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AN INTEGRATED PRODUCTION AND DELAY-TIME BASED PREVENTIVE MAINTENANCE PLANNING MODEL FOR A MULTI-PRODUCT PRODUCTION SYSTEM

MODEL INTEGRUJĄCY PLANOWANIE OPARTEJ NA POJĘCIU CZASU OPÓŹNIENIA KONSERWACJI ZAPOBIEGAWCZEJ ORAZ PLANOWANIE PRODUKCJI DLA SYSTEMÓW PRODUKCJI WIELOASORTYMENTOWEJ

This paper integrates preventive maintenance and medium-term tactical production planning in a multi-product production system. In such a system, a set of products needs to be produced in lots during a specified finite planning horizon. Preventive maintenance is carried out periodically at the end of some production periods and corrective maintenance is always performed at failures. The system's available production capacity can be affected by maintenance, since both planned preventive maintenance and unplanned corrective maintenance result in downtime loss during the planning horizon. In addition to the time used for preventive and corrective maintenance, our model considers the setup time and the product quality, as these are affected by the defects and failures of the system. Procedures are proposed to identify the optimal production plan and preventive maintenance policy simultaneously. Our objective is to minimize the sum of maintenance, production, inventory, setup, backorder costs and the costs of unqualified products within the planning horizon. A real case from a steel factory is presented to illustrate the model.

Keywords: *Production planning, preventive maintenance, delay-time, integration, multi-product.*

W niniejszej pracy zintegrowano proces konserwacji zapobiegawczej z procesem średnioterminowego taktycznego planowania produkcji w odniesieniu do systemu produkcji wieloasortymentowej. W takim systemie, zestaw wyrobów jest produkowany partiami w określonym, skończonym horyzoncie planowania. Konserwacja zapobiegawcza prowadzona jest okresowo pod koniec wybranych okresów produkcyjnych, natomiast w przypadku wystąpienia uszkodzenia wykonuje się konserwację korygującą. Konserwacja może mieć wpływ na dostępne moce produkcyjne systemu, jako że zarówno planowana konserwacja prewencyjna jak i nieplanowana konserwacja korygująca powodują straty związane z przestojem urządzeń w danym horyzoncie planowania. Oprócz czasu potrzebnego na konserwację zapobiegawczą i korygującą, nasz model uwzględnia czas konfiguracji urządzeń oraz jakość produktów, ponieważ one również zależą od defektów i awarii systemu. Zaproponowano procedury, które pozwalają na jednoczesne określenie optymalnego planu produkcji i optymalnej strategii konserwacji prewencyjnej. Naszym celem jest minimalizacja sumy kosztów konserwacji, produkcji, zapasów, konfiguracji urządzeń oraz zamówień oczekujących a także kosztów produktów, które nie zostały zakwalifikowane do wprowadzenia do obrotu w danym horyzoncie planowania. Model zilustrowano na przykładzie rzeczywistego przypadku z fabryki stali.

Słowa kluczowe: *Planowanie produkcji, konserwacja zapobiegawcza, czas opóźnienia, integracja, wieloasortymentowy.*

1. Introduction

Production planning is an activity that considers the best use of production capacity in order to satisfy customer demand during a specified finite planning horizon. There are three types of production planning, based on the different lengths of the planning horizon: the strategic planning level (long-term), the tactical planning level (medium-term) and the operational planning level (short-term) [11]. In general, the length of the long-term planning is about one year or more. In this stage, the aggregate demands need to be forecasted, and some strategic decisions, such as product category, equipment and resource planning, need to be made. The length of the medium-term planning is about one month or more. This stage often involves making decisions on production quantities or lot sizing and on the maintenance policy over the planning horizon. The length of the short-term planning is about one week or less, and this stage involves making decisions on the day-to-day scheduling of production operations based on customer orders and tactical planning. In this paper, our focus is on the medium-term tactical production planning, since at this level the production quantities or lot sizing should be determined, and the preventive maintenance (PM) decisions should be scheduled. Throughout this paper, we will use the term 'production planning' instead of 'tactical production planning', for simplicity.

