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## OPERATION RELIABILITY ANALYSIS BASED ON FUZZY SUPPORT VECTOR MACHINE FOR AIRCRAFT ENGINES

### ANALIZA NIEZAWODNOŚCI EKSPLOATACYJNEJ SILNIKÓW LOTNICZYCH W OPARCIU O METODĘ ROZMYTEJ MASZYNY WEKTORÓW NOŚNYCH (FSVM)

*The aircraft engine is a complex and repairable system, and the diversity of its failure modes increases the difficulty of operation reliability analysis. It is necessary to establish a dynamic relationship among monitoring information, failure mode and system reliability for achieving scientific reliability analysis for aircraft engines. This paper has used fuzzy support vector machine (FSVM) method to fuse condition monitoring information. The reliability analysis models including Gamma process model and Winner process model, respectively for different failure modes, have been presented. Furthermore, these two models have been integrated on the basis of competing failures' mechanism. Bayesian model averaging has been used to analyze the effects of different failure modes on aircraft engines' reliability. As a result of above, the goal of an accurate analysis of the reliability for aircraft engines has been achieved. Example shows the effectiveness of the proposed model.*

**Keywords:** aircraft engine, reliability analysis, competing failure, Bayesian model averaging, data fusion.

*Silnik samolotu to złożony system naprawialny, a różnorodność przyczyn jego uszkodzeń zwiększa trudność analizy niezawodności eksploatacyjnej. Istnieje konieczność ustalenia dynamicznych związków pomiędzy monitorowaniem informacji, przyczynami uszkodzeń i niezawodnością systemu, których znajomość pozwoliłaby przeprowadzać naukową analizę niezawodności silników lotniczych. Do integracji danych z monitorowania informacji, w pracy wykorzystano metodę rozmytej maszyny wektorów nośnych (FSVM). Dla różnych przyczyn uszkodzeń, przedstawiono odpowiednie modele analizy niezawodności – model procesu Gamma i model procesu Wienera. Przedstawione modele zintegrowano na podstawie mechanizmu uszkodzeń konkurujących. Do analizy wpływu różnych przyczyn uszkodzeń na niezawodność silników lotniczych wykorzystano procedurę bayesowskiego uśredniania modeli. Dzięki powyższym krokom, osiągnięto założony cel dokładnej analizy niezawodności silników samolotowych. Przykład pokazuje skuteczność proponowanego modelu.*

**Słowa kluczowe:** silnik samolotu, analiza niezawodności, uszkodzenie konkurujące, bayesowskie uśrednianie modeli, fuzja danych.

#### 1. Introduction

The level of the aircraft engines' reliability affects flight safety directly. Estimating the reliability level scientifically and objectively is the foundation of reliability management and decision-making of maintenance for aircraft engines. The difficulties of operation reliability analysis for aircraft engines lie in three aspects. First, there are less failure data and rich condition monitoring data. Second, there is a problem of competing failures caused by the diversity of failure modes arising from the complexity of the system. Third, the operational reliability is dynamic change.

Extracting reliability information from a large amount of monitoring information is a common concern issue in the current theoretical and engineering field. Researchers in the United States, Britain, Australia and other countries promote using HUMS (health and usage monitoring systems) to monitor the health and use of engines, structure, etc, which can provide full-time health information and on-line monitoring, in order to make the diagnosis and prediction of the remaining life of the equipment, structure and operation [10]. HP Engine Company has developed an advanced life prediction system for gas turbine engines, which integrates fault prognostics and health management capacity [22]. Sugier J, Anders GJ [24] described the deterioration process by a Markov model, developed the equipment

life curve using its various characteristics and quantified other reliability parameters. Cobel proposed using data fusion method, which fuses condition monitoring data and fault data effectively, to predict the remaining life, used genetic algorithm to select optimal monitoring parameters, applied GPM (General path model, GPM) to achieve that transform the traditional reliability analysis based on failure time to analysis based on failure process [7]. For the operation reliability or on-line reliability analysis, Lu H et al. presented a evaluation model of real-time performance based on time series method, and researched the reliability prediction of the bit excessive wear failure by regarding drill thrust as a performance monitoring parameters [15]. Elwany and Gebraeel presented a model for predicting system performance reliability based on Bayesian, and applied to parts replacement and inventory decisions [8]. Li et al. discussed the multi-state coherent system composed of multi-state components [14]. Chinnam made use of the reliability condition of some parts which performance degenerate signals were monitored and adopted a general polynomial regression model to describe performance change [6].

