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Adrian GILL
Adam KADZIŃSKI

THE DETERMINATION PROCEDURE OF THE ONSET OF THE OBJECT WEAR-OUT PERIOD BASED ON MONITORING OF THE EMPIRICAL FAILURE INTENSITY FUNCTION

PROCEDURA WYZNACZANIA POCZĄTKU STARZENIA SIĘ OBIEKTÓW NA PODSTAWIE MONITOROWANIA EMPIRYCZNEJ FUNKCJI INTENSYWNOŚCI USZKODZEŃ*

The estimation of the number of failures of technical objects is of key importance throughout the object life cycle, particularly in the wear-out period when the number of failures begins to grow significantly. In the literature related to this problem, examples exist of solutions (mathematical models) that can assist the estimation of the number of failures. For the description of the life cycles of objects, functions are usually used of known forms of probability distribution of the number of object failures. The procedure presented in this paper assumes the use of statistical data related to the failures of uniform population of nonrenewable technical objects, recorded in the form of empirical function of failure intensity. It specifically serves the purpose of determining the characteristic point of life of these objects i.e. the onset of the wear-out period. Within the procedure, a model of fuzzy inference has been applied that reflects the human reasoning (expert of the system) observing/investigating the objects. The results of the developed procedure may constitute a basis for forecasting of failures of mechanical nonrenewable technical objects.

Keywords: *failure intensity function, aging, nonrenewable objects, failure forecasting.*

Oszacowanie liczby uszkodzeń obiektów technicznych ma kluczowe znaczenie we wszystkich okresach cyklu życia obiektów, szczególnie w okresie uszkodzeń starzeniowych, kiedy to liczba uszkodzeń zaczyna znacząco rosnąć. W bibliografii tego zagadnienia przytoczone są przykłady rozwiązań (modeli matematycznych), którymi można wspomagać m.in. szacowanie liczby uszkodzeń. Do opisu cyklu życia obiektów technicznych wykorzystuje się zwykle funkcje o znanych postaciach rozkładów prawdopodobieństwa liczb uszkodzeń tych obiektów. Przedstawiona w niniejszym artykule procedura, zakłada korzystanie ze statystycznych danych o uszkodzeniach jednorodnej zbiorowości nieodnawianych obiektów technicznych, zapisanych w postaci empirycznej funkcji intensywności uszkodzeń. Służy ona w szczególności do wyznaczenia charakterystycznego punktu życia tych obiektów tj. chwili rozpoczynania się okresu uszkodzeń starzeniowych. W ramach procedury zastosowano model wnioskowania rozmytego, który odwzorowuje rozumowanie człowieka (eksperta systemu) obserwującego/badającego obiekty. Wyniki opracowanej procedury mogą stać się podstawą prognozowania uszkodzeń nieodnawianych obiektów technicznych typu mechanicznego.

Słowa kluczowe: *funkcja intensywności uszkodzeń, starzenie, obiekty nieodnawiane, prognozowanie liczby uszkodzeń.*

1. Introduction

In all periods of the object life cycle the number of failures needs to be forecasted. It is required by the demand estimation processes related to renewable objects, the need to configure maintenance systems that allow for the number of renewals as well as object proactive behavior, the lack of which may generate unacceptable hazard.

This problem is particularly conspicuous when analyzing the wear-out period and the usually surging number of failures in that period (assuming that the investigated population is sufficiently large). Relevant literature presents mathematical models that assist the process of reaction to object failure e.g. [2, 3, 10, 12], procedures that describe and compare classes of forecast models (e.g. [4, 17]) or assist in the forecasting of failures of nonrenewable technical objects in the wear-out period (e.g. [10, 12]). In [2] a method is proposed of detecting the onset of the object wear-out period and determining the maintenance efficiency based on, as the authors of [2] would call it, a step model of aging and the Bayes techniques. In [3] a new reliability model is presented of complex repaired technical objects/systems based on the bathtub curve.

