1. Introduction

Effective performance of logistics systems/networks and supply chains requires a proper definition of e.g. time relations which occur between the system’s facilities and its processes. This issue has gained particular importance over the last 30 years. On the one hand, this is connected with the increased awareness of managers regarding the need to control the operating costs of technical systems and the logistics systems that support them. On the other hand, the increased availability of methods and tools to support the modelling process provides the opportunity to study and solve new problems within the analysed scientific area [87].

The problem of time delays is typical for many physical or technical systems, and has been raised e.g. in biology, mechanics and economics [38]. Time in logistics systems is traditionally seen with respect to [10]:

Symbols used in the paper

- $c_b$ – expected cost of repair
- $c_{con}$ – the expected consequence costs for the supported system and its environment caused by failure occurrence in logistics system
- $c_i$ – cost of inspection action performance
- $c_{ir}$ – expected repair costs during the implementation of inspections
- $C(T)$ – function of the expected maintenance costs of a system
- $C_{ir}(T)$ – function of the expected consequences costs for the supported system and its environment
- $C_{opt}$ – the total expected maintenance costs of a system in one inspection cycle
- $d$ – time of inspection action performance
- $d_b$ – random time of system downtime caused by its failure
- $E(T)$ – function of system expected downtime
- $E(h)$ – expected value of delay time
- $F(T)$ – cumulative distribution function of the time between failures of a system
- $f(T)$ – probability density function of the time between failures of a system
- $f_1(T)$ – probability density function of the time to the first failure
- $F_{opt}(h)$ – cumulative distribution function of the random variable $h$
- $f_{opt}(h)$ – probability density function of the random variable $h$
- $h$ – random variable for delay time
- $k$ – constant intensity of system failures occurrence
- $MTBF$ – Mean Time Between Failures
- $MTTR$ – Mean Time To Repair
- $P_a(T)$ – function for system downstate probability
- $R(T)$ – system reliability function
- $T$ – the time between system consecutive inspection actions performance
- $T_{opt}$ – the optimal time between system consecutive inspection actions performance

Keywords: delay-time concept, maintenance, logistic system.

W artykule przedstawiono zagadnienia związane z modelowaniem utrzymania systemów logistycznych w stanie zdatności z wykorzystaniem koncepcji opóźnień czasowych. Przedstawiono przegląd literatury z badanego obszaru obejmujący okres 1984-2012. Następnie został omówiony algorytm postępowania w procesie implementacji koncepcji opóźnień czasowych w obszarze utrzymania w stanie zdatności systemów logistycznych. W ostatnim punkcie, został przedstawiony przykład zastosowania opracowanej metodyki do oceny niezawodności i oczekiwanych kosztów obsługiwania dziesięciu wózków widłowych funkcjonujących w wybranym systemie.

Słowa kluczowe: koncepcja opóźnień czasowych, utrzymanie zdatności, system logistyczny.
• execution time of (internal and external) placed customer’s order - specifying the time between the moment the order has been placed by the customer and its completion. In this case, the concept of time is relative to the level of supported enterprise performance as perceived by the customer,
• resource efficiency in the basic processes of the supported enterprise. For instance, in the case of manufacturing systems, the concept of time is directly related to minimizing downtime and optimizing capacity utilization,
• duration of various fundamental processes, covering the period from the time when all resources necessary to perform the process are ready for use, until the moment the result of the process has been achieved. The concept of time directly depends on the thorough identification of the process structure.

Therefore, accurate determination of the time relations in the logistic system will depend e.g. on the type of the modelled system (storage, transport, etc.), the type of operating task, the definition of the efficiency type that is essential in the system (e.g. whether one is interested in the system’s efficiency within a certain time horizon), and the behaviour of the system during its downtime (whether a single failure should affect the reliability of the whole system). The basic classification of system models with a time resource, taking into account the above features, has been presented e.g. in [88, 89].

The article focuses on the issues associated with the maintenance modelling of logistics systems using Delay Time Analysis approach, with particular emphasis on the methodology applying the concept of delay time in practice. The second part of the article focuses on the analysis of an internal transport system of a manufacturing company in the metallurgical sector. The proposed methodology was used to assess the reliability and the expected maintenance costs of the operating forklifts.

2. Modelling of technical and logistical systems maintenance processes using delay time analysis

The papers published over the last 50 years cover a wide range of problems in the area of modelling and design of maintenance processes in technical facilities (maintenance theory). Issues often discussed in the literature regard the areas of reliability modelling or necessary supplies providing for operational processes performance of e.g. production and transportation systems (e.g. works [6, 46, 47, 65, 72, 75, 76]), or the uncertainty of operating data (e.g. works [7, 57, 93]). A basic overview of these research works was presented in one of the first articles – [62], and later developed and extended by the authors Valdez-Flores & Feldman in [74]. The importance of the tasks related to the analysis of the research area as well as its diversity is supported by many studies (e.g. [18, 52, 53, 54, 55, 61, 69, 77]), where the authors provide an overview of basic models of selecting an optimal maintenance strategy for both single- and multi-element systems. At the same time, papers focused on the issue of maintaining single-element systems have been analysed e.g. in [5, 36, 69, 77], and those focused on the optimization of operation processes of multi-element systems are given e.g. in [18, 34, 55, 59, 67, 71].