For complex systems, the reliability evaluation of single failure mode or single point of failure is an ideal assumption. But in terms of practical situation of aircraft engines, the failure modes are various and multi-failure modes often coexist. The failure modes can be divided into degradation failure and sudden failure only on the basis

of major categories of classification. Different failure modes interact each other, constantly change their forms of expression and mechanism of action in different stages of the running system. It is a problem of competing failures in essence, increasing the complexity of the reliability evaluation. The problem of competing failures has drawn a lot of concern in the field of reliability engineering. Lehmann surveyed some approaches to model the relationship between failure time data and covariate data like internal degradation and external environment models [13]. Bagdonavičius et al. made use of the half updating process of the linear degradation model to study the non-parameter estimation method of competing failure model, and to simplify the calculation, the model used decomposition method [1]. Pareek et al. studied the problem of censored data processing for competing failures [16]. Bedford et al. presented a competing risks reliability model for a system that releases signals each time of its condition deteriorates and provided a framework for the determination of the underlying system time from right-censored data [2]. Su et al. regarded the incidence of sudden failure as the function of performance degradation amount, made use of Wiener process to describe the degradation process, and proposed a reliability evaluation model for competing failures [23]. Bocchetti et al. proposed a competing risk model to describe the reliability of the cylinder liners of a marine Diesel engine, in which the wear process is described by a stochastic process and the failure time due to the thermal cracking is described by the Weibull distribution [3]. Park et al. [17] and Kundu et al. [12] considered the analysis of incomplete data in the presence of competing risks among several groups. Chen et al. developed methods for competing risks when individual events are correlated with clusters [4]. Wang et al. used Bivariate exponential models to analyze competing risks data involving two correlated risk components [26]. Xing et al. presented a combinatorial method for the reliability analysis of system subject to competing propagated failures and failure isolation effect [27]. Salinas-Torres et al. [20] and Polpo et al. [19] proposed the Bayesian nonparametric estimator of the reliability of a series system under a competing risk scenario. Peng et al. developed reliability models and preventive maintenance policies for systems subject to multiple Dependent Competing Failure Process (MDCFP) [18].

For the characteristics of aircraft engines' operation reliability, the information fusion technology will be referenced to the aircraft engines' reliability modeling and the input parameters of the reliability model will be determined by information fusion. The impacts of competing failure modes on system reliability will be analyzed through data. The paper will use fuzzy support vector machine to fuse on-line condition monitoring data. Further, Bayesian model averaging method are used to study the data, to select the optimal model, and to propose a reliability analysis model for aircraft engines based on competing failures.

## 2. The modeling framework of operation reliability analysis for aircraft engines

The paper intends to combine the recent research results concerning operation reliability analysis and competing failures, and to propose operation reliability analysis methods based on competing failures for aircraft engines. The monitoring information characteristics of aircraft engines are reflected in the following aspects. First, there are multi-source monitoring information to monitor aircraft engines operation reliability. Second, there are insufficient monitoring information because of monitoring cost consideration. Third, the monitoring information often has some noise because of operation environment change and sensor reliability. Therefore, it can be concluded that for the analysis and prediction of aircraft engines performance, the problem consists in the data analysis of small sample and high noise. Fuzzy support vector machine has been selected to model operation condition of aircraft engines. Failure mechanism of aircraft engines

should be considered, establishing the reliability analysis models respectively for the different failure modes. In the case of different failure modes coexist, the reliability analysis model based on competing failures is established. The modelling process is shown in Fig. 1.

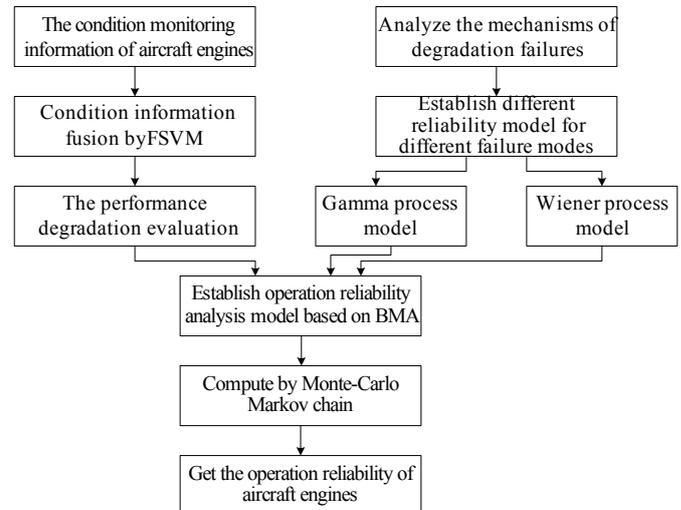


Fig. 1. The flow diagram of operation reliability analysis for aircraft engines based on FSVM

## 3. Performance degradation analysis of aircraft engines

In the aircraft engine operation process, performance degradation is the main reason leading to reliability decrease. So, it is necessary to evaluate performance degradation. By evaluating the performance degradation level, performance degradation reliability of aircraft engines can be evaluated. The performance degradation evaluation is based on condition monitoring information, through processing and fusion condition monitoring information of aircraft engines.