Production and maintenance departments are usually located separately in modern companies. The production department has to make production planning based on the maximum production capacity, and to satisfy customers' demands, whereas the maintenance department has to ensure the proper functioning of the production system through maintenance actions. Conflict is inevitably generated since the two departments share the same system. If the production department ignores the occupation of maintenance time, or just considers it subjectively or empirically, the system may experience idle time, production delays and even shortages. On the other hand, if the maintenance department schedules the maintenance without considering the production planning, excessive maintenance or inadequate maintenance may occur, so that the productive capacity cannot be fully used. Thus, it is necessary to find an integrated model to optimize the production and maintenance planning simultaneously.

There are only a few related researches at the tactical level in this integrated area. Weinstein and Chung [22] studied the integration of production and maintenance decisions in a hierarchical planning environment, and evaluated an organization's maintenance policy using a three-part model, where an aggregate planning model was described using a mixed-integer linear programming in stage one, a master production scheduling model was proposed to minimize the weighted deviations from the goals given at the aggregate level in stage two, and the master production schedule and the maintenance plan were simulated in stage three, which is the only stage studying the system failures. Their work was further researched by Aghezzaf et al. [1], which considered the reliability parameters of the production system at the early stage of the planning process, and developed a multi-item capacitated lot-sizing problem based on a system that was subjected to random failures, to minimize the expected total costs of production and maintenance. This work was extended by Aghezzaf and Najid [2], which discussed the issue of integrating production planning and PM in a production system composed of parallel failure-prone production lines. Two mathematical programming models for the problem were proposed, where the first model assumed that each production line of the system implements a cyclic preventive maintenance policy, the second model relaxed the cyclic restriction, but the backorder cost was not considered both in [1] and [2]. Fitouhi and Nourelfath [8] developed an integrated model for planning production and non-cyclical PM for a single machine. The backorder cost is considered and an enumeration method is used to get all of the PM solutions in the model, which was later extended in [9] to multi-state systems. Nourelfath et al. [15] also developed an integrated model for production and PM planning in multi-stage systems, where the preventive maintenance selection task in the integrated planning model is solved using a genetic algorithm. This work was then extended by Nourelfath and Chatelet [14] to a parallel system with dependent components, and a simulated annealing algorithm was developed. These works were well studied about the integrated problem, but some assumptions can be relaxed. For example, they did not consider the production quality influenced by the system, that is, they implicitly assumed that all of the products were qualified whereas unqualified products are common in reality. Besides, the setup time was ignored in the model, and the downtime caused by failures was not formulated by real production time. These gaps are filled in this paper.

The purpose of this paper is to develop an integrated production and PM model at the tactical level. The objective of the model is to determine the production and maintenance plan that minimizes the expected total cost over a finite planning horizon, which includes several equal-length periods. A set of products must be produced in lots during this planning horizon, and the demand for the same product may vary from one period to another. For example, the length of the planning horizon may be half a year, which may be thought of as six separate monthly periods, and the demand of the same product may vary from month to month. Since the production capacity is influenced by both PM and failures – and is therefore random – the production quantity of each product in each period will be calculated with consideration of the inventory and backorder. Since the system is subject to random failures, the PM is carried out periodically at the end of some production planning periods, whereas the corrective maintenance is always performed at a failure. The innovative points of this paper are 1) Previous works usually assume that the downtime caused by failures can be ignored, whereas our model considers this type of downtime when formulating the expected number of system failures over each period, 2) the downtime caused by setup is also studied, whereas no other literature has attempted, to our best knowledge, 3) we also consider how the product quality is affected by the defects and failures in system, and this was also not studied in previous works, 4) to model the PM decision for the system, we use the delay-time concept.

The concept of the delay-time has been widely applied in maintenance modeling and optimization; see [17] for a detailed review. The failure process of a component is regarded as a two-stage process: the period from new to the initial point of the defect, usually referred to as the normal stage or time-to-defect; and the period from the initial point to the component failure, referred to as the delay-time stage [3], as shown in Fig. 1. If the PM, which is considered to be perfect, is carried out during the delay-time, the defect can be identified and removed by repair or replacement. The delay-time models have been applied to single-component systems [4, 16, 24] and complex systems with many components [5, 6, 18–21]. In this paper, we use the complex system delay-time model since typical production systems are equipped with many components. For complex systems, multiple defects can be present at one time, and the arrival process of the defects can be approximated by a Homogenous Poisson Process (HPP) – see [17]. Fig. 2 illustrates the defect arrival and failure processes of a complex system where PM interventions were carried out at points A and B. It is clear from Fig. 2 that three defects could be identified and removed if the defect identification and removal are perfect. The delay-time-based PM models differ from other PM models in that they directly model the relationship between the PM and the number of system failures. Many case studies have shown the validity of the delay-time-based models [10, 13, 23].