One of the basic tasks associated with technical systems maintenance involves inspection process implementation [18, 62]. In this area it is possible to use the delay-time concept, enabling the modeling of the consequences of the Inspection Policy implementation for technical facilities performance [23].

The presented approach, used to this day in the renewal processes theory in order to optimize the technical systems downtime caused by undetected failure (optimization of time period between consecutive inspection actions performance), has been the subject of analysis since the 1970s, e.g. by authors Chrisler, Waller and Whiteley (e.g. [22, 23, 28, 29, 30, 32]). In this concept, it is assumed that the system/component failure does not occur suddenly, but is preceded by certain symptoms indicative of future damage [24]. The time between the moment when the first detectable signals of the forthcoming failure appear, and the moment the system is failed is called the time delay and is denoted by $\theta$ [21, 24]. This issue has been investigated e.g. in the papers [43, 44, 58].

Known in the literature delay-time models can be classified into two main groups [81]:
• models for single-element systems and models for complex systems, treated as a single technical facility,
• models for multi-element systems.

This topic has been widely studied in literature, e.g. in [2, 8, 19, 20, 21, 24, 25, 48, 58, 64, 77, 78], where reviews of the literature in the area of applying the delay time concept are given, and in [43, 44, 58, 86], where attention was paid to the possibilities of using of delay time models in multi-element systems performance.

One of the first papers focused on modelling the time delay in single-element systems is [27]. In this paper, the author analyses the decision-making process of system maintenance (item replacement) based on a survey research implementation. Furthermore, in this area the authors analysed e.g. systems reliability issues (e.g. papers [4, 16, 22, 41, 94]), time delay parameter estimation problems (e.g. paper [9]), risk analyses (e.g. [82]), the implementation of semi-Markov processes (e.g. [31]), or system safety issues (e.g. [84]).

The basic delay time model for multi-element systems was presented e.g. in the papers [24, 29, 30, 82]. The basic assumptions of the model include a perfect inspection performance, independence of the operating system components, Poisson process of defects occurrence in the system, known $f_{ih}(h)$ function, and a fixed period of inspection process $d$. For such assumptions, it is possible to determine the system failure probability $P_{dh}(T)$, or a function of the expected system downtime $E_d(T)$.

Development of the mentioned model includes e.g. the assumption of a non-perfect inspection performance (e.g. [14, 21, 79, 80]) or a non-Poisson process of defect appearance in the system (e.g. [3]). Moreover, the problem of model parameters estimation has been presented (e.g. [21, 80, 81, 85]).

3. Methodology of delay time concept use in the modelling of technical and logistics systems maintenance processes performance

Literature provides many papers devoted to the application of the concept of time delays in the real-life systems performance. The main areas of model application include e.g. the production systems performance (e.g. [1, 45]), failure processes of gearboxes (e.g. [49]), modeling operation processes of: vehicle fleets (e.g. [28, 35, 40, 68]), fishing vessels (e.g. [63]) or medical equipment (e.g. [26]). An important area of potential application of the presented approach is the logistics systems performance.

The basic functions of logistics include effective and efficient management of: the flow of resources and storage of goods (raw materials, semi-finished and finished goods) and services, from their sources of origin to the place of consumption, and the information associated with the material flow in order to meet the needs and requirements of customers [91]. Therefore, in the area of business logistics, efficiency of logistics processes of actors participating in the logistics chain is based on maintaining a balance between: the increasing demand for basic and support resources, the shorter duration of the logistic tasks, and control of the costs throughout the entire chain [11, 90, 91], based on the three main logistics pillars given e.g. in [56].

At the same time, the problems of logistics can also be defined in the area of operation of technical systems [88]. Based on the basic literature on logistics engineering (e.g. [12, 13, 17]), using a systemic approach in logistics, there can be defined the logistical support sys-
tem, which according to [12, 17] is defined as *purposely organized technical system's subsystem to support its primary (operational) process performance through the integration of all activities associated with the effective and beneficial flows of material resources and the necessary information*, and providing necessary for this process logistics resources (supporting equipment and control and measurement equipment). The presented definition refers on the one hand, to the life cycle of the system, on the other, includes both business logistics and military logistics characteristics.

The logistical considerations can be divided into two basic concepts of logistic support system reliability: upstate and its downstate, which can lead to e.g. [50]:

- disrupting or even preventing the execution of the current logistic task,
- inability to undertake new logistic tasks.

Taking into account the downstate of the support system, there should be defined a new perspective of the efficient and effective performance of the supported system, which requires analysing the logistics system inability for specific tasks performance, under certain conditions and at certain times, when a logistic request is randomly occurred in the system.