### 3.1. Condition monitoring information of aircraft engines

The performance monitoring of aircraft engines includes three categories, namely, gas path monitoring, oil monitoring and vibration monitoring. The engines' performance degradation (or reduced efficiency) will usually be reflected in changes of monitoring parameters.

- I Gas path monitoring. Gas path system is the key composition of aircraft engines, which includes air-compressor, combustor and turbine, etc. Gas path monitoring consists of some subsets of inter-stage pressure and temperatures, spool speeds and fuel flow. The main monitoring parameters of gas path are exhaust gas temperature and fuel flow.
- II Oil monitoring. Oil monitoring includes various oil system temperature, pressures, fuel temperature, and delivery pressure. Oil monitoring are the auxiliary instruments for aircraft engines, which can be used for monitoring components of lubrication system and its sealing. The main oil monitoring parameters are oil pressure, oil temperature, and oil consumption rate.
- III Vibration measurements. High and low spools of aircraft engines are composed of blades, plates, axis and bearings. There are some vibration signals while wear and damage occur during rotation. The key parameters of vibration measurements include low pressure vibration and high pressure vibration.

The performance degradation is usually reflected on the change of condition monitoring parameters. For example, if exhaust gas temperature exceeds the standard, oil consumption rate will increase, or

high pressure rotor speed deviation will occur, and a conclusion can be drawn that the aircraft engine is deteriorating.

### 3.2. Performance degradation evaluation based on fuzzy support vector machine

Comprehensively using above parameters to reflect the performance degradation of aircraft engines from the multi-dimensional perspective will be more realistic. In the paper, the problem can be solved by using fuzzy support vector machine. The advantages of fuzzy support vector machine method are shown as followed:

First, support vector machine (SVM) is presented as the study system of supposed linear function space which is used in high-dimension feature space by Vapnik and others [25] according to principles of structure risk minimization in study theory of statistics; it finds the best compromise between complexity of method and study ability according to limited sample information; it takes advantage of supporting vector machine in dealing with small sample and prediction to make it come true that it can be a highly effective conducting inference from training data sample to prediction sample, and to solve the reality problems of small sample, nonlinear, high dimension and local minimum point more sufficiently [21].

Second, fuzzy support vector machine can excellently handle samples with noise and outlier rejecting. It can achieve the goal of eliminating the influence of noise and outlier rejecting samples by applying fuzzy technology to support vector machine, using different punishment weight coefficients for different samples, giving smaller weight to samples with noise and outlier rejecting [9, 11]. There are a lot of applications by FSVM prediction. For example, Cheng et al utilized weighted SVM, fuzzy logic and fast messy Genetic Algorithm (fmGA) to handle distinct characteristics in Estimate at Completion (EAC) prediction [5].

### 3.3. Fuzzy support vector machine algorithm

#### (1) Method of support vector machine

Note that training sample is denoted by  $\{(x_i, y_i) | i = 1, 2, \dots, l\}$ . In the sample,  $x_i \in R^n$  is input variables,  $y_i \in R$  is output variables. Nonlinear mapping  $\phi(\cdot)$  is to input the sample from late space mapping to high-dimension feature space, and construct best decision function in this feature space:  $f(x) = (\omega \cdot \phi(x) + b)$ , where  $\omega \cdot \phi(x)$  is set as the scalar product of vector and mapping function,  $b$  is the bias. Thus, the corresponding constraint optimization problem can be shown as:

$$\min_{\omega, b, \xi, \xi^*} \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^l (\xi_i + \xi_i^*) \quad (1)$$

$$\text{s.t.} \begin{cases} y_i - \omega^T \cdot \phi(x_i) - b \leq \varepsilon + \xi_i \\ \omega^T \cdot \phi(x_i) + b - y_i \leq \varepsilon + \xi_i^*, \quad i = 1, 2, \dots, l \\ \xi_i, \xi_i^* \geq 0 \end{cases}$$

In Eq.(1),  $C$  is a penalty factor, it has made a compromise between empirical risk and trust scope,  $\xi_i, \xi_i^*$  is relax quantum, each of them shows the upper and low limit of training error  $(|y_i - [\omega^T \cdot \phi(x_i + b)]| < \varepsilon)$  in the constraint of error  $\varepsilon$ ;  $\varepsilon$  is the defined error of insensitive pricing function Vapnik- $\varepsilon$ . Eq.(1) has set the optimization problem which is a typical convex quadratic programming problem. According to Lagrange theory, weight vector equals to the linear combination of training data:

$$\omega = \sum_{i=1}^l (\alpha_i - \alpha_i^*) \phi(x_i) \quad (2)$$

Putting Eq.(2) in Eq.(1), the prediction values of the unknown point can be got as followed.

$$f(x) = \sum_{i=1}^l (\alpha_i - \alpha_i^*) K(x_i, x) + b \quad (3)$$

Where  $K(x_i, x) = \phi(x_i) \cdot \phi(x)$  is denoted as kernel function.