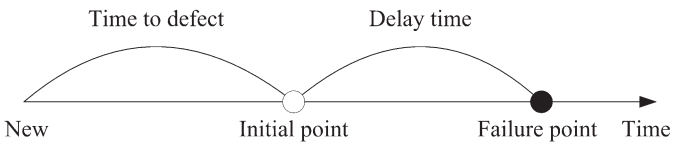

Fig. 1. The delay-time for a defect. '○' initial point, '●' failure point

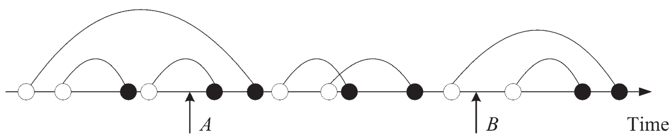

Fig. 2. The defect arrival and failure processes of a complex system

This paper is organized as follows. The model assumptions and notations are given in Section 2. Section 3 formulates the problem. Numerical examples are presented in Section 4. Section 5 concludes the paper.

2. Assumptions and notations

2.1. Assumptions

(1) The defects of the system arrive independently according to an HPP.
(2) The delay-time of all defects is independent and identically distributed.
(3) The PM is perfect and renews the system.
(4) A corrective maintenance is always performed at a failure, which is minimal in the sense that it will only repair the failed component while the defect arrival rate of the system is unaffected.
(5) The setup structure is sequence-independent.
(6) The PM may be carried out at the end of some production periods.

(7) The percentage of unqualified products is proportional to the number of failures.

Assumptions (1) and (2) have been used in previous delay-time models. Assumption (3) is for modeling simplification. Assumption (4) is commonly used in maintenance modeling, such as in [12], where – due to the time constraint and the need to resume the production as soon as possible – only the failed component is repaired or replaced, while the defect and failure rates of the system are unchanged after the repair. Assumption (5) is one of the two setup structures. Usually, production changeover between different products can incur setup time and setup costs. If the setup time and cost in a period are independent of the sequence, it is termed as a sequence-independent structure. Alternatively, the sequence-dependent structure refers to the case where the setup time and cost depend on the sequence of the products, and is more complex than the sequence-independent situation – see [11]. In this paper, we consider the simpler situation, and the more complex situation will be studied in a separate paper. Assumption (6) is practice-based since periodic PM is still the main form of preventive maintenance used in factories throughout the world. The rationale in assumption (7) lies in the fact that more failures mean more defective components within the system, which furthermore leads to the production of more unqualified products, given that the failure arrival process is a Non-Homogeneous Poisson Process (NHPP) [17].

2.2. Notations

(1) Model parameters

H The length of the planning horizon.
Z The number of periods during H.
T The length of each period.
n The number of different products.
d_{ij} The demand for product i in period j, $i=1,2,\cdots,n$, $j=1,2,\cdots,Z$.
t_i Production time for each unit of product i.
ξ_j The actual production time for all products needed to be produced in period j.
u The defect arrival point.
$f(\cdot)$ The probability density function (pdf) of the delay-time.
$F(\cdot)$ The cumulative distribution function (cdf) of the delay-time.
λ The rate of the occurrence of defects.
d_s^i The average time per setup for product i.
d_d The average time to repair per defect identified at a PM.
d_f The average time to repair per failure.
d_p The average time per inspection at a PM.
C_s^i The average setup cost of producing product i.
C_d The average cost of repairing a defective component that is identified at a PM.
C_f The average cost of repairing a failure, which contains the cost of repairing or replacing the failed component and the additional cost of unavailability.
C_p The average cost of an inspection at a PM.
C_{dp} The average cost of one unit of unqualified product.
p_i The cost of producing one unit of product i.
h_i Inventory holding cost per unit of product i per unit time.
b_i Backorder cost per unit of product i.
$E_N(dp)$ The expected number of unqualified products.
$E_C(dp)$ The expected total cost of unqualified products.
$E_C(m)$ The expected total cost of maintenance.