The available literature on reliability theory provides a number of papers on the subject of modelling and assessing the logistics systems performance, designed to support technical facilities that are subject to maintenance processes implementation. The developed models, however, are primarily limited to the analysis of the supply process providing the technical system with the necessary spare parts (e.g. [15]), taking into account the problem of estimating the necessary number of repairmen and multi-echelon issues (e.g. [37, 66, 70]), without investigating the impact of the operation of other logistics elements on the supported system’s reliability, dependability or availability.

Other aspects of the analysed research area, which require further analysis, regard to e.g. the area of assessment of the supply sources, inventory management problems, consideration of the storage limitations, and integration of logistic tasks with the objectives of the technical facilities maintenance strategy [51].

At the same time, over the last twenty years there has been observable an increase in interest in the issues of time management and analysis of the time relations observed in technical systems/facilities, including those involved in logistics. A review of the fundamental research issues related to the modelling of time relations in logistic chains are presented e.g. in [60].

Following these considerations and taking into account the complexity of the logistic support systems, proper modelling of their operation can be based on the delay time concept implementation. In this case, effective development of a logistics system performance model requires using appropriate methodology [45].

This problem is analysed e.g. in the paper [63], which presents an algorithm of applying the delay time concept in the operation of complex systems. Then, in [83], the authors proposed a modelling algorithm for inspection processes in multi-element systems performance, where the process of defects occurrence is of the NHPP type.

Methodology of modelling time delays in production systems was discussed in [45]. The authors proposed an algorithm of maintenance optimization for the case of complex systems, which supplements the estimation of expected maintenance costs and model of expected period of downstate with the so-called model for assessing the consequences of failure (*environmental model*), which defines the effect of a malfunction of the production machine in a manufacturing company (e.g. production downtime).

The methodology of modelling the technical systems maintenance processes including the delay time concept when using the Monte Carlo simulation techniques were developed in [33]. The authors have proposed algorithms for the case of perfect/imperfect inspection and for the case of imperfect repair.

While developing the methodology for modelling the processes of technical systems maintenance, taking into account the delay time concept, it should be noted that the purpose of the model is usually to minimize the expected duration of system downtime $E_d(T)$, or the expected maintenance costs $C_d(T)$. At the same time, in the case of logistics systems performance, the impact of their components failures on the level of supported system performance and its environment is also important. The level of this impact can be expressed by the function of the expected logistic support system failure consequences costs $C_e(T)$. On the other hand, further considerations omit the possibility of performing a multi-criteria analysis.

The algorithm for the determination of the optimal time between the operation actions performance $T$ includes the following steps (Fig. 1) [33, 45, 83]:

- understanding the process of maintaining the selected system (e.g. determining the type of the involved maintenance operation, the relationship between the system’s components),
- identifying the problems in the selected system operational process (e.g. long-term repair operations, frequent damage),
- preliminary definition of model assumptions (e.g. single-/ multi-element system), specifying the type of operational and diagnostic data that can and should be collected in order to carry out the maintenance optimization,
- data collection and analysis – based on using subjective and objective methods,
- determining the assumptions of the model – based on the data and knowledge about the performed operational processes,
- estimating the basic parameters of the model – for example, functions $f_i(h)$ and $F_i(h)$ for the time delay parameter $h$, selecting the probability distribution for the time between failures or the repair time,
- defining optimization criteria – depending on the available operational data and model assumptions,
- estimating the function of the optimization criterion, designating the optimal period $T_{opt}$,
- determining the relationship between the expected value of the time delay parameter $h$ and the optimal period $T_{opt}$ – in order to determine whether the delay time concept implementation allows for optimum solution obtainment (more info e.g. in [43, 44]).

![Fig. 1. The algorithm for assessment of optimal period T using the delay time concept. Own contribution based on [33, 45, 83]](image-url)
One of the important issues is the method of determining the model’s criterion functions. The form of the function will depend on the assumptions of the system. For example, in the case of a complex system, one of the basic models of the periodic inspection, based on the use of the delay time concept is the model of technical object periodic inspection presented in [30]. The basic assumptions of this model include:

a) inspections are held at a fixed time \( T \) and last \( d \) time units,
b) the cost of inspection action performance is \( c_i \) units,
c) inspection operations are perfect, which means that all defects that can be detected during the inspection are identified,
d) inspections are carried out independently of each other,
e) failures are independent and occur in the system with a constant intensity \( k \),
f) all defects detected during the inspection operation are removed during the inspection process,
g) system failure lasts a short period of time \( d_a \) compared to periods \( T \) and \( d \),
h) the time delay \( h \) is independent of the failure intensity and has the known form of the functions \( f_i(h) \) and \( F_i(h) \).

