1. Introduction

“Replacement decision” is a classical operation research topic in the industrial engineering. The replacement theory can indicate the optimal equipment life. “Optimal life” can be defined as the period between the time when the equipment comes into service and the time when it should be replaced due economic reasons. The operating cost of an equipment or asset generally rises as their condition deteriorates over time. When the cost reaches a certain level, the long-run costs associated with investment in a new equipment become less than those if keeping the old equipment [6]. At this point, replacement is carried out. Thus, a basic replacement analysis usually examines both the trend in operating and maintenance costs (O&M) and the net cost of replacement, which is defined as the difference between the cost of the new equipment and the salvage value of the old one. In some cases, the replacement analysis also considers the resale value of the equipment at various stages of its service life.

For fleet replacement, the literature suggests two kinds of models: economic engineering (EE) and operational research (OR) models[12, 16]. EE models are restricted to economic and financial aspects, with technological, management and strategic variables considered as exogenous. This limitations force management to avoid formal investment analysis and to use unstructured subjective analysis [5]. Traditional OR models focus on a single objective to be maximized/minimized by modeling multiple variables. These methodologies are complemented by a management tool used for decision-making known as conventional Life Cycle Cost Analysis (LCC)[2]. According to Nowakowski [15] and other authors [7, 18], mathematical models of life cycle costs can be classified into three basic groups:

- models dedicated to technical objects’ manufacturers that are designed to minimize the costs that occurred in the early stages of its lifetime,
- models aimed to minimize the lifetime cost of the facilities already in operation,
- models oriented to customers willing to purchase a new technical object.

The main area of the authors’ interest is the last group of models, which is useful to define the future costs. Furthermore, in this for transport fleet applied model, the LCC is based on engineering economics to identify a point of a given asset’s life at which the cumulative cost of operating (O), maintenance (M) and ownership reaches its minimum value. According to Fan and Jin [9], the most widely accepted approach is called the “cost minimization method”. Granberg and O’Connor [11] describe it as “the most appropriate analysis method” and proposes that it “yields an optimum replacement timing cycle and a corresponding Equivalent Annual Cost (EAC)”. In order to establish their useful life, particularly for buses, it is of key importance the understanding of the concepts explained in the following figure 1.
The theoretical optimum service life is the point at which cumulative costs are at the minimum and it defines the economic life. From a financial standpoint, the cost object of minimum life cycle is the ideal age of retirement and/or replacement. This age can be shown in years of life or mileage travelled. This analysis has to be subject to the same working conditions, in order to observe a similar trend. According to the asset type, design specifications and the service to be performed, it is clear that in most cases total vehicle mileage is a better indication of asset aging than the vehicle’s age.

A viable alternative to conduct a LCC study on vehicles is to use an economic engineering criterion in conjunction with optimization models [4, 13, 14]. Depending on the type of vehicles in a fleet, optimization models can be divided into two categories: homogeneous and heterogeneous [3, 10]. In homogeneous models, the main objective is to find the best time for the replacement of a set of identical vehicles (same type and age) that must be replaced together; this is also known as the “no cluster splitting rule”. These models are usually developed using a dynamic programming approach. Heterogeneous models are more appropriate when different types of vehicles need to be optimized simultaneously or when there are budget constraints. These models can solve more practical problems and the input variables are generally deterministic.

LCC analysis always includes elements of uncertainty because a part of the input data has to be defined on the basis of different estimations assumptions about the development of costs and revenues in a long term. It has been recognized that probability models are useful in handling uncertainty in cost models. Thus, instead of treating the input variables as fixed, such as performance, quality, costs and price requirements, it is more appropriate to quantify them in terms of probability distribution functions. The Monte Carlo method includes all random modeling will be used for estimation. The problem is defined as (x,y) ∈ (Ω,F,P), in which “Ω” is the sample space of events, “F” the algebra of events, and “P” a probability measure. From the following hypotheses are necessary:

1) The total cost function is differentiable;
2) The random variables that define the cost function are limited and have no predictability or a well-defined pattern. Thus, random modeling will be used for estimation. The problem is defined as (x,y) ∈ (Ω,F,P), in which “Ω” is the sample space of events, “F” the algebra of events, and “P” a probability measure.