(2) Model variables

x_{ij} Quantity of product i produced in period j.
I_{ij} Inventory of product i at the end of period j; I_{i0} denotes the inventory of product i at the beginning of period 1.
y_{ij} Binary variable, which is equal to 1 if product i is produced in period j, and 0 otherwise.
B_{ij} Backorder level of product i at the end of period j; B_{i0} denotes the backorder of product i at the beginning of period 1.
T_{PM} The length of the PM interval, $T_{PM}=kT$, k is a positive integer, $1\le k\le Z$

3. The models

In this section, we first describe the research problem, then we evaluate the production time constraint in each period, the expected total cost of maintenance and the expected total cost of unqualified products, at last the integrated model is derived.

3.1. Problem description

The integrated production and maintenance strategy is illustrated in Fig. 3. The length of the planning horizon is H, including Z periods of fixed length T, and a set of products need to be produced during this planning horizon. For each period j, the demand for product i, d_{ij}, needs to be satisfied either by production in this period or by carrying inventories from earlier periods, but the backorder may happen due to setup and PM times, so we consider not only inventory holding cost but also backorder cost in our model. The number of PMs during the planning horizon is $\lfloor Z/k \rfloor$, since the length of PM interval T_{PM} is equal to kT, where $\lfloor Z/k \rfloor$ is the largest integer smaller than or equal to Z/k. The planning horizon can be divided into two parts, with the first part $\left[0,\left(\lfloor Z/k \rfloor k\right)T\right]$ including $\lfloor Z/k \rfloor$ equal PM intervals $\left[\hat{z}kT,(\hat{z}+1)kT\right]$, where $\hat{z}=0,1,\cdots,\lfloor Z/k \rfloor-1$, and the second part $\left[\left(\lfloor Z/k \rfloor k\right)T,ZT\right]$, the length of which is shorter than T_{PM}. We must decide which products to produce in which periods, the exact production quantities, and PM interval, in order to minimize the sum of PM, production, unqualified products, setup, backorder and inventory holding costs.

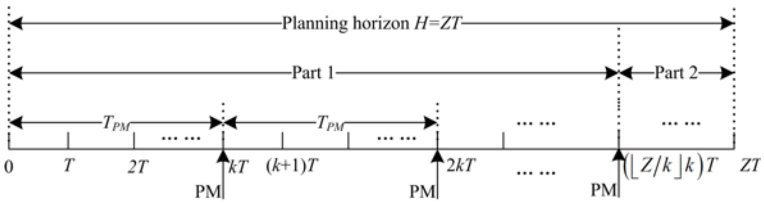

Fig. 3. Integrated production and maintenance strategy.

3.2. The production time constraint

(1) The production time constraint in $\left[0,\left(\lfloor Z/k \rfloor k\right)T\right]$

As the quantity of product i produced in period j is x_{ij}, the actual total production time in period j is $\xi_j=\sum_{i=1}^{n} t_i x_{ij}$. Specifically, in PM interval $\left[\hat{z}kT,(\hat{z}+1)kT\right]$, the period number j ranges from $\hat{z}k+1$ to $\hat{z}k+k$. We use $L_j\left(x_{ij},y_{ij},T_{PM}\right)$ to denote the production time constraint in period j, that is, $\sum_{i=1}^{n} t_i x_{ij} \le L_j\left(x_{ij},y_{ij},T_{PM}\right)$.

The first period in PM interval $\left[\hat{z}kT,(\hat{z}+1)kT\right]$ is $\left[\hat{z}kT,(\hat{z}k+1)T\right]$. In this period, $j=\hat{z}k+1$, the expected number of failures is $\int_0^{\xi_{\hat{z}k+1}}\lambda\int_0^t f(t-u)dudt=\int_0^{\xi_{\hat{z}k+1}}\lambda F(t)dt$, the expected downtime caused by failures is $d_f \int_0^{\xi_{\hat{z}k+1}} \lambda F(t)dt$, [13], and the downtime caused by setup in this period is $\sum_{i=1}^{n} d_s^i y_{i,\hat{z}k+1}$. Thus, we have $\xi_{\hat{z}k+1} + d_f \int_0^{\xi_{\hat{z}k+1}} \lambda F(t)dt + \sum_{i=1}^{n} d_s^i y_{i,\hat{z}k+1} \le T$, that is,

$$L_{\hat{z}k+1}\left(x_{i,\hat{z}k+1}, y_{i,\hat{z}k+1}, T_{PM}\right) = T - d_f \int_0^{\xi_{\hat{z}k+1}} \lambda F(t)dt - \sum_{i=1}^{n} d_s^i y_{i,\hat{z}k+1} \,. \quad (1)$$