With taking into account the assumptions defined as above, the function for system downtime probability \( P_a(T) \) is given by [30]:

\[
P_a(T) = \frac{1}{T} \int_0^T (T - k) f_i(h) dh
\]  

(1)

The expected downtime, defined by the function \( E_d(T) \), can be described as follows [30]:

\[
E_d(T) = \frac{kTdT_p(T) + d}{T + d}
\]  

(2)

At the same time, assuming the expected cost of repair \( c_b \), the expected repair costs during the inspection actions performance \( c_r \), the expected maintenance cost within the period \( T \) can be described by the function \( C(T) \) in the form [30, 45]:

\[
C(T) = \frac{1}{T + d} \left\{ kT \left[ c_i P_b(T) + c_r (1 - P_b(T)) \right] + c_i \right\}
\]  

(3)

Moreover, the expected consequences costs for the supported system and its environment caused by failure occurrence in logistics system can be presented as follows:

\[
C_c(T) = \frac{1}{T + d} \left\{ kT_{con} P_b(T) \right\}
\]  

(4)

where: \( c_{con} \) – the expected consequence costs for the supported system and its environment caused by failure occurrence in logistics system (e.g. related to the loss of life, degree of damage to the supported system, the delay in the implementation of fundamental processes, etc.)

Simulation model for multi-element systems performance with taking into account delay time concept implementation was developed e.g. in the papers [42, 43, 44].

4. Case study

In order to demonstrate the applicability of the discussed methodology for using the concept of time delays in logistics system performance, an analysis of a manufacturing company (non-ferrous metal smelter) was carried out. The study focused on the process of operation of forklift trucks, which are used in the manufacturing plant.

The analysed forklift trucks primarily support manufacturing processes performance, take deliveries of materials and support warehouses, where goods are stored after leaving the production line. The most important workplace for forklifts is the production line. A forklift is used in the smelter batch storehouse next to the elevator that supplies the furnace. The operator’s task is to take the batch with a forklift from the ramp or the yard (according to the production requirements), insert it into the elevator, which takes the batch to the smelter. The work here is done continuously and remains uninterrupted under a three-shift work. Around 350 tons of feed material is melted during one shift. Two tracks directed to support the production work about 14 mth per one shift. The working conditions are variable, as in the case of taking deliveries. Additionally, forklifts which support production often dispose of liquid material at high temperatures, reaching up to 900°C. In such instances, the temperature inside the cabin rises rapidly to about 65°C.

Another place of forklift work is the unloading ramp, where it takes batches of material delivered by rail and the yard, where supplies delivered by car fleet are stored. 4-6 forklifts are simultaneously deployed at the railway platform to unload wagons with batches for production, which include 5-24 metal sheets weighing about 1800 kg each. The daily delivery by rail consists of 19 wagons.

An equivalent workplace for operators working on the railway platform is the yard, where they unload trucks loaded with the same batches as railway wagons. This is where batch weighing about 2500 kg each are delivered. About 300 tons are delivered daily. Work in these areas takes place in variable and severe weather conditions. Forklifts must deal with huge overloads associated with heat, snow and frequent glaze when wheels lips are frequent. The plant is usually dusty. The situation is similar on the loading ramp, which is why forklifts have a triple filtration system and are inspected more frequently.

Apart from the main places of the work, which involve handling production and receiving supplies, forklifts carry out a number of other tasks related to the maintenance of other departments of the plant. For example, their work involves daily supply of departments with technical gases, salts and chemicals, technical materials delivered from storage, necessary for maintaining production, supplying the packing departments with steel tapes and stretch tapes, which are used to pack finished goods.

The data necessary for the reliability analysis performance include the operational and maintenance data of ten electric forklifts of the selected brand, covering the period of operation from January 2000 to February 2013. These data contain the exact information about operational performance of forklifts, durations of repairs and planned maintenance actions, identification of the items replaced and all defects that occurred during that time.

4.1. The operational process of forklifts and analysis of operational data

The routes of forklift movement are variable and depend in particular on the tasks that the forklift performs. The most frequent and regular operations include loading the furnace – production support and reception of supplies. After a short briefing at the beginning of each shift, the operator lists the equipment they collect in the so-called forklift work report. In this report, the operator makes the following entry (operator entry based on the example of forklift truck no. 4 dated...
The study of the 10 trucks allowed for the determination of e.g. the time to first failure. The probability distribution of working time to the first failure can be described by the Weibull distribution (Fig. 4). It was also possible to determine the reliability function $R(t)$, Cumulative distribution function $F(t)$, or the probability density function of time to the truck’s failure $f(t)$. The data analysis was based on the program Weibull++ v. 6 (ReliaSoft Co., USA) implementation, which allowed e.g. choosing the probability distribution of the random variable for the lifetime. It is a log-normal distribution (Fig. 5).

In the analysed time period, there were 748 defects observable in the systems of the frame and carriage body, electrical installations and equipment, hydraulic system, as well as lifting, drive, steering, and braking systems.