#### (2) Model of fuzzy support vector machine

Fuzzy support vector machine introduces fuzzy theory into support vector, and adds a membership attribute  $\mu(\delta \leq s \leq 1)$  to every sample  $(x, y)$ , where  $\delta$  is arbitrarily small positive to show the subordination degree between sample  $x$  and type  $y$ .

Denoting the training sample as:

$$U = \{(x_i, y_i, u_i) | x_i \in R^n, \varepsilon \leq u_i \leq 1\}, \quad i = 1, 2, \dots, l \quad (4)$$

Thus, its corresponding optimization problem is:

$$\min_{\omega, b, \xi, \xi^*} \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^l (\mu_i \xi_i) \quad (5)$$

As is known from Eq. (5), this kind of fuzzy support vector machine, which makes samples with different membership, plays different roles in point training just by fusing the penalty factor  $C$ . The bigger  $\mu_i$  is, the more important the sample is, and the less possible the classification mistake is. When it is noise or outlier rejecting, the sample will reduce the effort on training by giving very small value to its membership so that it can widely reduce the influence of noise or outlier rejecting to support vector machine.

#### (3) Method of determining membership

In order to determine the relationship between effective sample with outlier rejecting or noise, the paper uses affinity to determine membership function. After having found the smallest hyper-sphere of the sample set in feature space, marking the core and radius of the smallest hyper-sphere with  $a$  and  $r$ , thus the membership of the sample can be described as followed:

$$\mu(x_i) = \begin{cases} 0.6 \times \left[ \frac{1 - \|\phi(x_i) - a\|/r}{1 + \|\phi(x_i) - a\|/r} \right] + 0.4, & \|\phi(x_i) - a\| \leq r \\ 0.4 \times \left[ \frac{1}{1 + (1 + \|\phi(x_i) - a\|/r)} \right], & \|\phi(x_i) - a\| > r \end{cases} \quad (6)$$

This method makes a distinction between effective sample from noise or outlier rejecting more effectively and takes different calculation method of membership to effective sample and noise or outlier rejecting, and under the situation of maintaining support vector and membership big enough, it also can reduces the membership of noise and outlier rejecting so that it reduces the sensitivity of fuzzy support vector machine for noise. The calculation of membership is achieved by determining the radius of the smallest hyper-sphere.

#### 4. Performance degradation analysis of aircraft engines

Considered the multi-failure mode, sole reliability model is difficult to analyze aircraft engines reliability. The paper chooses Gamma process and Wiener process to describe aircraft engines operation reliability. Gamma process model mainly applied in the gradual performance degradation and reliability monotone decreasing. Wiener process model mainly applied in the structural reliability analysis, which has the function of describing multi-factors random disturbance effect on reliability. Using above two reliability analysis model, the reliability change law can be described comprehensively.

##### 4.1. Gamma process reliability analysis model

Let  $y(t)$  be the amount of performance degradation failures at time  $t$  and  $l$  be the failures threshold. When  $y(t) \geq l$ , aircraft engines will come up with performance degradation failures. Aircraft engines' performance degradation is irreversible, that is, the performance gradually decreases and the amount of performance degradation is constantly increasing with the use of time. Therefore, the Gamma process can be applied to describe the degradation process. Assume that  $y_0$  is aircraft engines' initial performance, so  $w(t) = y(t) - y(t_0)$  represents the accumulated deterioration at time  $t$ . Because degradation amount increases monotonically, for any  $t_i, t_j$ , if  $t_j > t_i$ , there must be  $w(t_j) - w(t_i) > 0$ . Assume that degradation amount  $w(t)$  obey  $Ga(\mu(t), \lambda)$ , its density function can be expressed as follows:

$$f_g(\xi, \alpha(t), \lambda) = \frac{\lambda^{\alpha(t)}}{\Gamma(\alpha(t))} \xi^{\mu(t)-1} e^{-\lambda \xi} \quad (7)$$

where,  $\alpha$  and  $\lambda$  represent shape parameter and scale parameter respectively;  $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$  is Gamma function.