The problem studied in this work considers variations in the cost function that have no predictability or a well-defined pattern. Thus, random modeling will be used for estimation. The problem is defined as (x,y) ∈ (Ω,F,P), in which “Ω” is the sample space of events, “F” the algebra of events, and “P” a probability measure. From this the following hypotheses are necessary:

1) The total cost function is differentiable;
2) The random variables that define the cost function are limited and statistically independent [1].

As a result, the problem P1 is reformulated as:

\[ \text{Find } (x^*, y^*) \in [R^2, P], \text{ such as } (x^*, y^*) = \arg \min[H(x, y, w)]; \]

The problem defined in Eq. (4) will be solved through Monte Carlo simulation-based methods. This kind of method is developed in three stages:

1) Generate, according to the probability functions of each parameter, N – samples of random variables that model the uncertainty on the parameters that define the function total cost;
2) For the sample of the parameters, solve the following problem:
and the variables

\[ \begin{cases} \text{Find } (x^*_i, y^*_i) \in [R^2, P], \text{ such as } \\ (P3) \quad (x^*_i, y^*_i) = \arg \min H(x, y, w) \\ (x, y, w) \in R^2 \times (\Omega, F, P) \end{cases} \] (5)

III With all these results, analyze the final result with the graphical distribution from the function \( H^*_i (x^*_i, y^*_i) \) and the variables \( x^*_i \) and \( y^*_i \).

In order to generate a model that forecasts the total cost of maintenance and operating simultaneously in function of age and mileage \( f(x, y) \), a regression analysis was developed. Besides, this analysis employing algorithms of minimum distance was carried out using the software "Minitab" in order to find out the best arrangement through a mathematical function. The kind of equation elected to be used in this model was made with respect to the quality and quantity of available data. For the Spanish fleet, in which the data were more abundant in terms of years, a quadratic function was applied (formula 6). Nevertheless, as the Brazilian fleet lacks the same quantity of data, an exponential function was employed in order to have a representative model was made with respect to the quality and quantity of available data. For the Spanish fleet, in which the data were more abundant in terms of years, a quadratic function was applied (formula 6). Nevertheless, as the Brazilian fleet lacks the same quantity of data, an exponential function was employed in order to have a representative function (formula 8). In order to analyze the parameter’s variations that the deterministic approach can’t assess, the Monte Carlo simulation will be employed.

### 2.2.1. The Spanish Urban Transport Fleet

\[ f(x, y) = a x^2 + b y^2 + c x y + d x + e y + f \] (6)

Where:
- \( a, b, c, d, e, f \in R \)

According to this, the constants from the quadratic function will be considered as uniform random variables with a “p” variation of the data field:

\[ X_i \in [(1 - p) x_i; (1 + p) x_i] \] (7)

Where:
- \( X_i \): Set of possible results for a random variable \( x_i \).
- Where: \( x_i=a; x_i=b; x_i=c; x_i=d; x_i=e; x_i=f \) and \( p \in [0; 1] \).

### 2.2.2. The Brazilian Urban Transport Fleet

\[ f(x, y) = a x e^{b y} \] (8)

Where:
- \( a, b \in R \)

The constants from the exponential function will be considered as uniform random variables with a “p” variation of the data field:

\[ X_i \in [(1 - p) x_i; (1 + p) x_i] \] (9)

Where:
- Set of possible results for a random variable .
- Where: \( x_i=a; x_i=b \) and \( p \in [0; 1] \).

### 2.3. Database analysis

#### 2.3.1. The Spanish Urban Transport Fleet

A sample of 34 vehicles was selected and named as “Type A” vehicles. The buses belonging to this sample have the same technical characteristics, mechanical configurations, fuel, and they were exposed to similar operating conditions, such as average speed, stops per mileage, passenger loading, climate conditions, and the largest mileage during lifespan. The analysis period considered was of 10 years (2005-2014), and all the costs were converted and updated using the Spanish economic indicators. The results obtained were extrapolated to the entire lifespan of the vehicles. The following restraints were applied and considered:

1) Averaged fuel consumption was considered as a constant along the vehicle’s lifespan.
2) Annual averaged mileage was constant and determined through all sample selection buses. For this work, the annual averaged mileage was taken as 61.597 km/year.
3) In order to determine the operational costs, fuel, insurance and tax costs were summed up.
4) Resale Value (VR) was calculated by a linear model used by the company and based on its own experience, which is obtained by formula 10:

\[ VR = R + \frac{V_C - R}{N} \times 0.7778 \times R_v \] (10)

Where:
- 0.7778 Factor dependent on service conditions.
- \( R \): Remaining vehicle’s life.
- (R) Residual Value: In accounting, residual value is another name for salvage value, the remaining value of an asset after being fully depreciated. The formula to calculate the residual value for this case study was established in 10% of the purchase cost.
- (Vc) Purchase Cost: the investment cost considered to acquire a new vehicle, similar as type A. To simplify, the investment was considered paid in full at the purchase moment.
- Value: € 240.000,00.

