referred to the local coordinate system \{Sxyz\};
- \( m \) – vehicle mass;
- \( \vec{p} \) – vector from the origin of the inertial coordinate system to the centre of vehicle mass, with vector coordinates (x, y, z) being expressed in terms of the reference frame \{Oxyz\};
- \( I_i \) – moments of inertia of the bus body relative to the centre of mass, expressed in terms of the reference frame \{Sxyz\}, i.e. \( I_x, I_y, I_z \);
- \( F_{xj} \), \( F_{yj} \), \( F_{zj} \) – external forces and moments, as appropriate, acting on the bus body;
- \( \dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z \) – projections of the vector of angular speed of the bus body in terms of the reference frame \{Sxyz\};

The bus behaviour at the input applied by the driver model depends on the interaction between the tyres and the road surface [8].

The description of the forces acting on the tyres has been based on a semi-empirical tyre model named TMeasy [20], which makes it possible to approximate the actual forces and moments generated by the tyre based on experimentally determined slip characteristics of the pneumatic tyre. In the TMeasy model, the characteristics of longitudinal reactions \( X_k \) vs. longitudinal slip and of lateral reactions \( Y_k \) vs. lateral slip are taken into account. In this work, the said characteristics were determined with the use of 255/70 R22.5 tyre test results [9].

The values of the road surface reactions acting on each of the wheels were determined at each calculation step in the \{Sxyz\} coordinate system, in accordance with the current state of the motion and with the local coefficient of adhesion being taken into account, i.e.
where $X_K$ and $Y_K$ are the tangent reactions, $\mu$ is the coefficient of adhesion, and $Z_K$ is the normal reaction at the contact between the tyre and the road surface (Fig. 9; $Z_K$ is there perpendicular to the tyre drawing plane).

The analysis of the bus behaviour when an obstacle having suddenly sprung up is rapidly avoided is based on simulation calculations carried out with the use of the PC Crash 9.0 computer software. This program makes it possible not only to simulate the movement of a bus on wheels but also to simulate the vehicle rollover process and the movement of the bus sliding on its side or roof [11, 20].

The sequence of photographs presented in Fig. 10 shows that the bus rollover process is preceded by significant side-slip of the rear axle wheels (shadowed tyre traces) and increasing tilt of the bus body. The calculation results have been shown where the selected characteristics of the suspension system and the tyres were treated as nominal (W0).

8. Main calculations. The impact of drive speed on bus behaviour at the avoidance of an obstacle

Simulation calculations were carried out for the bus moving without passengers (AP1) and with full load of passengers (AP2), with a constant speed, which was raised at successive simulations to a level at which the bus was found to roll over. Figs. 11 through 16 (in the part denoted by “A”) show examples of a few characteristic graphs of changes in the physical quantities describing the bus behaviour during the avoidance of an obstacle, namely:

- Trajectory of the centre of mass;
- Turning angle of the steered wheels;
- Bus body tilt angle;
- Lateral (side) acceleration of the centre of mass;
- Bus wheel pressure on the road;
- Slip angle of bus wheels (tires).

An analysis of changes in these quantities vs. time or distance travelled provided grounds for defining the parameters that might characterize the bus behaviour during the avoidance of an obstacle and simultaneously be a basis for determining the critical drive speed value. In each of the Figs. 11 through 16, graphs of changes in the said characteristic parameters as functions of bus speed have been added in the form of part B of individual drawings. As the characteristic parameters, the extreme values of the following quantities were chosen:

- In Fig. 11B, the maximum values of deviations of the actual trajectory of the centre of mass of the bus from the present bus trajectory (denoted by “d”);
- In Fig. 12B, the maximum values of the turning angle of the steered wheels in the initial phase of the manoeuvre of avoiding an obstacle (observed in the period from 0.1 to 0.4 s from the start of carrying out the manoeuvre and denoted by “$\delta_M$”);
Fig. 12. Examples of changes in the steered wheel turning angle vs. time during the avoidance of an obstacle (A) and a graph showing the maximum values of this angle ($\alpha_M$) vs. bus speed (B).

Fig. 13. Example curves representing the bus body tilt angle vs. time during the avoidance of an obstacle (A) and a graph showing the extreme values of this angle ($\beta_M$) vs. bus speed (B).

Fig. 14. Example curves representing the lateral acceleration of the bus body centre of mass vs. distance travelled during the avoidance of an obstacle (A) and a graph showing the extreme values of this acceleration ($a_B$) vs. bus speed (B).
Fig. 11 and 12 show graphs of the calculated quantities vs. time or the distance travelled. Since the driver model kept a preset value of the bus speed for the whole manoeuvre time, the relation between these arguments (time and distance) was constant and unequivocal. Actually, time was chosen as the argument where the frequencies of vibrations of the vehicle body seated on the suspension system might substantially affect the function output.

