Optimization of Complex Systems Reliability by Firefly Algorithm

Optymalizacja niezawodności złożonych systemów za pomocą algorytmu świetlik*

Algorithms based on swarm intelligence are more and more frequently applied to problems of systems reliability. The article presents the application of a firefly algorithm to the reliability optimization of two systems: bridge and 10-unit, with minimal paths set, minimal cuts set and decomposition methods. The obtained results are presented and compared with the available literature data.

Keywords: system reliability optimization problems, reliability optimization methods, RRAP system, firefly algorithm.

2. Reliability of complex systems

When designing a highly reliable system, it is very important to achieve a balance between reliability, and other resources, such as cost, volume or weight. The problem of optimizing reliability with respect to redundancy (RRAP, reliability redundancy allocation problem) is treated as a nonlinear programming problem which has one or more resources constraints. Among these known systems, two cases were considered: a bridge system and a system consisting of 10 elements.

2.1. Bridge system

The bridge system shown in Figure 1 can be formulated as follows [12, 14]:

\[
\begin{align*}
\text{Max } f(r, n) &= R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 + \\
&- R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \\
\end{align*}
\]

(1)

with constraints taking into account the upper limit of the total volume and weight (V), cost (C) and system weight (W):

\[
\begin{align*}
g_1(r, n) &= \sum_{i=1}^{m} w_i n_i^2 - V \leq 0 \\
g_2(r, n) &= \sum_{i=1}^{m} \alpha_i \left( -\frac{1000}{\ln(n_i)} \right)^p - C \leq 0 \\
g_3(r, n) &= \sum_{i=1}^{m} w_i n_i^{0.25n_i} - W \leq 0 \\
0 &\leq i \leq m, \quad 0 \leq n_i \leq 1, \quad n_i \in \mathbb{Z}^+
\end{align*}
\]

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie www.ein.org.pl
where:

- \( m \) – the number of subsystems in the system,
- \( n_i \) – the number of components in subsystem \( i \),
- \( r_i \) – the reliability of each component in subsystem \( i \),
- \( R_i \) – the reliability of subsystem \( i \),
- \( \alpha \), \( \beta \) – physical features of components,
- \( w_x \), \( v_x \), \( c_x \) – the weight, volume, cost of element in subsystem \( i \).

The parameter settings of the bridge system can be found in the literature. According to [14] the following values shown in Table 1 were selected.

**Table 1. Data used in the bridge system**

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( w_x )</th>
<th>( v_x )</th>
<th>( c_x )</th>
<th>( V )</th>
<th>( C )</th>
<th>( W )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>1.5</td>
<td>1</td>
<td>7</td>
<td>110</td>
<td>175</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.450</td>
<td>1.5</td>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.541</td>
<td>1.5</td>
<td>3</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8.050</td>
<td>1.5</td>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.950</td>
<td>1.5</td>
<td>2</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.2. The system consists of 10 elements

The reliability structure of a 10-unit system is shown in Figure 2 [14].

### 2.2.1. The diagram of 10-unit system

Assuming by \( R_i(x_i) \) reliability of the subsystem \( i \) equals \( 1 - (1 - r_i)^{x_i} \) and \( Q_i = 1 - R_i \) this problem can be formulated as follows [1]:

\[
\max f(x) = R_1 R_2 R_3 R_4 + R_2 R_3 R_6 R_9 (Q_3 + R_3 Q_3) + R_3 R_4 R_9 R_5 (Q_4 + R_4 Q_4) + \ldots
\]

subject to \( m \) constraints:

\[
\begin{align*}
\sum_{i=1}^{10} x_i &\leq b_j, \quad y = 1, 2, \ldots, m, \quad x_i \in \mathbb{Z}^+ \\
\end{align*}
\]

For this system the coefficients \( c_y \), represent random numbers in the range \([0, 100]\), \( r_i \) are generated in the range \([0.65, 0.85]\), whereas the parameter \( b_y = r_{id} = \text{rand}(1.5, 3.5) \sum_{i=1}^{10} c_y x_i \). The values of setting parameters of the model are summarized in Table 2, based on the data available in the literature [14].