(N) Estimated Lifespan: The estimated age indicated by the company and adopted for this study was 14 years, which is similar to those ones used by other Spanish companies. Notice that this parameter is above the average value in other countries such as the United States, France and Italy, where the considered vehicles’ lifespan is 12 years. Probably, the very important economic crisis suffered by Spain on this period can be the explanation for this increase on the estimated lifespan.

#### 2.3.2. The Brazilian Urban Transport Fleet

A sample of 33 vehicles was selected and named as “Type B” vehicles. The buses belonging to the sample have the same technical characteristics, mechanical configurations, fuel, and they were exposed to similar operating conditions such as average speed, stops per mileage, passenger loading, climate conditions, and the largest mileage during lifespan. The analysis period considered was of 05 years (2011-2015). The results obtained were extrapolated to the entire lifespan of the vehicles. The following restraints were applied and considered:

1) Averaged fuel consumption was considered as a constant along the vehicle’s lifespan.
2. Annual averaged mileage was constant and determined through all sample selected buses. For this work, the annual average mileage was taken as 80,620km/year.
3. In order to determine the operational costs, only the fuel cost was taken into account, as the company previously decided.
4. Resale Value (VR) was taken in the same way as in the Spanish fleet formula 10, and:
   \[(V_c)\] Purchase Cost: Value € 103,926.10.
   \[(N)\] Estimated Lifespan: The estimated age indicated by the company and adopted for this study was 12 years, which is set by the local legislation.

3. Results

For a better understanding, the optimum replacement moment was at first determined by using the conventional Life Cycle Cost Analysis (LCC) methodology. After that, an analysis using the Monte Carlo method was performed and, finally, the results can be compared.

3.1. Life Cycle Cost Analysis

For this case study, some aspect should be taken into account:
1. The maintenance costs were selected and adjusted based on Extrapolation Mathematics Technique, in order to obtain a set of observations and to extend this pattern into the future.
2. The Total Accumulated Cost (TAC) until a certain year is the result of the total investment cost plus the maintenance and operating costs. All the costs were accumulated until that year less the resale value of the same year.

\[
\text{TAC} = \text{Investment Cost} + \text{Maintenance Cost} + \text{Operating Cost} - \text{Resale Value}.
\]

3. The Average Annual Cost (AAC) indicates the cost accrued until the vehicle’s life, divided by its lifetime thereof, so that the minimum average annual cost will determine the optimal time for the vehicle’s renewal, which presents the lowest possible cost for the vehicle operation (formula 10).

\[
AAC = \frac{\text{Investment + Maintenance + Operating} - \text{Resale Value}}{n\text{(year)}},
\]

Results of the LCC Method

The cost development and analysis for the Spanish and the Brazilian fleets are shown in Tables 1 and 2 respectively.

2.3.1. The Spanish Urban Transport Fleet

In summary, the results indicate that the minimal Average Annual Cost per kilometer (AAC) is shown to the Spanish fleet in the 7th year (1,02 €/km), to the Brazilian fleet in the 5th year (0,393 €/km).

3.2. Monte Carlo Simulation

The method was developed considering a constraint in the annual mileage between 55,000 km and 80,000 km for the Spanish fleet (Scenario 1), and between 60,000 km and 100,000 km for the Brazilian fleet (Scenario 2). This technique gives a reliable mathematical basis for solutions derived from individual scenarios, and it can be applied to linear problems in order to improve pure scenario analysis.

### Table 1. Spanish Fleet

<table>
<thead>
<tr>
<th>Year (n)</th>
<th>∑ Purchase Cost</th>
<th>∑ Maintenance Cost</th>
<th>∑ Operation Cost</th>
<th>VR Resale Value</th>
<th>TAC</th>
<th>AAC</th>
<th>AAC/km</th>
</tr>
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<tr>
<td>1</td>
<td>240,000.00</td>
<td>8,581.30</td>
<td>30,143.09</td>
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<td>129,365.81</td>
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<tr>
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<td>307,604.76</td>
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<td>501,120.04</td>
<td>24,086.90</td>
<td>936,354.70</td>
<td>96,946.73</td>
<td>1,09</td>
</tr>
</tbody>
</table>
The data used for the development of the Monte Carlo methodology were processed by using software Matlab.