Table 1. Operational parameters of analysed forklifts

<table>
<thead>
<tr>
<th>Forklift No.</th>
<th>Mileage (mth)</th>
<th>Number of defects</th>
<th>MTBF (h)</th>
<th>MTTR (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3685</td>
<td>97</td>
<td>38.0</td>
<td>5.6</td>
</tr>
<tr>
<td>2</td>
<td>3636</td>
<td>82</td>
<td>44.3</td>
<td>6.5</td>
</tr>
<tr>
<td>3</td>
<td>5513</td>
<td>100</td>
<td>55.1</td>
<td>6.3</td>
</tr>
<tr>
<td>4</td>
<td>7310</td>
<td>107</td>
<td>68.3</td>
<td>6.4</td>
</tr>
<tr>
<td>5</td>
<td>1876</td>
<td>26</td>
<td>72.2</td>
<td>5.5</td>
</tr>
<tr>
<td>6</td>
<td>3624</td>
<td>79</td>
<td>45.9</td>
<td>6.6</td>
</tr>
<tr>
<td>7</td>
<td>2684</td>
<td>51</td>
<td>52.6</td>
<td>7.3</td>
</tr>
<tr>
<td>8</td>
<td>4374</td>
<td>89</td>
<td>49.1</td>
<td>7.8</td>
</tr>
<tr>
<td>9</td>
<td>4990</td>
<td>50</td>
<td>99.8</td>
<td>6.6</td>
</tr>
<tr>
<td>10</td>
<td>5224</td>
<td>67</td>
<td>78.0</td>
<td>6.4</td>
</tr>
</tbody>
</table>

2 As defined in the PNTTE operation dictionary [92]
The weak link in the analysed forklift are the elements comprising the rear suspension of the truck. This is where elements of swivel connectors wear easily, as well as parts fastening crossovers between the rear wheel and the rear twist beam axle. The noticed problem concerned the rapid wear and tear of these elements without prior warning signs of future damage – no noticeable backlash occurs prior to failure. As a result, the rapid wear of these elements usually results in breakage and crumbling of connector bearings and swivel bolts, which leads to breakdowns and equipment downtime. Another drawback of the currently used forklift trucks is the fact that they do not have cabins fitted as standard. Cabins are fitted additionally in separate plants that deal with forklift facilities. These cabins also lack air conditioning and a suitable design to facilitate the work of the operator.

4.2. Maintenance costs for forklift trucks

The next step of research analysis covered an economic analysis of the forklifts. The obtained results are shown for the four trucks of the same type, equipped with a 4-cylinder engine with a displacement of 3200 cm³ and power of 63,000 KW, working in the same areas and performing the same operational tasks: warehouse work, production support and unloading supplies. Tables 2 and 3 summarize the total operating costs in 2010 and 2011. During this period, the unit cost of OT-1 and OT-2 maintenance amounted to 230 PLN and 750 PLN. The cost of testing made by Office of Technical Inspections allowing the truck to work amounted to another 520 PLN in 2010 and 560 PLN in 2011.

The average cost per man-hour for the service department employee was estimated at 20 PLN, which allows determining the average cost of labour for repair/replacement at 130 PLN. The average cost of repairing a forklift truck has been set at 2500 PLN, and the average cost of truck maintenance (repair/replacement) during inspection action performance including man labour - at the level of 1500 PLN.

Due to the lack of data, it is impossible to estimate the cost of the consequences of damage to a forklift truck for the supported system (production system) and its environment.

4.3. Analysis of the maintenance process of forklift trucks using the delay time concept

In order to use the algorithm shown in the Section 3, it is necessary to estimate a number of parameters. Due to the inability to estimate the cost parameters including the assessment of the impact of forklifts downstate on the production process, the process of selecting the optimal period $T$ shall be analysed, using the criterion of the expected period of system downtime and the expected maintenance costs. The analysed system is a multi-element system. The basic parameters were estimated as follows:

a) the inspection action performance time $d_i$ was estimated at 2 h,

b) the time required to remove the failures $d_r$ was 6.5h (mean time of all repairs registered for the ten forklifts),

c) the total operating time during the test period was (for 10 trucks) 42916 mth,

Table 2. General operating costs of a forklift truck in 2010

<table>
<thead>
<tr>
<th>Forklift</th>
<th>Cost of diesel consumption</th>
<th>Preventive maintenance costs</th>
<th>Tire replacement costs</th>
<th>The costs of handling operations*</th>
<th>Salary of the mechanic / conservator</th>
<th>The average annual cost of maintenance:</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-4</td>
<td>26 413.24</td>
<td>3 800.00</td>
<td>8 573.98</td>
<td>7 488.60</td>
<td>3 000.00 PLN</td>
<td>49 275.82</td>
</tr>
<tr>
<td>W-6</td>
<td>24 975.72</td>
<td>4 030.00</td>
<td>7 524.52</td>
<td>7 488.60</td>
<td>3 000.00 PLN</td>
<td>47 018.84</td>
</tr>
<tr>
<td>W-10</td>
<td>26 356.12</td>
<td>3 800.00</td>
<td>8 573.98</td>
<td>7 488.60</td>
<td>3 000.00 PLN</td>
<td>49 218.70</td>
</tr>
<tr>
<td>W-11</td>
<td>23 309.72</td>
<td>3 800.00</td>
<td>8 573.98</td>
<td>7 488.60</td>
<td>3 000.00 PLN</td>
<td>46 172.30</td>
</tr>
</tbody>
</table>