### 3. Methods of determining the reliability of systems

In systems defined as having redundant reliability structure, the case of incompatibility of some features of the system with the specified requirements does not lead to system’s failure. The two minimal subsets of elements can be distinguished, giving the possibility of estimating upper and lower bounds of the system reliability [2, 3, 8, 11]:

- minimal path - a set of components whose proper functioning (all) ensure the successful operation of the whole system, hence the failure of one of these elements will cause a failure state for the system as a whole; components of a minimal path are connected in series and the actual reliability structure of the system can be mapped to the structure of a parallel-series, in which the minimal paths are connected in parallel [3];

Denoting by \( P_1, \ldots, P_j \) the minimal paths set of the system, the structure function of the system is given as [2]:

\[
f(x) = 1 - \prod_{i \in \{1, \ldots, r\}} \left( 1 - \prod_{i \in P_i} R_i \right)
\]

- minimal cut - is a set of components, which being in a failure state cause the system malfunction, however damage to any subset of this elements set does not damage the system; elements of the minimal cut are connected in a parallel combination and the real reliability structure of system can be converted to an equivalent series-parallel structure, wherein the minimal cuts are connected in series [3]. If \( C_1, \ldots, C_j \) denote the set of minimal cuts, therefore we have:

\[
f(x) = \prod_{j \in \{1, \ldots, n\}} \left( 1 - \prod_{i \in C_j} (1 - R_i) \right)
\]

One of the known methods for determining the reliability of complex systems is called the decomposition method which consists of performing consecutive structural operations converting a \( n \)-elements object of any structure to a certain number of simple objects with the series-parallel structures for which the reliability can be determined.
3.1. Case: bridge system

There are four minimal paths in the bridge system shown in Figure 1, namely: $P_1 = \{1, 2\}$, $P_2 = \{3, 4\}$, $P_3 = \{1, 4, 5\}$ and $P_4 = \{2, 3, 5\}$. For these minimal paths the function which describes the reliability of the system in terms of its elements has following form:

$$f = 1 - (1 - R_{12}) (1 - R_{24}) (1 - R_{124R_3}) (1 - R_{23R_4})$$

Analyzing minimal cuts set method, the bridge structure is characterized by the following cuts: $C_1 = \{1, 3\}$, $C_2 = \{2, 4\}$, $C_3 = \{2, 3, 5\}$, $C_4 = \{1, 4, 5\}$, and on the basis of Eq.(4) the function is given as:

$$f = [1 - (1 - R_{12}) (1 - R_{13})][1 - (1 - R_{24}) (1 - R_{24})][1 - (1 - R_{12}) (1 - R_{13})][1 - (1 - R_{24}) (1 - R_{24})]$$

As can be seen the bridge system contains sets $\{2, 3, 5\}$, $\{1, 4, 5\}$, which are both the minimal path and minimal cut.

Applying the decomposition method, the reliability of the bridge system can be calculated in terms of decomposed structures reliabilities with respect to the chosen component 5 which are:

$$R_5^{(4)} = \left[1 - (1 - R_1) (1 - R_3)\right] \left[1 - (1 - R_2) (1 - R_4)\right]$$

$$R_5^{(2)} = R_2 R_4 + R_3 R_4 - R_2 R_3 R_4$$

For these specific reliabilities, the total reliability of the bridge system is given by the following formula:

$$R_5^{(5)} = R_2 R_5^{(4)} + (1 - R_2) R_5^{(4)} - R_3 R_5^{(4)} + R_2 R_2 R_3 R_4 - R_2 R_3 R_5 R_4 + R_2 R_3 R_4 R_5 - R_3 R_5 R_4 R_5 + R_2 R_3 R_4 R_5$$

$$+ 2R_2 R_3 R_4 R_5$$

In the case of homogeneous system $(R_i = r)$, the total reliability reduces to form:

$$R_5^{(5)} = 2r^5 + 2r^3 - 5r^4 + 2r^5$$

3.2. Case: structure of 10 elements

In order to evaluate the total reliability of structure shown in Figure 2, methods of minimal cuts set and minimal paths set were used. Applying minimal paths set method one can find eight paths. Therefore we have four minimal paths of order 4: $P_1 = \{1, 2, 3, 4\}$, $P_2 = \{7, 8, 9, 10\}$, $P_3 = \{1, 5, 9, 10\}$, $P_4 = \{1, 2, 6, 10\}$ and their appropriate reliabilities: $\pi(P_1) = R_1 R_2 R_3 R_4$, $\pi(P_2) = R_7 R_8 R_9 R_{10}$, $\pi(P_3) = R_1 R_5 R_9 R_{10}$, $\pi(P_4) = R_1 R_2 R_6 R_{10}$. Then, for four paths of order 6: $P_5 = \{7, 8, 5, 2, 3, 4\}$, $P_6 = \{1, 5, 9, 6, 4, 3\}$, $P_7 = \{7, 8, 5, 2, 6, 10\}$, we have $\pi(P_5) = R_7 R_8 R_5 R_6 R_4 R_3$, $\pi(P_6) = R_1 R_5 R_9 R_6 R_4 R_3$, $\pi(P_7) = R_1 R_2 R_6 R_5 R_9 R_{10}$.

The reliability of the whole system can be described as follows:

$$f = \prod_{i=1}^{n} [1 - (1 - \pi(P_i))] = 1 - (1 - R_1 R_2 R_3 R_4)(1 - R_7 R_8 R_9 R_{10})(1 - R_1 R_5 R_9 R_{10})(1 - R_1 R_2 R_6 R_{10})$$

$$f = 1 - (1 - R_1 R_2 R_3 R_4)(1 - R_7 R_8 R_9 R_{10})(1 - R_1 R_5 R_9 R_{10})(1 - R_1 R_2 R_6 R_{10})$$

Structure of 10 elements is characterized by 16 minimal cuts, including:

- five of second order: $C_1 = \{1, 7\}$, $C_2 = \{1, 8\}$, $C_3 = \{2, 9\}$, $C_4 = \{3, 10\}$, $C_5 = \{4, 10\}$,
- six of third order: $C_6 = \{1, 5, 9\}$, $C_7 = \{2, 6, 10\}$, $C_8 = \{2, 5, 8\}$, $C_9 = \{3, 6, 9\}$, $C_{10} = \{2, 5, 7\}$, $C_{11} = \{4, 6, 9\}$,
- five of fourth order: $C_{12} = \{7, 6, 5, 3\}$, $C_{13} = \{8, 6, 5, 3\}$, $C_{14} = \{7, 5, 6, 4\}$, $C_{15} = \{8, 5, 6, 4\}$, $C_{16} = \{1, 5, 6, 10\}$.

Using Eq.(4), the reliability of the whole structure is determined from the following formula:

$$f = [1 - (1 - R_1) (1 - R_7)][1 - (1 - R_2) (1 - R_8)][1 - (1 - R_3) (1 - R_9)][1 - (1 - R_4) (1 - R_{10})][1 - (1 - R_5) (1 - R_6)]$$

In the case of the decomposition method, performed analysis on the new elements is repeated until structures resulting from replacement of new elements are sufficiently simple for the calculation. Hence the initial structure $R^{(10)}$ can be decomposed with respect to its element 5, which is replaced by “short circuit” and “break”. Therefore, the structure reliability is given as:

$$R^{(10)} = R_5 R_9^{(9)} + (1 - R_5) R_9^{(9)}$$

In the next step, structures $R_9^{(9)}$ and $R_9^{(9)}$ are decomposed with respect to the chosen component 6, for which:

$$R_9^{(8)} = R_9 R_6^{(8)} + (1 - R_9) R_6^{(8)}$$

$$R_9^{(8)} = R_9 R_6^{(8)} + (1 - R_9) R_6^{(8)}$$

The structures $R_6^{(8)}$, $R_6^{(8)}$, $R_6^{(8)}$, $R_6^{(8)}$ obtained in this way are simple structures, therefore their reliability can be easily computed.

4. Firefly algorithm

The firefly algorithm (FA), based on the behavior of fireflies flying towards a light source and their interaction with bioluminescent signals, is one of the algorithms belonging to the group of swarm algorithms. The phenomenon of a firefly moving towards the brighter individual is the basis of the algorithm. One of the rules used in the firefly algorithm is that all fireflies are unisex. Moreover, attractiveness of fireflies is proportional to the intensity of their emitted light, wherein the light intensity determined by the value of the objective function (it is proportional to this value.