The procedure presented in this paper is dedicated to uniform nonrenewable mechanical objects. As a starting point, reference to the forecasting tool of the failure of nonrenewable technical objects was assumed. These tools were developed by one of the authors of this paper (*i.a.* [10, 11, 12]). The basis for the failure forecasting models is the estimation of the parameters of object operating time distribution until wear-out failure occurs. It was assumed that the parameters of this distribution are estimated based on statistical data related to:

- number of object failures occurring in the period between the onset of the wear-out period and the end of the observation time,
- number of objects that are forecasted to fail due to aging.

A troublesome point of the said models is the determination of the onset of the wear-out period that is necessary for the estimation of the

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** parameters of the operating time distribution until failure. The significance of this issue is also supported by other authors ([1]) by mentioning the return point of the failure intensity function as useful in terms of maintenance and risk analysis related to failures. They present a certain way of solving problems basing on the modified function of Weibull distribution.

In relevant literature it is difficult to indicate formal algorithms allowing the determination of the onset of wear-out periods. Few publications in this matter pertain mainly to the attempts to find new forms of functions describing the processes of object operation [6, 9] or focus on modeling of the entire course of the function of failure intensity ([19]). A reliable solution is proposed by the authors of [2], but only for known continuous distributions. What is missing is the solution to the problem if the failure intensity function is a non-continuous characteristics and its form as a function is unknown.

The intention of the authors of this paper is to present the procedure of estimation of the onset of the wear-out period of objects based on the empirical function of failure intensity without having information on the reliability function.

2. Formal description of the procedure

2.1. Concept and main assumptions

The solution to the problem is the analysis of the data on the number of failures in subsequent periods of time of their investigation/observation and then selection of the moment when the number of failures begins to grow significantly. If information in the form of non-continuous functions is used, the top or bottom limit of a given interval group is assumed (interval in which the number of failures grows significantly and its growth continues in further intervals).

The simplest but least accurate and informal method to solve the problem is intuitive choice of the onset of the wear-out period by the researcher (expert) based on the analysis of the course of selected reliability functions. It is advantageous in the case of functions of untypical courses and allows (due to lack of other tools) a quick obtainment of a satisfactory result.

This enabled an adoption of the following concept of the procedure: a man well acquainted with the modeled system/object, i.e. system expert, may correctly indicate the onset of the wear-out period of objects even if he infers having limited (partial) information on the object failures. Such a subjective choice is usually made because of experience and knowledge about the object. From the observations conducted by the authors we know that a man who is not a system expert but has all the necessary needed information related to the time of the loss of object worthiness (data in the form of courses of functions of failure intensity) is also capable of deciding about the onset of the wear-out period.

This paper reproduces (through fuzzy inference models) the system expert's reasoning that leads to the indication of the onset of the wear-out period based on observations of the course of the empirical function of failure intensity. Literature mentions applications of elements of fuzzy inference to solve a variety of problems related to reliability of objects. For example [18] presents the application of fuzzy sets in the problem of matching curves to the reliability data, [20] describes its use in reliability analysis of elements while [7, 13] discuss the application of fuzzy inference in methods designed to determine the measures of reliability.

The concept of the procedure consists in determining moment t_p – the onset of the wear-out period of technical objects through mathematical models that reproduce (simulate) the reasoning of the system expert. The mathematical model was developed based on the following main assumptions:

- Uniform population of technical objects is analyzed,
- The number of failures of technical objects in time is known (observation time intervals) and the statistical data on the failures are stored in the form of stemplot,
- The type of probability distribution of the operating time of objects until failure is unknown,
- Fuzzy inference is possible based on the results of monitoring of the empirical value of the function of failure intensity $\lambda_N(t)$,
- The onset of the wear-out period t_p falls between moment t_{p0} of the first increase of function $\lambda_N(t)$ and moment t_k – the end of object observation,
- The failure intensity function is a constant interval non-decreasing function in the wear-out period.