Total: 191 685.66

* costs include e.g. periodic replacements of hydraulic oil and testing made by Office of Technical Inspections
**d)** the fixed intensity $k$ of system failures occurrence has been set at $k = 0.017155$ per moto hour,

**e)** MTBF at 58.29 mth, with standard variation of 88.24 mth.

In the present case, it is assumed that the process of the system failures is a Poisson process with a constant intensity $k = 0.017155$ failure/mth.

After estimating the parameter data, there should be also specified the time delay $h$, which is assumed to be independent of the failure intensity and has a well-known form of the function $f_h(h)$ and $F_h(h)$. In the case of the operation of the analysed technical facilities, there is no historical data about the $u$ time moments of the occurrence of symptoms of forthcoming failures. Therefore, there is no reliable information as to which probability distribution should be used to model the time delay parameter to obtain the best results. On the other hand, based on the paper [42] it is crucial to know the expected time delay period of $E(h)$, while the probability distributions of the variable have a lesser impact on the model results. Therefore, the article focuses on the case, in which the time delay parameter is described by exponential distribution, and the probability density function is as follows:

$$f_h(h) = \lambda e^{-\lambda h} \quad (5)$$

Using the designated parameters and substituting function (1) in the equation (2), the expected system downtime $E_d(T)$ can be described as follows:

$$E_d(T) = \frac{\int_0^T (T-h) f_h(h) dh}{T + d} \quad (6)$$

Substituting the system parameters and the formula (5) to equation (6) will give the following:

$$E_d(T) = \frac{(0.017155T) \int_0^T (T-h) \lambda e^{-\lambda h} dh}{T + 2} \quad (7)$$

Taking into account the average failure rate of $\lambda$ in the equation (7) (in the analysed case, $\lambda = k = 0.017155$) the results are presented in the graph shown in Fig. 6. For this particular case, the optimal time period between the consecutive inspection actions performance is 61 mth.

Then, substituting functions (1) and (5) to the formula (3), the expected maintenance costs of forklifts per time unit in the operating cycle can be described by the following formula:

$$C(T) = \frac{1}{T + 2} \left[ kT \left( c_{ir} + \frac{1}{T} \left( c_{ir} + c_{iv} \right) \right) \int_0^T (T-h) f_h(h) dh \right] + c_i \quad (8)$$

which allows reaching the following:

$$C(T) = \frac{1}{T + 2} \left[ \int_0^T \left( 1500 + \frac{1}{T} \left( 2500 + 1500 \right) \right) (T-h) \lambda e^{-\lambda h} dh \right] + 250 \quad (9)$$

Figure 7 shows the cost results of the system performance. In the present case, the optimal time period $T_{opt}$ is 45 mth, with the total expected costs of system maintenance at the level of 1 602 PLN per operating cycle ($C(45)$ equal to 35.60 PLN). In the case of leaving the optimal time period $T_{opt}$ at 61 mth (according to the formula (7)) - the total expected maintenance costs amounted to approximately 2 531 PLN per operating cycle, which gives a nominal increase of 929 PLN (costs $C(61) = 35.88$ PLN).

**Table 3. General operating costs of a forklift truck in 2011**

<table>
<thead>
<tr>
<th>Forklift</th>
<th>Cost of diesel consumption</th>
<th>Preventive maintenance costs</th>
<th>Tire replacement costs</th>
<th>The costs of handling operations*</th>
<th>Salary of the mechanic / conservator</th>
<th>The average annual cost of maintenance:</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-4</td>
<td>28 728.00</td>
<td>5 470.00</td>
<td>10 851.20</td>
<td>7 488.60</td>
<td>3 000.00</td>
<td>55 537.80</td>
</tr>
<tr>
<td>W-6</td>
<td>37 700.25</td>
<td>7 890.00</td>
<td>7 524.52</td>
<td>8 516.80</td>
<td>3 000.00</td>
<td>64 631.57</td>
</tr>
<tr>
<td>W-10</td>
<td>40 446.00</td>
<td>7 140.00</td>
<td>11 900.66</td>
<td>10 991.80</td>
<td>3 000.00</td>
<td>73 478.46</td>
</tr>
<tr>
<td>W-11</td>
<td>32 382.00</td>
<td>6 680.00</td>
<td>10 851.20</td>
<td>8 338.60</td>
<td>3 000.00</td>
<td>61 251.80</td>
</tr>
<tr>
<td>Total:</td>
<td>254 899.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* costs include e.g. periodic replacements of hydraulic oil and testing made by Office of Technical Inspections