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for maximization problems) decreases with increasing distance between the fireflies. If there is no more attractive individual, a firefly moves randomly [16, 17]. Each firefly has a certain light intensity $I$, which varies according to the distance $r$ between two individuals, and attractiveness $\beta$, which is proportional to the light intensity seen by the neighboring fireflies. Therefore, attractiveness ($\beta$) is dependent on distance and the light absorption coefficient $\gamma$ [17]:

$$\beta(r) = \beta_0 e^{-\gamma r^m}, \quad m \geq 1$$

(13)

where $\beta_0$ denotes the attractiveness at $r = 0$.

The movement, during which the firefly $i$ being in the position $x_i$ tries to get closer to the more attractive individual $j$ in the position $x_j$ is determined by the following formula [17]:

$$x_i = x_i + \beta_0 e^{-\gamma r^2_j} (x_j - x_i) + \alpha (\text{rand} - \frac{1}{2})$$

(14)

where $x_i$ is the current position of a firefly $i$, the second term denotes attractiveness and the third term is due to random movement ($\text{rand}$ is a random number generator uniformly distributed in the range $[0, 1]$, and $\alpha \in [0, 1]$).

The general structure of the FA is as follows [7, 16, 17]:

1. Initialize algorithm’s parameters ($\beta_0$, $\gamma$; stopping criterion) and randomly generate initial population of $n$ fireflies; define the objective function $f(x)$.
2. Compute the light intensity of each individual, whereby the light intensity of $i$th firefly $I_i$ is determined by the value of the objective function $f(x_i)$.
3. While the stopping criterion has not been met, do the following:
   - compare all pairs of fireflies in terms of light intensity: if $f_i > f_j$ then move firefly $i$ towards another firefly $j$,
   - determine new values of the objective function $f(x_i)$, evaluate new solutions, update the light intensity.
4. If the stopping criterion has been met, determine the best solution.

The firefly algorithm was originally developed for the continuous optimization problems. Applying it to reliability optimization of selected structures with continuous and discrete decision variables requires certain additional operations. Correctly determining the distance and ways of movement of individuals, in order to ensure the validity of the solutions are the main elements of the algorithm, which should be adapted. We assume that the distance between two fireflies is determined as the norm of the difference between values of the decision variables assigned to the two individuals. The movement of each firefly in the direction of the brighter individual consists in performing the specified number of steps, in which the length of the step does not exceed the predefined maximum changes of values for the continuous variables ($\text{STEP}_\text{MAX}_\text{CV}$) and for the discrete variables ($\text{STEP}_\text{MAX}_\text{DV}$). If after the performed step the firefly finds itself outside the acceptable area, the maximum length of the step is reduced (multiplied by a random number from the range $[0.5, 0.99]$). If after a specified number of trials ($\text{MAX}_P$) the solution does not find itself in the acceptable area the firefly will not move.

### 5. Results of experiments

Using the set values of various parameters in the selected two systems, listed in Tables 1 and 2, many experiments have been performed to investigate the suitability of the firefly algorithm in solving selected reliability problems. As we know, the reliability for minimal cuts (i.e. lower bound) is less than for minimal paths (i.e. upper bound), which represents the basis for seeking out optimal values. Within the confines of the testing, for the chosen set parameters, the efficiency of the firefly algorithm was checked and the obtained results were compared to the best previously known solutions. The presented results of the applied algorithm to solve the problem of the reliability of system with 10 elements were limited to discussing the results where $m = 5$. The firefly algorithm was implemented in the Matlab 2015a environment. During the experiments, to verify the quality of the results of the algorithm, the following values of its parameters were established:

### Table 3. Results for the bridge system after 50 runs

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of fireflies</th>
<th>Best value</th>
<th>Worst value</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>decomposition</td>
<td>30</td>
<td>0.999989027392830</td>
<td>0.999692113944072</td>
<td>0.999867770075797</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.99982704854672</td>
<td>0.99535345770864</td>
<td>0.99789510790645</td>
</tr>
<tr>
<td>cuts</td>
<td>30</td>
<td>0.99987373640587</td>
<td>0.999709686550381</td>
<td>0.999839910069411</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.99981295104186</td>
<td>0.99561725825725</td>
<td>0.99797540150406</td>
</tr>
<tr>
<td>paths</td>
<td>30</td>
<td>0.999998825015460</td>
<td>0.9999599869590</td>
<td>0.999997854290027</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.99999874719315</td>
<td>0.9999266893783</td>
<td>0.999997639615957</td>
</tr>
</tbody>
</table>