Generally assume that the scale parameter does not change in a performance monitoring process. Shape parameter changes with the change of the degradation process, because the extent and rate of the performance degradation experience an increasing trend, so we assume that shape parameter is proportional with expected degradation degree and time power, that is:

$$\alpha(t) = kt^v \quad (8)$$

Further, Eq.(7) can be transformed as following:

$$f_g(\xi, \alpha(t), \lambda) = \frac{\lambda^{kt^v}}{\Gamma(kt^v)} \xi^{kt^v-1} e^{-\lambda \xi} I_{(0, \infty)}(\xi) \quad (9)$$

Based on the theory of system reliability, the reliability for degradation failures can be depicted as following:

$$R_g(t) = P\{T > t\} \Rightarrow P\{w(t) < \varepsilon\} \quad (10)$$

where,  $\varepsilon$  is the failure threshold for performance degradation of an aircraft engine.

Then, the reliability evaluation for performance degradation of an aircraft engine can be depicted as following:

$$R_g(t) = \int_0^{\varepsilon} f_w(\xi) d\xi = \int_0^{\varepsilon} \frac{\lambda^{kt^v}}{\Gamma(kt^v)} \xi^{kt^v-1} e^{-\lambda \xi} d\xi \quad (11)$$

##### 4.2. Wiener reliability analysis model

The Wiener process from Brown motion in the physics. Denoted degradation amount  $w(t)$  as the following:

$$w(t) = \eta t + \delta B(t), \quad t \geq 0 \quad (12)$$

The stochastic process is defined as  $\{W(t)\}$ , If  $t > 0$ ,  $\{W(t)\}$  is defined as Wiener process and satisfy the following assumptions:

- I  $w(0) = 0$ ;
- II  $\{w(t)\}$ ;  $t > 0$  with a stationary independent increments;
- III For any  $t > 0$ ,  $\{w(t)\}$  is normal random variable, whose mean is 0, the variance is  $\delta^2 t$ ;

For any  $0 \leq s < \infty$ ,  $[W(t) - W(s)]$  follow Gaussian distributions  $N[\eta(t-s), \delta^2(t-s)]$ .

Assuming that the aircraft engine failure threshold is, the failure time of aircraft engine is described as the following:

$$T = \inf \{t; w(t) > w\} \quad (13)$$

Accordingly, at this time aircraft engine operation reliability is:

$$R_{wi}(t) = P\{T > t\} = 1 - F(t) = \Phi\left(\frac{\eta t - \varepsilon}{\delta \sqrt{t}}\right) - \exp\left(\frac{2\eta \varepsilon}{\delta^2}\right) \Phi\left(\frac{-\eta t - \varepsilon}{\delta \sqrt{t}}\right), \quad t > 0 \quad (14)$$

The probability distribution function is described as the following:

$$f_{wi}(t) = \frac{1}{\sqrt{2\pi\delta^2 t^3}} \exp\left(-\frac{(\varepsilon - \eta t)^2}{2\delta^2 t}\right) \quad (15)$$

Distribution form for the above is Inverse Gaussian, denoted as  $t \sim IG(\mu, \beta)$ .

##### 4.3. Reliability analysis of aircraft engines using Bayesian model averaging(BMA)

For the complex system like aircraft engines, single reliability model is difficult to objectively describe the reliability change process. It is necessary to use multi-model technology to comprehensively analysis multi-modes of aircraft engines, which will improve the accuracy of reliability analysis and prediction. To study the mechanism of different reliability models, this paper will use Bayesian model averaging method. Bayesian model averaging (Bayesian model averaging, BMA) is a probability forecast approach that is proposed recently and is used in multi-mode collection. The forecast probability density function (PDF) of a particular variable in BMA, is a weighted average of a single model forecast probability distribution after deviation correction, and the weight is the corresponding model's posterior probability which represents each model's relative forecast skill in the model training phase. The secondary use of condition monitoring data and event data can be achieved through BMA technology. And this not only solves the problem of reliability analysis based on a single failures mode, but also solves the problem of interaction of multiple failures modes. Based on the data re-learning, the goal of an accurate analysis of civil aircraft system reliability can be achieved.

The multi-model is described as the following:

$$p(X, \theta_k, M_k) = P(M_k) p(\theta_k | M_k) p(X | \theta_k, M_k) \quad (16)$$

$k \in \kappa$  represents the index of model,  $P(M_k)$  represents the prior density function of  $M_k$ ,  $p(\theta_k | M_k)$  represents the conditional probability of  $\theta_k$  under the model  $M_k$ ,  $p(X | \theta_k, M_k)$  represents the conditional probability function of event  $X$  under the model  $M_k$  and the parameter  $\theta_k$ .