2.2. General mathematical model

The structure of the inference models is formed by properly written rules of inference i.e. fuzzy implications R_k ($k = 1,2,\ldots,l$) [8, 14, 15]. These are the if-then type of rules that in a general form can be written as follows:

$$R_k\text{: If } x_{(1)} \text{ is } A_{1j} \text{ and } x_{(2)} \text{ is } A_{2j} \text{ and} \ldots \text{and } x_{(m)} \text{ is } A_{mj} \text{ then } y \text{ is } B_j \quad (1)$$

where:

$x_{(i)}$ – input variables of the inference model forming the m-dimensional input vector **x**. It has been assumed that at the first stage of the calculations, variables $x_{(i)}$ assume the values of the empirical function of failure intensity of objects $\lambda(t_i)$ in subsequent i-th *intervals* (t_{i-1}, t_i) *of the function monitoring* $\lambda_N(t)$ i.e. $x_{(i)} = \lambda_N(t_i)$, ($i = 1,2,\ldots, m$);

A_{ij} – ($i = 1,2,\ldots, m$; $j = 1,2,\ldots, n$) denote the linguistic values (parameters of the inference model) defined in a fuzzy manner by appropriate functions of membership $\mu_{A_{ij}}$ determined in spaces X_i. If A_{ij} is a fuzzy set in a given space X_i then value $\mu_{A_{ij}}(x_{(i)})$ will denote the degree of membership $x_{(i)} \in X_i$ in set A_{ij};

y – output variables of the inference model;

B_j – fuzzy sets of the conclusion of the inference rules.

Graphic interpretations of the assumptions and the understanding of some of the elements of the presented procedure have been shown in Figure 1.

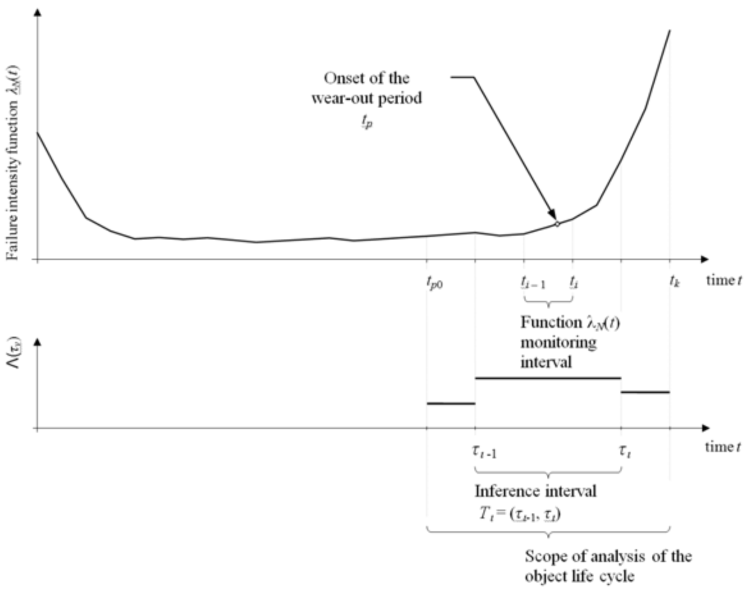

Fig. 1. Graphic interpretation of the assumptions of the determination procedure of the onset of the wear-out period based on the monitoring of the failure intensity function

In further considerations, the constructive model of inference was applied [8]. In models of this type the value of $\mu_A(\mathbf{x})$ of the function of membership related to the degree of rule activation, is interpreted in the form of a logical product of fuzzy sets. In the fuzzy sets the product operation (as well as the sum of these sets) can be performed in different ways. In the literature, many different relations for each of these operations have been presented. For a logical sum of the fuzzy sets there is a group of relations referred to as the s-norm operators and for the logical products - a group of t-norm operators. For example, to calculate the logical product of fuzzy sets a minimum (*MIN)* operator can be used – relation (2):

$$\mu_A(\mathbf{x}) = \mu_{A_{1j} \cap A_{2j} \cap \ldots \cap A_{mj}}(\mathbf{x}) = \min\{\mu_{A_{1j}}(x_{(1)}), \mu_{A_{2j}}(x_{(2)}), \ldots, \mu_{A_{mj}}(x_{(m)})\} \tag{2}$$

The *MIN* operator has many disadvantages (as described in detail in [15]), which is why the product operator is more frequently used. The calculation of the function of membership of the product of fuzzy sets with the use of this operator is done according to the following formula:

$$\mu_A(\mathbf{x}) = \mu_{A_{1j} \cap A_{2j} \cap \ldots \cap A_{mj}}(\mathbf{x}) = \mu_{A_{1j}}(x_{(1)}) \cdot \mu_{A_{2j}}(x_{(2)}) \cdot \ldots \cdot \mu_{A_{mj}}(x_{(m)}) \tag{3}$$

It was assumed that the aggregation on the implication level is realized as an algebraic product of the degrees of membership of the fuzzy sets (relation (3)) for both the implication premise and the consequent.