**Fig. 6. Function $Ed(T)$ when the time delay parameter is described exponentially**

**Fig. 7. Function $C(T)$ when the time delay parameter is described exponentially**
4.4. Maintenance costs for forklift trucks

In order to analyse the influence of model parameters’ levels changes on the level of the analysed functions \( E_d(T) \) and \( C(T) \), a sensitivity analysis has been conducted. In the first step, the effect of time parameters on the level of the expected period of operating forklifts downtime has been explored. One of the parameters that can affect the results of the model is the average time of system inspection \( d \). Prolonging the period \( d \) will affect the extension of period \( T_{opt} \) and result in a higher expected period of system downtime, as shown in Table 4 and Figure 8.

The second parameter, whose effect on the level of function \( E_d(T) \) was analysed, is the mean repair time \( d_b \). The results of the analysis are shown in Table 5 and Figure 9. As expected, the shorter mean system repair time, the longer the period \( T_{opt} \) and the lower the expected period of downtime. A similar effect is obtained when extending the mean time between failures, but in this case the changes are almost imperceptible – extending MTBF by nearly 100% extends the \( T_{opt} \) by only 9 mth (Table 6).

In the second part of the analysis, the effect of economic parameters on the level of the expected system maintenance costs per time unit \( C(T) \) has been explored. The evaluation analysis includes the change in the mean cost \( c_{ir} \), mean cost \( c_{ir} \) and mean cost \( c_{ir} \).

As shown in Table 7 and in Figure 10, the increase of mean cost \( c_{ir} \) to the level of costs of preventive inspections allowing forklift operation, the optimal period \( T_{opt} \) has been increased by 100% while changing the expected maintenance costs per time unit by approximately 5 PLN (and changing the total expected cost of system maintenance per operating cycle by more than 2 000 PLN).

In accordance with the opinion of experts, in the case of assessing the change in the average cost \( c_{ir} \), the mean value of this cost in the long term is rather expected to decrease. This in turn will result in shortening an optimal of the period \( T_{opt} \) (Table 8, Figure 11).

The last analysed parameter is the average cost of system repair \( c_{ir} \). In the event of forklift trucks failure occurrence, the operations involve the replacement/repair of items such as rod ends, actuators, or computers, where the repair costs far outweigh the amount of 2 500 PLN. On the other hand, there are many repairs whose cost does not exceed 1 000 PLN. Therefore, the change in the cost has been estimated at - 500 PLN to + 1 000 PLN relative to the base level of this input parameter (Table 9, Figure 12). As expected, the more expensive the repair operation, the shorter the period \( T_{opt} \) should be. If the expected cost of repair increase to 3 500 PLN, there is a noticeable shortening of the optimal period \( T_{opt} \) by 33%, which will reduce the total expected costs of maintaining the system in the operational cycle.

Table 4. The sensitivity analysis of function \( E_d(T) \) to changes in the mean period of system inspection \( d \)

<table>
<thead>
<tr>
<th>( d ) [h]</th>
<th>( T_{opt} ) [mth]</th>
<th>( E_d(T_{opt}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>0.055939</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>0.074431</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>0.086954</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.096334</td>
</tr>
<tr>
<td>5</td>
<td>140</td>
<td>0.103483</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>0.109056</td>
</tr>
</tbody>
</table>

Table 5. The sensitivity analysis of function \( E_d(T) \) to changes in the mean repair time \( d_b \)

<table>
<thead>
<tr>
<th>( d_b ) [h]</th>
<th>( T_{opt} ) [mth]</th>
<th>( E_d(T_{opt}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>120</td>
<td>0.046513</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>0.055834</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>0.063951</td>
</tr>
<tr>
<td>6</td>
<td>65</td>
<td>0.071084</td>
</tr>
<tr>
<td>6.5</td>
<td>61</td>
<td>0.074431</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>0.077679</td>
</tr>
<tr>
<td>8</td>
<td>52</td>
<td>0.083695</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>0.089396</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
<td>0.094662</td>
</tr>
</tbody>
</table>

Table 6. The sensitivity analysis of function \( E_d(T) \) to changes in the mean time between failures

<table>
<thead>
<tr>
<th>MTBF [h]</th>
<th>( T_{opt} ) [h]</th>
<th>( E_d(T_{opt}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>55</td>
<td>0.086446</td>
</tr>
<tr>
<td>45</td>
<td>58</td>
<td>0.082585</td>
</tr>
<tr>
<td>50</td>
<td>58</td>
<td>0.079197</td>
</tr>
<tr>
<td>58.29</td>
<td>61</td>
<td>0.074431</td>
</tr>
<tr>
<td>65</td>
<td>63</td>
<td>0.071165</td>
</tr>
<tr>
<td>70</td>
<td>64</td>
<td>0.069003</td>
</tr>
<tr>
<td>80</td>
<td>65</td>
<td>0.065251</td>
</tr>
<tr>
<td>100</td>
<td>70</td>
<td>0.059305</td>
</tr>
</tbody>
</table>
to around 1 200 PLN with an expected cost per time unit at 39.97 PLN (an increase by 4.17 PLN/mth).