Results of the Monte Carlo Method

i Age Indicated
The ages indicated for replacement fleet that came from this methodology are graphed in a histogram (figure 2).

The analysis of the histograms for both scenarios show us which of them is the best option for the replacement decision, taking into account the previously defined restrictions. For the Spanish fleet, the 8th year is clearly the most convenient option. For the Brazilian fleet, the 3,98th year is the highest value, although for practical reasons the 4th year is an accurate approximation.

ii Mileage Indicated
The mileages indicated for replacement fleet were obtained using this method and they are graphed as shown in the histogram (figure 3).

The histogram’s analysis of Scenario 1, which presents the best mileage to be used by the vehicle, shows that the values in km are inclined to the established value of 80.000 km (simulation’s restraining condition), and the same trend occurs on the second scenario, which obtains 100.000 Km for ideal annual milleage. This indicates that the restraining conditions don’t represent the minimal cost real points of an unrestrained scenario. An unrestrained analysis shouldn’t be performed because the database had no information about these regions, thus, any inference would not be precise.

iii General Cost Analysis
This analysis shows which parameter(s) would be the best for the fleet’s vehicle use during its lifespan in order to optimize its costs. For this analysis, some parameters are correlated: Annual Mileage, Age of the vehicle and the Annual Average Cost (AAC). Figure 4 shows one sample out of ten thousand possibilities obtained in the scenario 1 (Spanish Fleet). Besides, figure 5 shows the similar analysis for the scenario 2 (Brazilian Fleet). For both figures, the age of replacement is represented in the axis of the abscissa while the average annual mileage is represented in the axis of the ordinates and the annual average cost (AAC), calculated by the function above mentioned in equation 2, is represented by color grading of the intersection point, ranging from dark blue for lower costs to red for higher costs. Furthermore, the figure 4 and 5 indicate the best fit between age and mileage, that minimizing the value of the AAC.

Despite the capacity of indicating the optimal point, this analysis also demonstrates other combinations over its lifespan, and it might effectively support fleet managers to develop their strategies and activities, especially when the fleet managers need to access the trade-off between costs and benefits related to different parameters. Furthermore, a comparison between the Spanish and the Brazilian results could be realized, as it’s shown below (table 3), based on figure 4 and 5.
### Table 3. Fleets Comparison

<table>
<thead>
<tr>
<th></th>
<th>Spanish Fleet</th>
<th>Brazilian Fleet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>8 years</td>
<td>4 years</td>
</tr>
<tr>
<td>Annual Mileage</td>
<td>80,000 km</td>
<td>100,000 km</td>
</tr>
<tr>
<td>Cost per Km</td>
<td>€1,00</td>
<td>€0.50</td>
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</table>

### 4. Conclusion

The association of these two models allows not only the evaluation of the proper replacement moment, but also a general cost analysis over the fleet’s useful life, which indicates the age, mileage driven, and the unit cost per kilometer in an organized and simultaneous way over the vehicle’s useful life. Therefore, this research provides the management with the evaluation of the vehicle’s life cycles, since it is a tool that assesses the replacement decision of the vehicles for a similar one. The final replacement decision should take into account not only economic criteria, but also a variety of factors which are different from those previously studied, such as fleet size, real mileage, number of workers and passengers, service quality, governmental transport policies, environment, annual budget, among others. For this case study, the cost per kilometer is much lower to the Brazilian fleet probably due to the lower purchase price, lower costs with fleet labour, fuel, as well as lower direct costs for maintenance, if in comparison to the costs in Spain. Besides, it is important to consider the devalued exchange rate Real/Euro (€1.00 = R$4.33), and the inferiority in terms of available technology and luxury of the Brazilian fleet compared to the Spanish.

Regarding replacement, it may occur sooner in the Brazilian fleet, basically because of the difficult environmental conditions and the traffic that the fleet is subjected to. Therefore, these features and a higher level of use overload the fleet. Finally, another possible function of this model, which will be studied on future researches, could be used to improve the management decision process with an ideal replacement strategy, regarding how many and which vehicles should be replaced.

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