The output of the inference models is made by the superposition of the outputs of individual inference rules. It consists (based on the R_k rules) in the reproduction of the realization of the input variables $x_{(i)}$ into a certain output quantity *y* representing moment $\hat{t}_p$.

The first step of this procedure consists in combining (for certain input data) the premises (antecedents) of the *k*-th fuzzy rule. We may use the operation of the product of sets – relation (2) or (3). In this way we determine ζ – degree of rule activation (activity) of the rule. Since, the inputs are non-fuzzy values the degree of rule activation ζ of each of the rules forming the database of inference rules can be determined as follows:

$$\zeta = \mu_A(\mathbf{x}) \, . \tag{4}$$

Assuming that the database of inference rules is composed of *l*-th number of inference rules, another step of the procedure is the determination of the fuzzy sets C_k (k = 1, 2,…, l) derived by the *k*-th rule. Let sets C_k be determined in certain space *y* in the following way:

$$\mu_{C_k}(y) = \zeta_k \cdot \mu_{B_j}(y) \tag{5}$$

where:

ζ_k – denotes degree of rule activation (activity) of the *k*-th inference rule determined according to relation (4).

By performing the aggregation of sets C_k we may obtain value *C* for the output value *y* as a relation:

$$y \; is \; C \tag{6}$$

wheras *C* is a fuzzy subset determined in space *y*.

The aggregation of fuzzy sets C_k can be performed in many ways [8, 15, 16]. For example, one may use the operation of logical sum of the fuzzy sets i.e.:

$$C = \bigcup_{k=1}^{l} C_k \, , \tag{7}$$

$$\mu_C(y) = \mu_{C_1 \cup C_2 \cup \ldots \cup C_l}(y) = \max[\mu_{C_1}(y), \mu_{C_2}(y), \ldots, \mu_{C_l}(y)] \tag{8}$$

2.3. Detailed mathematical model and the method algorithm

The first stage of determining of moment t_p is evaluating the input value $x_{(i)}$ of the inference models. It was assumed that this evaluation can be done by a minimum number (two) linguistic terms (w = 2). A finite set Φ of these terms takes the form:

$$\Phi = \{small, large\} \, . \tag{9}$$

Linguistic terms are written in the form of fuzzy sets A_j (j = 1, 2) of polygonal [8, 15] (triangular and trapezoidal) functions of membership. These sets are regular convex fuzzy sets [16] of the support limited with values *a*, *b*, *c*, *d*.

The database of inference rules, with a relatively large number of $\lambda_N(t)$ function monitoring intervals may have an excess number of rules. In order to reduce this number, it is proposed to search for moment t_p in a limited range. This is referred to as the *life cycle analysis range* (marked in Fig. 1). The range covers the period between moment t_{p0} – of the first increase of function $\lambda_N(t)$ and moment t_k – end of object observation. It was assumed that moment t_{p0} equals the onset of the monitoring interval where the first positive increment of function $\lambda_N(t)$ takes place.

In the range of analysis (t_{p0}, t_k) *four inference intervals* are then introduced $T_\iota = (\tau_{\iota-1}; \tau_\iota)$, ($\iota$ = 1, 2, 3, 4). They are created by a combination (Fig. 1) of the subsequent *monitoring intervals* (t_{i-1}; t_i) (i = 1,2,…, m). In such a case number s must be determined i.e. the number of monitoring intervals that compose a single inference interval ($\tau_{\iota-1}$; τ_ι). The preliminary number *s* of monitoring intervals is obtained from relation:

$$s = \mathrm{ent}\left(\frac{t_k - t_{p0}}{4 \cdot \Delta t_{i-1,i}} + \frac{1}{2}\right), \tag{10}$$

where:

$\Delta t_{i-1,i}$ – length of the $\lambda_N(t)$ function monitoring interval.