The probability of event  $X$  under the model  $M_k$  can be computed as following:

$$P(X | M_k) = \int_{\Theta_k} p(\theta_k | M_k) p(X | \theta_k, M_k) d\theta_k \quad (17)$$

Using Bayes theorem, the posterior density can be described as following:

$$P(M_k | X) = \frac{P(M_k) P(X | M_k)}{P(X)} \quad (18)$$

The prediction effect is compared the actual value with the prediction value. The weight can be assigned by the Monte-Carlo Markov chain method.

For the aircraft engines,  $M = \{M_1, M_2\}$  represents the reliability analysis models  $M_1$  represents the gamma reliability analysis model,  $M_2$  represents the Wiener process reliability analysis model. Operation reliability analysis model for aircraft engines can be described as following:

$$p[R | (M_1, M_2, R^T)] = \sum_{j=1}^2 \rho_j p_j(R | (M_j, R^T)) \quad (19)$$

$M_j (j=1,2)$  represents the reliability analysis model which should be averaged,  $p_j(R | (f_j, R^T))$  represents the probability density function of single failure model, which is the probability density function of failure mode  $j (j=1,2)$  under known reliability  $R$ .  $\rho_j$  represents the posterior probability of failure mode  $j$  being best failure mode, which is negative and  $\sum_{j=1}^2 \rho_j = 1$ , which reflect the contribution degree of each failure mode on reliability analysis.

The posterior expectation and variance of reliability by BMA can be described as following:

$$E(R | D) = \sum_{j=1}^2 p(M_j | D) \cdot E[R | M_j, D] = \sum_{j=1}^2 \rho_j M_j \quad (20)$$

$$Var(R | D) = \sum_{j=1}^2 \rho_j \left( M_j - \sum_{i=1}^2 \rho_i M_i \right)^2 + \sum_{j=1}^2 \rho_j \sigma_j^2 \quad (21)$$

Where,  $\sigma_j^2$  is the prediction error of model  $M_j$  under data set  $D$ . From eq. (21) the prediction variance of BMA include two item, the first is the dispersion degree in the set, the second is the variance of prediction model itself.

In the paper, we use Markov Monte-Carlo (Markov Chain Monte Carlo, MCMC) simulation algorithm to calculate the failure mode weighting model in BMA model. MCMC is an important method to deal with complex statistical problems, especially for high dimensional integrals by Monte Carlo to compute the posterior distribution density. MCMC algorithm uses a number of different Markov chain, random sampling got based on BMA weights and variance in the likelihood function of weight variables. Considering the model weight itself is random, we assumes the weight is normal distribution. Using Metropolis-Hastings sampling technique, for the probability density function  $\pi(\rho_i)$  for the unknown parameters  $\rho_i (i=1,2)$ . Choose start point  $\rho_i^{(0)}$ , meet  $\pi(\rho_i^{(0)}) > 0$ , produce Markov chain according to the following steps iteration:

- A. Suppose condition value  $\rho_i^{(m-1)}$  at time  $m-1$  and get a candidate from suggested density  $\pi(\rho_i^* | \rho_i^{(m-1)})$ ;
- B. Compute acceptance probability of candidate point  $\theta^*$ ,

$$\pi(\rho_i^{(m-1)}, \rho_i^*) = \min \left\{ 1, \frac{\pi(\rho_i^*) p(\rho_i^{(m-1)} | \rho_i^*)}{\pi(\rho_i^{(m-1)}) p(\rho_i^* | \rho_i^{(m-1)})} \right\} \quad (22)$$

- C. Get a random  $u$  from  $U(0,1)$ , if  $u < \pi(\rho_i^{(m-1)}, \rho_i^*)$ , the candidate point is accepted, and  $\rho_i^{(m)} = \rho_i^*$ . Otherwise,  $\rho_i^{(m)} = \rho_i^{(m-1)}$ .

After enough iterations, M-H algorithm makes the Markov chain converges to the target distribution.

### 5. Example

Table 1 shows the 36 samples which have repaired and replaced engines. There are six parameters have been monitored, which are DEGT (the deviation exhaust gas temperature), DWF (the deviation of fuel flow), DOP (the deviation of oil pressure), DHPRS (the deviation of high pressure rotor speed), DLPRV (the deviation of the low pressure rotor vibration value) and DHPRV (the deviation of high-pressure rotor vibration value). The engines' TSI (Time Since Installation) and FH (Fight Hour) from the beginning of the monitoring moment can be obtained. From the data of Table 1, the relationship between PDD (Performance Degradation Degree) and the various monitoring parameters can be extracted by the FVSM. In Table 1, the former 24 samples are as training samples and the latter 12 samples are as test samples.