Similarly, the time constraint in the subsequent periods until the penultimate period can be expressed as

$$L_{\hat{z}k+k'}\left(x_{i,\hat{z}k+k'}, y_{i,\hat{z}k+k'}, T_{PM}\right) = T - d_f \int_{\sum_{a=1}^{k'-1} \xi_{\hat{z}k+a}}^{\sum_{a=1}^{k'} \xi_{\hat{z}k+a}} \lambda F(t)dt - \sum_{i=1}^{n} d_s^i y_{i,\hat{z}k+k'}, \quad (2)$$

where $k' = 2, \cdots, k-1$.

The PM is carried out in the last period, where $j = \hat{z}k + k$; the number of defects identified at PM is $\int_0^{\sum_{a=1}^{k} \xi_{\hat{z}k+a}} \lambda\left[1 - F(t)\right]dt$, so the expected downtime caused by PM is $d_p + d_d \int_0^{\sum_{a=1}^{k} \xi_{\hat{z}k+a}} \lambda\left[1 - F(t)\right]dt$. The time constraint in this period can be given by:

$$L_{\hat{z}k+k}\left(x_{i,\hat{z}k+k}, y_{i,\hat{z}k+k}, T_{PM}\right) = T - d_f \int_{\sum_{a=1}^{k-1} \xi_{\hat{z}k+a}}^{\sum_{a=1}^{k} \xi_{\hat{z}k+a}} \lambda F(t)dt - d_p - d_d \int_0^{\sum_{a=1}^{k} \xi_{\hat{z}k+a}} \lambda\left[1 - F(t)\right]dt - \sum_{i=1}^{n} d_s^i y_{i,\hat{z}k+k}. \quad (3)$$

(2) The production time constraint in $\left[\left(\lfloor Z/k \rfloor k\right)T, ZT\right]$

In $\left[\left(\lfloor Z/k \rfloor k\right)T, ZT\right]$, j can be denoted as $\lfloor Z/k \rfloor k + 1, \cdots, Z$ orderly, in each period. We only consider the downtime caused by failures and setup, since PM is not carried out during this interval. The model of the production time constraint is similar to Equations (1) and (2), so we will not express it here.

3.3. The cost model of maintenance and unqualified products

(1) The expected maintenance cost

Based on Section 3.2, during each PM interval $\left[\hat{z}kT, \left(\hat{z}+1\right)kT\right]$, the actual production time is $\xi^{\hat{z}} = \sum_{j=\hat{z}k+1}^{\hat{z}k+k} \xi_j = \sum_{j=\hat{z}k+1}^{\hat{z}k+k} \sum_{i=1}^{n} t_i x_{ij}$, the expected number of failures is $\int_0^{\xi^{\hat{z}}} \lambda F(t)dt$, and the expected number of defects identified at PM is $\int_0^{\xi^{\hat{z}}} \lambda\left[1 - F(t)\right]dt$. The failure and the repair cost of the defects are $C_f \int_0^{\xi^{\hat{z}}} \lambda F(t)dt$ and $C_d \int_0^{\xi^{\hat{z}}} \lambda\left[1 - F(t)\right]dt$, respectively. The PM cost is C_p.

The actual production time during $\left[\left(\lfloor Z/k \rfloor k\right)T, ZT\right]$ can be expressed as $\xi' = \sum_{j=\lfloor Z/k \rfloor k+1}^{Z} \xi_j = \sum_{j=\lfloor Z/k \rfloor k+1}^{Z} \sum_{i=1}^{n} t_i x_{ij}$, and the expected failure repair cost is $C_f \int_0^{\xi'} \lambda F(t)dt$.