Such a method of creating inference intervals results in a situation when, in some cases, the sum of the lengths of these intervals $\Delta\tau_{\iota-1,\iota}$ exceeds the end of the *range of life cycle analysis*. If such a situation occurs, a shortening of each of intervals T_ι is admissible by length $\Delta t_{i-1,i}$, i.e. by the length of one monitoring interval. It is also proposed that the shortening be realized starting from the last (ι = 4) of intervals T_ι. As a result of such an operation, lengths $\Delta\tau_{\iota-1,\iota}$ of intervals T_ι that will eventually be used in the inference model, may differ from one another. Number *s* (relation (10)) will thus be dependent on the T_ι interval number, which is further marked as $s^{(\iota)}$.

In further calculations, the values of functions $\Lambda(\tau_\iota)$ obtained according to relation (11) were assumed as input variables of the inference model:

$$\Lambda(\tau_\iota) = \frac{1}{s^{(\iota)}} \cdot \sum_{i=p0-s^{(\iota)}+S}^{p0+S-1} \lambda_n(t_i) \, , \text{ oraz } S = \sum_{j=1}^{\iota} s^{(j)} \;\; (\iota = 1, 2, 3, 4), \tag{11}$$

where $p0$ is the subsequent number of interval (t_{i-1}; t_i) in which the first increase of function $\lambda_N(t)$ was observed (monitoring).

Moment t_p was described with fuzzy numbers $L_{\iota\, w}$ related to individual inference intervals T_ι. Fuzzy numbers $L_{\iota\, w}$ were written as follows:

$$"after\ \mathrm{T}_\iota" = L_{\iota 1} \quad \text{and} \quad "near\ \mathrm{T}_\iota" = L_{\iota 2}\,, \qquad (12)$$

and expressed through appropriate fuzzy sets $B_j(y)$, (j = 1, 2,…, 8):

$$\underset{\iota=1,2,3,4}{\forall} \; L_{\iota 1} \rightarrow B_j(y) \quad \text{and} \quad L_{\iota 2} \rightarrow B_{j+1}(y)\,. \qquad (13)$$

To describe fuzzy sets $B_j(y)$, j = 1, 2,…, 8, triangular forms of the membership function were used. The sets support points (a, b, c) are within the limits of relevant monitoring intervals, which, using earlier adopted symbols, can be written as follows:

$$a = \begin{cases} t_{p0-s^{(\iota)}+S} & \text{dla} \quad L_{\iota 1} \\ t_{p0-s^{(\iota)}+S} + \dfrac{\Delta\tau_{\iota-1,\iota}}{2} & \text{dla} \quad L_{\iota 2} \end{cases},$$

$$b = \begin{cases} t_{p0-s^{(\iota)}+S} + \dfrac{\Delta\tau_{\iota-1,\iota}}{2} & \text{dla} \quad L_{\iota 1} \\ t_{p0-1+S} & \text{dla} \quad L_{\iota 2} \end{cases}, \qquad (14)$$

$$c = \begin{cases} t_{p0-1+S} & \text{dla} \quad L_{\iota 1} \\ t_{p0-1+S} + \dfrac{\Delta\tau_{\iota,\iota+1}}{2} & \text{dla} \quad L_{\nu 2} \end{cases},$$

In the case of the fuzzy number „near T_ι" = L_{12}, trapezoidal extreme function of membership was applied.

The general algorithm of the procedure in a graphical form has been shown in Figure 2.