The calculation results presented in Figs. 11 through 16 show the substantial impact of changes in the steered wheel turning angle (i.e. reactions of the driver model) on, first of all, bus body tilt angle and lateral acceleration. In this input, the presence of components with frequencies of 0.8 to 1.2 Hz can be noticed for a few periods (see Fig. 12). This input frequency is close to the natural frequency of lateral vibrations of the bus body (cf. Table 5), which has an additional and adverse impact on the lateral tilt of the bus body during the avoidance of an obstacle. The proximity of the resonance frequency of bus body vibration is also indicated by big changes in the values of normal wheel pressure on the road surface (cf. the simulation results shown in Fig. 15).

9. Recapitulation

It is not easy to define a direct relationship between the simulation results and the analytical calculations and some comments must be added here regarding this issue:

- The analytical calculations are only applicable to steady motion with fixed control along a road bend with constant radius;
- The process of avoiding an obstacle is a motion with dynamically changing radius of the vehicle trajectory curvature;
- The results of simulation of an obstacle-avoiding manoeuvre were obtained with taking into account the continuous reaction of the driver model to deviations of the actual vehicle trajectory from that having been present;
- The manoeuvre under consideration is so complex that it shows the favourable and unfavourable effects of the dynamic processes taking place (including those resulting from strong reactions of the driver model), in particular the processes of tyre slip, side-slip of driving axle wheels, and bus body tilt up to the bus rollover.
An analysis of changes in the characteristic parameters chosen for analysing the behaviour of a double-deck bus has led to the following findings (see Tables 8 and 9):

- The extreme values of the slip angle $\delta$ for the equivalent driving axle wheels and of the lateral acceleration $a_y$ of the bus body centre of mass grew with increasing bus speeds;
- The extreme values $d$ of deviations of the actual bus trajectory from the present one began to rapidly increase at a speed of 106÷108 km/h for the bus without passengers and of 81÷82 km/h for the bus fully loaded with passengers;
- The distance travelled by the bus driven with a speed close to its critical value from the instant when bus wheels were lifted off to the instant when the bus body tilt angle reached its critical value was 12 to 15 m and the time of travelling this distance was 0.4 to 0.6 s, depending on the bus speed;
- The rollover process was preceded by rear wheels lift-off on one bus side, which already occurred at a speed lower by 6÷12 km/h than the critical rollover speed.

In the analytical calculations carried out for the bus motion along a road bend, the rollover process was also found to be preceded by wheel lift-off on one bus side, which occurred at a speed lower by 10÷15 km/h than the critical rollover speed (Table 9).

Table 8. Summary of the characteristic values of bus speed [km/h], determined from the computer simulation of the obstacle-avoiding manoeuvre

<table>
<thead>
<tr>
<th>The minimum bus speed at which:</th>
<th>Bus without passengers</th>
<th>Bus fully loaded with passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel lift-off took place [km/h]</td>
<td>102÷104</td>
<td>74÷76</td>
</tr>
<tr>
<td>Bus rollover took place [km/h]</td>
<td>109÷110</td>
<td>82÷84</td>
</tr>
<tr>
<td>The actual bus trajectory deviated from the present one by more than 1 m [km/h]</td>
<td>106÷108</td>
<td>81÷82</td>
</tr>
<tr>
<td>The extreme value of the slip angle for the equivalent driving axle wheels exceeded 6 deg [km/h]</td>
<td>93÷95</td>
<td>74÷76</td>
</tr>
<tr>
<td>The extreme value of the bus body tilt angle exceeded 30 deg [km/h]</td>
<td>108÷109</td>
<td>82÷84</td>
</tr>
<tr>
<td>The lateral acceleration of the bus body centre of mass exceeded 6 m/s² [km/h]</td>
<td>104÷106</td>
<td>80÷81</td>
</tr>
</tbody>
</table>

The calculations carried out have revealed the following as regards the features and values that characterize the bus behaviour:

- The critical bus rollover speed is 82÷110 km/h, depending on the number and arrangement of passengers (the lowest and the highest value of this speed is applicable to the bus fully loaded with passengers and to the bus without passengers, respectively). The imminent rollover is “signalled” by significant tyre slip, which simultaneously results in rapidly increasing deviation of the bus trajectory from that intended by the driver and both of these factors clearly warn the driver against the increasing hazard;
- The time elapsing from the instant of wheel lift-off to the achieving of the critical bus tilt angle value is 0.4÷0.6 s at the highest bus speeds.

A part of this work was done within research project No. N N509 502438 “Analysis of the possibility of modifying the superstructure of a double-deck coach,” and N N509 554440 “Analysis of the possibility of minimizing dynamic loads and injuries for adults and children on a bus during a road accident” where Andrzej Muszyński, D. Eng. (PIMOT, Warsaw) was the Project Manager.

References


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