### Table 4. Results of 10-unit system ($m = 5$)

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of fireflies</th>
<th>Best value</th>
<th>Worst value</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>decomposition</td>
<td>30</td>
<td>0.999124934817144</td>
<td>0.998712767969089</td>
<td>0.99929684217294</td>
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<tr>
<td></td>
<td>10</td>
<td>0.999124934817144</td>
<td>0.997639045897561</td>
<td>0.99870655489805</td>
</tr>
<tr>
<td>cuts</td>
<td>30</td>
<td>0.999123179843347</td>
<td>0.9998581286534003</td>
<td>0.99959187420550</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.999123179843347</td>
<td>0.9997349205284400</td>
<td>0.99869753072659</td>
</tr>
<tr>
<td>paths</td>
<td>30</td>
<td>0.999999983601514</td>
<td>0.99999996168195</td>
<td>0.99999997369618</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.999999983601514</td>
<td>0.999999967299759</td>
<td>0.999999978215736</td>
</tr>
</tbody>
</table>
the presented algorithm using minimal paths versus other methods. The results of the research suggest an advantage of independent repetitions of the algorithm.

Unfortunately, such a conclusion cannot be drawn when comparing the FA using other methods. In the case of the 10-unit system, the firefly algorithm can clearly be seen to have an advantage over the cuckoo search. It should be noted, however, that for the considered examples, the results, obtained during maximization with the minimal paths method, differ significantly from the results of both the decomposition method and minimal cuts set method. Therefore, we can conclude that in the design of a variety of real systems, the safest approach is to adopt the lower estimated value for reliability.

6. Conclusions

The paper presents research results obtained using the firefly algorithm in reliability-redundancy allocation problems. In order to examine the effectiveness of the algorithm, two systems and three methods of determining the reliability were chosen, i.e. the minimal paths set, the minimal cuts set and the method of decomposition. Analyzing the results, it can be concluded that for the considered systems, significantly better results for the firefly algorithm were obtained in conjunction with the use of the minimal paths set method. It is worth noting that the results concerning the use of swarm algorithms presented in the literature turned out to be worse than those that managed to obtain by proposed implementation of the firefly algorithm.

Table 5. Compilation of best obtained results from the literature data

<table>
<thead>
<tr>
<th>Structure: bridge system</th>
<th>Algorithm</th>
<th>Best result</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO [4]</td>
<td>0.99988957</td>
<td>0.99988594</td>
<td></td>
</tr>
<tr>
<td>PSO [15]</td>
<td>0.99988963</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPSO [10]</td>
<td>0.999889376</td>
<td>0.9998891423</td>
<td></td>
</tr>
<tr>
<td>ABC [18]</td>
<td>0.99988962</td>
<td>0.99988362</td>
<td></td>
</tr>
<tr>
<td>CS-GA [6]</td>
<td>0.9998864</td>
<td>0.9998854</td>
<td></td>
</tr>
<tr>
<td>CS [13, 14]</td>
<td>0.99988964</td>
<td>0.99987998</td>
<td></td>
</tr>
<tr>
<td>BAT [9]</td>
<td>0.999889376</td>
<td>0.9998894767</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structure: 10-unit system</th>
<th>Algorithm</th>
<th>Best result</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACO [1]</td>
<td>0.999991</td>
<td>0.9980477</td>
<td></td>
</tr>
<tr>
<td>CS [13, 14]</td>
<td>0.67189992</td>
<td>0.67189992</td>
<td></td>
</tr>
</tbody>
</table>

As is evident from the calculations, in the case of the bridge system the firefly algorithm with minimal paths set was the one method, which enabled the obtainment of results (0.999998825015460), exceeding the results of PSO, MPSO, ABC, CS, CS-GA and BAT. Unfortunately, such a conclusion cannot be drawn when comparing the FA using other methods. In the case of the 10-unit system, the firefly algorithm can clearly be seen to have an advantage over the cuckoo search. It should be noted, however, that for the considered examples, the results, obtained during maximization with the minimal paths method, differ significantly from the results of both the decomposition method and minimal cuts set method. Therefore, we can conclude that in the design of a variety of real systems, the safest approach is to adopt the lower estimated value for reliability.

References


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