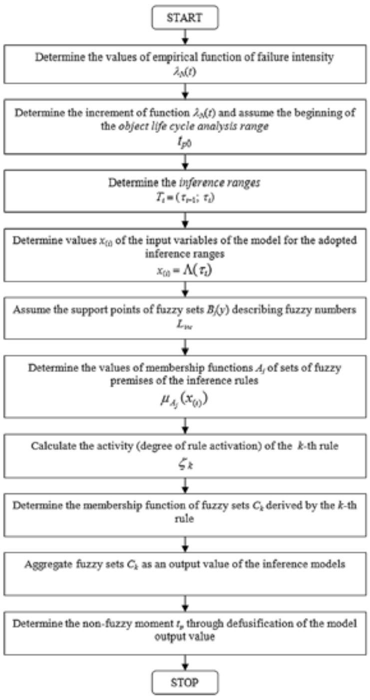

Fig. 2. General algorithm of the determination procedure of the onset of the wear-out period based on the monitoring of the failure intensity function

3. Example of procedure realization

The example of the monitoring of the course of the function of object failure intensity was performed for 100 nonrenewable railroad objects (locomotives). The objects were observed for the time corresponding to the mileage of 600.000 km. During the investigations, in the subsequent intervals $\Delta l_{i-1,i}$ = 50000 km of the locomotive mileage, the number of failures was recorded. The results (number of failures and the value of the empirical function of failure intensity) have been shown in Table 1.

Table 1. Record of failure information of nonrenewable railroad objects

Interval i	Bottom interval limit t_{i-1}	Top interval limit t_i	Number of failures $n(\Delta t_{i-1,i})$	Accumulated number of failures $n_{sk}(t_i)$	Values of the empirical function of object failure intensity $\lambda_N(t_i)$
1	0	50	17	17	0.0000034
2	50	100	11	28	0.0000027
3	100	150	9	37	0.0000025
4	150	200	7	44	0.0000022
5	200	250	6	50	0.0000021
6	250	300	5	55	0.0000020
7	300	350	5	60	0.0000022
8	350	400	4	64	0.0000020
9	400	450	3	67	0.0000017
10	450	500	4	71	0.0000024
11	500	550	6	77	0.0000041
12	550	600	8	85	0.0000070

Source: based on [5]

In further part of the paper the results of the realization of selected steps of the procedure algorithm have been presented. Figure 3 shows the course of the function of object failure intensity.

In the initial stage of the calculations the life cycle analysis range is determined based on the value of function $\lambda_N(t)$. To this end, the increment of the function must be determined. Its first increase was recorded in the intervals from 300000 to 350000 km of the object operation i.e. 7th (p0 = 7) failure record interval. For this interval the increment of function $\lambda_N(t)$ was 2.22222E-07. The subsequent increments of the function were: 7.57576E-07 (for the 10th interval), 1.71369E-06 (for the 11th interval) and 2.81859E-06 (for the 12th interval). The beginning of the life cycle analysis range was thus assumed to be 300000 km.

Based on the assumed beginning of the analysis range, numbers $s^{(\iota)}$ of the combined intervals (t_{i-1};t_i) were determined following relation (10). Numbers $s^{(\iota)}$ were: $s^{(1)}$ = 2, $s^{(2)}$ = 2, $s^{(3)}$ = 1, $s^{(4)}$ = 1. The values of the input variable of the inference model $\Lambda(\tau_\iota)$ were also determined (relation (11)). Some of the results have been shown in Table 2.

The output value of the inference model (fuzzy onset of the wear-out period of nonrenewable railroad vehicle objects that were subjected to analysis) has been shown in figure 4 in a graphical form.

In order to determine the non-fuzzy $\hat{t}_p$, an operation of defusification of the model output value by the COA (*Center Of Area*) method was performed. The non-fuzzy onset of the wear-out period obtained according to the said procedure is 428215 km.