Then, the total maintenance costs during the planning horizon can be expressed as:

$$E_C(m) = \sum_{\hat{z}=0}^{\lfloor Z/k \rfloor - 1} \left\{ C_f \int_0^{\xi^{\hat{z}}} \lambda F(t)\mathrm{d}t + C_d \int_0^{\xi^{\hat{z}}} \lambda\left[1 - F(t)\right]\mathrm{d}t + C_p \right\} + C_f \int_0^{\xi'} \lambda F(t)\mathrm{d}t. \quad (4)$$

(2) The expected cost of unqualified products

We then consider the quality of products affected by the defects and failures in the system. Based on Assumption (7), a coefficient β will be used to construct the relationship between the expected number of unqualified products, $E_N\left(dp\right)$, and the number of failures: we have $E_N\left(dp\right) = \beta\left[\sum_{\hat{z}=0}^{\lfloor Z/k \rfloor - 1} \int_0^{\xi^{\hat{z}}} \lambda F(t)\mathrm{d}t + \int_0^{\xi'} \lambda F(t)\mathrm{d}t\right]$. The expected cost incurred due to unqualified products in the planning horizon can be given as:

$$E_C(dp) = C_{dp} E_N\left(dp\right). \quad (5)$$

3.4. The integrated model

Based on the models proposed in Section 3.2 and Section 3.3, the integrated cost model can be expressed as follows:

$$\text{Minimize } \sum_{i=1}^{n} \sum_{j=1}^{Z} \left(p_i x_{ij} + b_i B_{ij} + h_i I_{ij} + C_s^i y_{ij}\right) + E_C(m) + E_C(dp) \quad (6)$$

$$\text{Subject to: } I_{ij} - B_{ij} = I_{i,j-1} - B_{i,j-1} + x_{ij} - d_{ij}, \quad (7)$$

$$x_{ij} \le \left(\sum_{j^* \ge j} d_{ij^*} + B_{i,j-1}\right) y_{ij}, \quad (8)$$

$$\sum_{i=1}^{n} t_i x_{ij} \le L_j\left(x_{ij}, y_{ij}, T_{PM}\right), \quad (9)$$

$$y_{ij} \in \left\{0,1\right\}, \quad (10)$$

$$x_{ij},\ I_{ij},\ B_{ij} \ge 0. \quad (11)$$

The objective function (6) consists of the total production cost, the total backorder cost, the total inventory cost, the total setup cost, the expected total maintenance cost as given by Equation (4) and the total expected cost of unqualified products as given by Equation (5) during the planning horizon. Equation (7) relates backorder or inventory at the start and the end of period j to the production quantity and the demand in that period; clearly $I_{ij} > 0$ and $B_{ij} > 0$ cannot hold simultaneously. Equation (8) sets the upper limit of x_{ij}, and if $y_{ij} = 0$ then $x_{ij} = 0$, and if $y_{ij} = 1$ then $x_{ij} \geq 0$. Equation (9) corresponds to the available production time constraint in period j, the detail of which was proposed in Section 3.1. Equation (10) indicates that y_{ij} is a binary variable. Equation (11) indicates that x_{ij}, I_{ij} and B_{ij} are all non-negative numbers.

The integrated cost model, Equations (6)–(11), can be solved by some mathematical programming software (e.g., LINGO or MATLAB). In our paper, we use MATLAB to solve it.

3.5. Validation

The integrated cost model is composed of two parts, the linear function $\sum_{i=1}^{n}\sum_{j=1}^{Z}\left(p_i x_{ij} + b_i B_{ij} + h_i I_{ij} + C_s^i y_{ij}\right)$, and the nonlinear function $E_C(m)$ and $E_C(dp)$. The linear part is commonly used in literature so we pay attention to validate $E_C(m)$ and $E_C(dp)$. Both $E_C(m)$ and $E_C(dp)$ were derived based on assumptions (1) and (2). Here we explain the rationale of the two assumptions. For complex systems, an approximation was made based on the delay-time concept: defect arrivals from all components are grouped and modelled by a stochastic point process, such as an HPP, and the delay times of all defects follow one identical distribution. This HPP approximation has been justified mathematically in [7], and the grouped delay time case was verified by simulation studies and a case study in [6].

4. Numerical example

In this Section, the pdf of the delay-time is assumed to follow an exponential distribution with parameter α. Previous delay-time-based case studies have used an exponential distribution as the delay-time [4], and this was chosen based on the best fit to the actual data. Subsequently, a simulation test was carried out to confirm that, for a complex system with many components, the pooled delay-time followed an exponential distribution approximately [6].