The training samples comparison between predictive values and real values are shown in Fig. 2 and the test samples comparison between predictive values and real values are shown in Fig. 3. As far as the comparison between the real value and predictive value by Fig. 2 and Fig. 3, there are good effectiveness of prediction. By computation, the total predictive error is below 10%, and the result satisfy the basic demand of performance degradation evaluation of aircraft engines.

By collecting 9 samples condition monitoring parameters of some on-wing aircraft engines, the performance degradation degree can be calculated. Then, the performance degradation degree can be used as input variable, the gamma reliability model and the Wiener reliability model are used to analyze reliability of aircraft engine, respectively.

Table 1. Key performance monitoring parameters for some aircraft engines

Monitoring point	DEGT	DWF	DOP	FHPRS	DLPRV	ZVB2R	TSI(FH)	PDD
1	7.51	2.54	1.89	-7.27	1.06	0.32	4055	0.1192
2	-4.74	3.52	1.92	-5.16	0.52	0.55	7095	0.0459
3	-0.03	2.03	1.19	-8.33	0.57	0.37	7801	0.0378
4	8.04	5.16	1.69	-7.74	0.24	0.57	3331	0.1176
5	7.77	7.80	2.12	-6.81	0.86	0.46	3832	0.1207
6	4.69	2.66	1.83	-3.76	0.31	0.51	3282	0.1028
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
32	4.23	4.83	1.96	-58.92	0.17	0.62	3397	0.096
33	14.28	5.25	1.63	-2.03	0.78	0.94	1422	0.1572
34	11.38	3.14	1.63	4.19	0.23	0.74	3330	0.1465
35	8.24	3.17	2.18	9.78	1.05	0.76	1954	0.1185
36	9.12	3.37	1.78	6.89	0.31	0.46	6752	0.1290

Table 2. The reliability parameters and reliability analysis results by different reliability models

No	Running time $t_i$ (FH)	Gamma reliability model				Wiener reliability model				$R_{BMA}$
		$a_i$	$b_i$	$\rho_G$	$R_G$	$\eta_i$	$\delta_i$	$\rho_W$	$R_W$	
1	493	1.31	36815	0.179	0.9970	$1.042 \times 10^{-5}$	$1.556 \times 10^{-3}$	0.821	0.9970	0.9970
2	1052	1.28	34921	0.358	0.9904	$1.711 \times 10^{-5}$	$1.163 \times 10^{-3}$	0.642	0.9909	0.9907
3	1707	1.26	33028	0.419	0.9796	$1.757 \times 10^{-5}$	$8.823 \times 10^{-4}$	0.581	0.9726	0.9755
4	2109	1.25	32392	0.525	0.9720	$1.660 \times 10^{-5}$	$7.608 \times 10^{-4}$	0.475	0.9686	0.9704
5	2805	1.23	36053	0.576	0.9630	$1426 \times 10^{-5}$	$6.432 \times 10^{-4}$	0.424	0.9608	0.9621
6	3206	1.22	35216	0.647	0.9541	$1.560 \times 10^{-5}$	$5.289 \times 10^{-4}$	0.353	0.9525	0.9535
7	3924	1.19	36893	0.751	0.9401	$1.626 \times 10^{-5}$	$3.777 \times 10^{-4}$	0.249	0.9370	0.9393
8	4740	1.18	37029	0.415	0.9243	$1.688 \times 10^{-5}$	$2.060 \times 10^{-4}$	0.585	0.9207	0.9222
9	5595	1.16	38351	0.392	0.9080	$1.698 \times 10^{-5}$	$5.142 \times 10^{-5}$	0.608	0.9030	0.9050

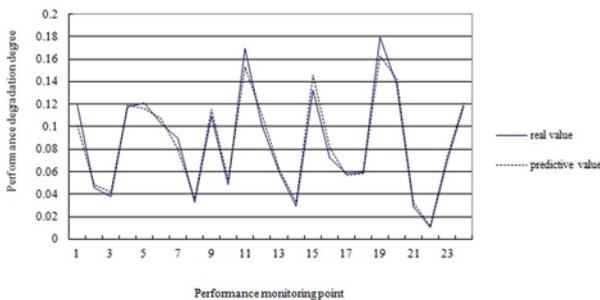


Fig. 2. Performance degradation degree comparison between predictive values and real values on training samples

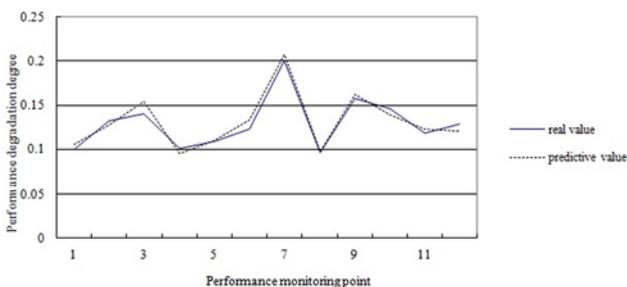


Fig. 3. Performance degradation degree comparison between predictive values and real values on test samples