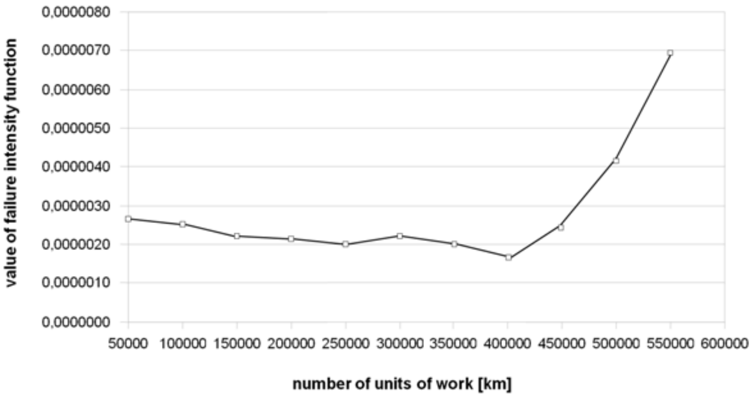

Fig. 3. Course of the function of failure intensity of example nonrenewable rail vehicles

Table 2. Values of the membership function of the fuzzy sets related to the premises of the inference rules in the example problem of determination of the onset of wear-out period of nonrenewable railroad objects

Subsequent number of the inference interval ι	Bottom and top values of the inference interval $T_\iota = (\tau_{\iota-1};\tau_\iota)$		Values of the input variable of the inference model $\Lambda(\tau_\iota)$	Values of the membership function of the fuzzy sets	
	$\tau_{\iota-1}$	τ_ι		$\mu_{A_1}(x_{(i)})$	$\mu_{A_2}(x_{(i)})$
1	300000	400000	1.1111E-07	0.754	0.246
2	400000	500000	3.7879E-07	0.160	0.840
3	500000	550000	1.7137E-06	0.000	1.000
4	550000	600000	2.8186E-06	0.000	1.000

Source: Own findings

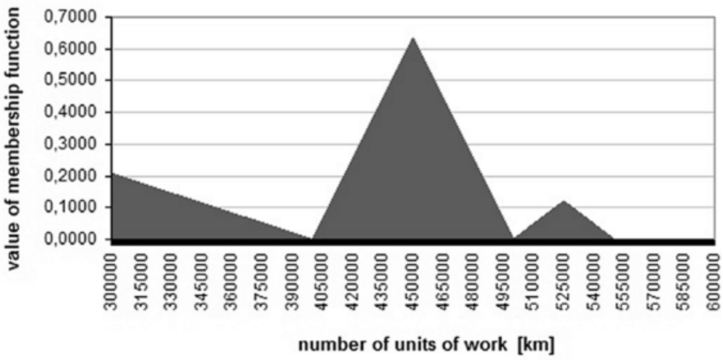

Fig. 4. Fuzzy onset of the example wear-out period of nonrenewable railroad objects

4. Conclusions

The authors attempted to present a concept of determination of the onset of the wear-out period of uniform nonrenewable technical mechanical objects. According to the analyses conducted, *inter alia,* by the authors of this paper, the determination of this moment is a major issue in forecasting of the number of failures of a variety of objects.

The presented concept of the procedure assumes the application of statistical data related to the time of object operation until failure, recorded in the form of empirical function of failure intensity. Usually, during observations/investigations, this sort of information about the failures (number of failures) is recorded in subsequent time intervals and then characteristics in the form of empirical reliability function are created.

According to the presented procedure, it is not necessary to explore the distribution of probability of the operating time until failure (and/or parameters) that characterizes the failures of the investigated objects. There is no need to use the continuous characteristics and known mathematical models reflecting the stage of wear-out period either. It has been confirmed that the determination of this moment is possible through the application of fuzzy inference model that reflects the reasoning of the human observing/investigating the objects (system expert) and does not require the information indicated herein.

The procedure has been designed to forecast failures of nonrenewable technical objects. It is particularly useful in determining the characteristics point in time of the object operation when the wear-out period begins. The presented approach constitutes a new, unique way of solving the problem. The versatility of the applied modeling originates in the possibility of utilization of the empirical equivalent of the theoretical function of failure intensity (rather than its estimation) as well as the method of fuzzy inference as input information on the observed objects. This, however, requires appropriate database of inference rules that accumulates the knowledge of the system expert.

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Adrian GILL
Adam KADZIŃSKI
Institute of Combustion Engines and Transport
Poznan University of Technology
Piotrowo 3, 60-965 Poznan, Poland
E-mails: adrian.gill@put.poznan.pl, adam.kadzinski@put.poznan.pl