We now present a real case to demonstrate the integrated cost model proposed in Section 3. The data in this section was collected from our recent visit to a factory producing steel gratings. There are several production systems in this factory, and we chose a system that produces two types of steel gratings, that is, n=2. The production unit is 'ton', and the demands of products are as shown in Table 1. The planning horizon is Z=6 months with T=30 days (the time unit is 'day'). Table 2 shows the time and cost parameters, and the other parameters are λ=0.0462, α=0.0833 and β=0.5.

Using Equations (1)–(11) and the parameters given above, we can calculate all of the optimal results corresponding to different PM intervals, as shown in Table 3. It can be seen that the minimum total cost is 1.1934e+07 when the PM interval is 2 months, that is, carrying out the PM bi-monthly is the best choice for this system.

Table 4 shows the detailed results with the optimal PM policy, and we can see that I_{1j}=0, B_{1j}=0, $I_{2j} \neq 0$ (j=1~4) and $B_{25} \neq 0$. Both the inventory and backorder occur for product 2, but neither occur for product 1. This is because $h_1 > h_2$ and $b_1 > b_2$. To confirm this, changing the inventory cost parameters to h_1=50 and h_2=60, we can get the optimal production plan as shown in Table 5, where $I_{1j} \neq 0$ (j=1~4) and I_{2j}=0. Table 4 also shows the production planning in every period. The factory can arrange its production and maintenance according to our model and results.

Table 1. Demands of products in every period.

	j=1	j=2	j=3	j=4	j=5	j=6
d_{1j}	700	1050	1000	880	1300	600
d_{2j}	1100	700	800	900	580	1200

Table 2. The time and the cost parameters

	t_i	d_s^i	C_s^i	p_i	h_i	b_i
i=1	0.0167	0.0139	55	1200	50	1400
i=2	0.0165	0.0135	50	1000	40	1200
d_d	d_f	d_p	C_p	C_d	C_f	C_{dp}
0.012	0.0417	0.0174	20	200	800	25

Table 3. The optimal total cost with different PM intervals.

T_{PM}	1 month	2 months	3 months	4 months	5 months	6 months
Total cost	1.1936e+07	***1.1934e+07***	1.1935e+07	1.1935e+07	1.1936e+07	1.1935e+07

Table 4. The optimal production plan with T_{PM}=2 months.

Period	Product 1				Product 2			
	x_{1j}	I_{1j}	B_{1j}	y_{1j}	x_{2j}	I_{2j}	B_{2j}	y_{2j}
1	700	0	0	1	1105.8	5.8	0	1
2	1050	0	0	1	749	54.8	0	1
3	1000	0	0	1	802.2	57	0	1
4	880	0	0	1	921	78	0	1
5	1300	0	0	1	498.5	0	3.5	1
6	600	0	0	1	1203.5	0	0	1

Table 5. The optimal production plan with h_1=50, h_2=60 and T_{PM}=2 months.

Period	Product 1				Product 2			
	x_{1j}	I_{1j}	B_{1j}	y_{1j}	x_{2j}	I_{2j}	B_{2j}	y_{2j}
1	705.8	5.8	0	1	1100	0	0	1
2	1098.3	54.1	0	1	700	0	0	1
3	1002.2	56.3	0	1	800	0	0	1
4	900.8	77.1	0	1	900	0	0	1
5	1222.9	0	0	1	576.5	0	3.5	1
6	600	0	0	1	1203.5	0	0	1

5. Conclusion

An integrated production and periodic PM planning model was proposed at the tactical level for a complex system, with the delay-time concept used to model the PM decision for the system. A real case was studied as a numerical example. Using this integrated model, we can decide which products need to be produced and their exact production quantities in each period, and the optimal PM interval can be calculated simultaneously. From the example, we can see that the production quantities were also influenced by the inventory cost and backorder cost of different products. Of course, this study has certain limitations, which leads to the need for further research: 1) considering the integrated production and PM planning with a sequence-dependent setup structure; 2) considering non-periodic PM policies; and 3) considering several types of defects and failures that may have different impacts on the cost and downtime of the production systems.

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