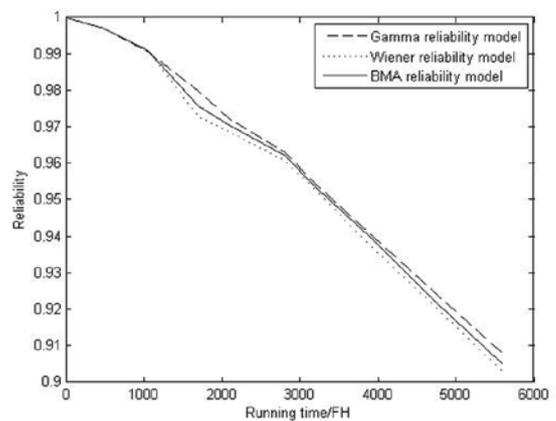


Fig. 4. Reliability curve of different reliability models on aircraft engines

The parameters of Gamma reliability model and Wiener reliability model are shown in Table 2.

At the same time, BMA are used to calculate the each model weight, then the reliability model of aircraft engine can be calculated integrated Gamma reliability model and Wiener reliability model into one framework. The computation results by different reliability model are also shown in Table 2. The reliability comparison among Gamma reliability model, Wiener reliability model and BMA reliability model are shown in Fig. 4. From Fig. 4, BMA reliability model are averaged by Gamma reliability model and Wiener reliability model. Fig. 5 gives the probability density function of Gamma reliability model

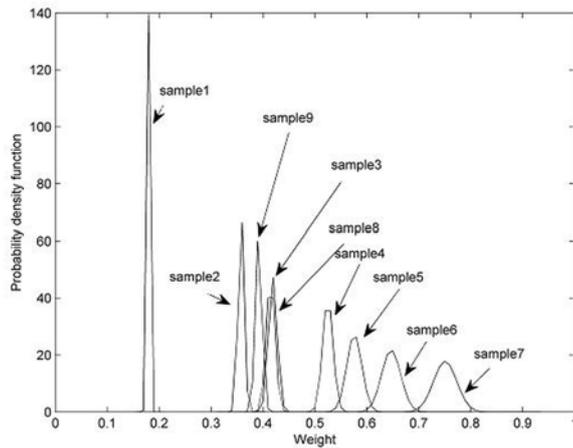


Fig. 5. Probability density function of weight on Gamma reliability model

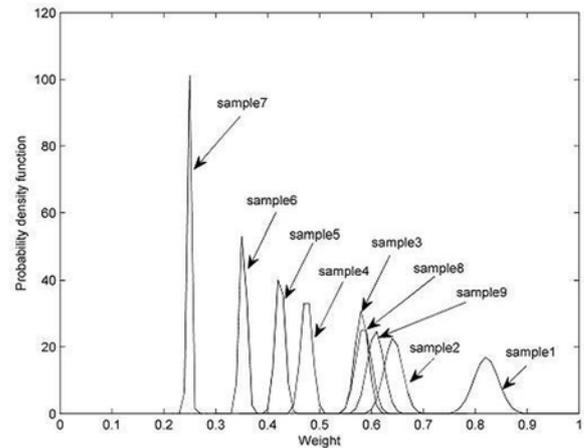


Fig. 6. Probability density function of weight on Wiener reliability model

weight. Fig. 6 gives the probability density function of Wiener reliability model weight.

For the three alternative models, the Gamma reliability model and Wiener reliability model represent the different failure mode. BMA reliability model represent the different failure modes into one framework by computing the different reliability model weight. So, BMA can really analyze the mechanism of action between the different failure modes through learning different data. The advantages of the model are that it has higher forecast accuracy and it can effectively avoid the reliability overestimate or underestimate.

## 6. Conclusion

In this paper, the mechanism of different failure modes aircraft engines has been analyzed. Fuzzy support vector machine (FVSM) method are used to fuse condition monitoring information. The reliability analysis models including Gamma process model and Wiener process model, respectively for different failure modes, have been

presented. Furthermore, these two models have been integrated on the basis of competing failures' mechanism. BMA has been used to analyze the impacts of different failure modes on aircraft engines' reliability. A reliability evaluation model for competing failures has been proposed, and the traditional model of competing failures has been transformed. This method not only can make full use of condition monitoring information, but also can analyze the mechanism and transforming relationship between different failure modes through data learning. The method should be studied further.

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