Table 7. The sensitivity analysis of function $C(T)$ to changes in the mean cost of system inspection $c_i$

<table>
<thead>
<tr>
<th>$c_i$ [PLN]</th>
<th>$T_{opt}$ [mth]</th>
<th>$C(T_{opt})$ [PLN]</th>
<th>$C_{total}$ [PLN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>25</td>
<td>32.21</td>
<td>805.32</td>
</tr>
<tr>
<td>250</td>
<td>45</td>
<td>35.59</td>
<td>1601.77</td>
</tr>
<tr>
<td>400</td>
<td>70</td>
<td>38.24</td>
<td>2677.08</td>
</tr>
<tr>
<td>560</td>
<td>90</td>
<td>40.22</td>
<td>3619.85</td>
</tr>
</tbody>
</table>

Table 8. The sensitivity analysis of function $C(T)$ to changes in the mean cost of system repair performed during inspection action $c_{ir}$

<table>
<thead>
<tr>
<th>$c_{ir}$ [PLN]</th>
<th>$T_{opt}$ [mth]</th>
<th>$C(T_{opt})$ [PLN]</th>
<th>$C_{total}$ [PLN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>35</td>
<td>29.62</td>
<td>1036.88</td>
</tr>
<tr>
<td>1500</td>
<td>45</td>
<td>35.59</td>
<td>1601.78</td>
</tr>
<tr>
<td>2000</td>
<td>70</td>
<td>41.03</td>
<td>2871.84</td>
</tr>
</tbody>
</table>

Table 9. The sensitivity analysis of function $C(T)$ to changes in the mean cost of system repair $c_{b}$

<table>
<thead>
<tr>
<th>$c_{b}$ [PLN]</th>
<th>$T_{opt}$ [mth]</th>
<th>$C(T_{opt})$ [PLN]</th>
<th>$C_{total}$ [PLN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>70</td>
<td>32.54</td>
<td>2277.96</td>
</tr>
<tr>
<td>2500</td>
<td>45</td>
<td>35.59</td>
<td>1601.78</td>
</tr>
<tr>
<td>3000</td>
<td>35</td>
<td>37.88</td>
<td>1325.80</td>
</tr>
<tr>
<td>3500</td>
<td>30</td>
<td>39.78</td>
<td>1193.46</td>
</tr>
</tbody>
</table>

Fig. 10. Function $C(T)$ upon changing the mean cost of system inspection $c_i$

Fig. 12. Function $C(T)$ upon changing the mean cost of facility repairs $c_{b}$

5. Summary

Analysis and proper selection of maintenance strategies for logistic support systems is one of the most important aspects discussed in literature. In the present case, the lack of reliability in the forklifts’ operational tasks performance would have prevented the proper performance of the metallurgical plant, which would expose the company to significant financial losses due to e.g. production downtime.

In the present case, there have been applied the algorithm for the selection of the optimal period $T_{opt}$ for the operation of forklift trucks in the selected production system. Under the adopted assumptions the optimal period $T_{opt}$ has been determined at 61 mth, taking into account the criterion of expected period of downtime $E_d(T)$, and at 45 mth when considering the economic criterion $C(T)$. However, the lack of data made it impossible to analyse the optimization of the period $T$ with consideration of the criterion of the expected consequence costs of the forklift failure. Therefore, future research directions should expand the presented analysis by a third evaluation process following the collection of the necessary operational and economic data.

Meanwhile, in the present case, the optimal period $T_{opt}$ equal to 61 mth, assuming a three-shift work system, and an average of 7 mth worked per shift, means that the period of inspection is approximately 3 days. Taking into account the working conditions of forklift trucks, this approximation appears to be credible.

The article focuses on presenting the applicability of the delay time concept to determine the optimal interval between inspections performance. The proposed methodology can be supportive for managers in their decision-making process, including determining the proper operational time of technical objects. This article is a continuation of work on the issue related to delay time modelling for multi-element systems, as presented in the papers [20, 30, 31, 38]. In their research work, the authors will focus in subsequent steps on identifying the opportunities for the application of delay time models to assess the performance of real-life technical systems (e.g. consideration of imperfect inspections), or the development of mathematical models for multi-element systems with time delays. This will allow defining the fundamental principles of selecting a preventive maintenance policy from the point of view of the person managing the operation of the technical system.

Acknowledgments

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References


64. Restel F. J. Train punctuality model for a selected part of railway transportation system. Proc. of 22nd Annual Conference on European Safety and Reliability ESREL 2013, 29 September-02 October 2013, Amsterdam, the Netherlands, 2014.

