The article presents the results of research on the operational properties of contactless power supply system used in the PRT (Personal Rapid Transit) vehicle demonstration model, made within the framework of the ECO Mobility project. The area of transport applications of the PRT automated rail transport system is presented. Elements of the ECO Mobility PRT Drive System have been described – an inductive linear motor, dynamic contactless power supply, and supercapacitor recuperation system. Electrical performance maps of the linear motor and contactless power system were presented. Also shown was the method of their use in calculation of traction energy consumption by means of theoretical journeys. The results of the simulation calculations for the trial track were presented. The results of design calculations of the power supply parameters for the planned line of the demonstrator with real dimensions are presented.

**Keywords:** PRT, contactless power supply, simulation, assumed operating traffic conditions.

Artykuł prezentuje wyniki badań własności eksploatacyjnych układu zasilania bezstykowego zastosowanego w modelu demonstracyjnym pojazdu PRT, wykonanym w ramach projektu ECO Mobilność. Przedstawiono obszar zastosowań transportowych systemu szynowego automatycznych środków transportu PRT. Opisano rozwiązanie układu napędowego pojazdu PRT konstrukcji ECO Mobilność – napeł za pomocą indukcyjnego silnika liniowego, zasilanie bezstykowe dynamiczne oraz układ rekuperacji z zastosowaniem superkondensatora. Zaprezentowano mapy sprawności elektrycznej silnika liniowego i układu zasilania bezstykowego. Przedstawiono wyniki obliczeń symulacyjnych dla toru próbnego w skali. Przedstawiono wyniki obliczeń projektowych parametrów układu zasilania dla planowanej linii demonstratora o wymiarach rzeczywistych.

**Słowa kluczowe:** PRT, układ zasilania bezstykowego, symulacja, zakładane eksploatacyjne warunki ruchu.

1. Introduction

Transportation solutions using PRT (Personal Rapid Transit) vehicles are not new [1]. The PRT is a mode of automatic rail transport. PRT vehicles usually run on rubber-tired wheels at special track systems [26]. Also in Poland, PRT transport applications have been subject to exhaustive analyses [5]. At present, however, there has been a renewed interest in such modes of transport, due to the concept of “pod car”, which is the idea of extending the use of this type of automatic rail vehicle to public circuits [27]. Introduction of this concept can be based on two different levels of motion control automatics. At the lower level, usually designated Level 3 [29], vehicles move on separate lanes of public roads called “virtual roads” due to the need to build a special V2I communication infrastructure and communication system [10]. At the highest level, marked Level 5, vehicles should move as autonomous vehicles. PRT vehicles moving in line with the idea of “pod car” also on public roads will become part of automated transport networks (ATN) in urban areas [8]. In the future, vehicles of this type will be the synthesis of an automatic rail and wheel vehicle. In dense areas with heavy traffic, they will be able to navigate the dedicated tracks built specifically for them so that traffic infrastructure can be used in suburban areas with low traffic.

This article is devoted to the description of the power supply used in the “Polish” PRT version, made as a physical model in scale within the framework of the ECO Mobility project. This project has not yet been implemented for transport applications but has nevertheless been tested on a test track for scale vehicles. A fragment of track in scale was recently presented at the Hannover Fair at the SciTech Poland “scientific” Polish stand [7, 28].

The already mentioned power supply is a contactless, dynamic power system, which means that it can deliver energy to the vehicle in motion as opposed to stationary systems where energy is delivered only when the vehicle is stationary. The drive motor is a linear induction motor. This system solution illustrates one of the many possibilities that can be applied to the driving and powering of this type of vehicle. The power supply can also be made as a contact one with power points at parking places. Propulsion motors can be made as brushless, induction and wheeled or as central units. On the other hand, the contactless power supply has the advantage of being a safe system [23, 30]. The supply energy is transmitted inductively from the primary winding distributed along the track - similarly to the third rail in the metro. The fundamental difference between the contact supply by means of the third rail and the contactless induction is that the contact rail “power” is isolated and thus safe.

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie www.ein.org.pl
2. Power Transmission System for PRT with contactless power supply, linear motor, and supercapacitor

The layout of the PRT drive system with the contactless power supply, the linear motor and the supercapacitor is shown in Fig. 1, where the following are marked:

- vehicle part: contactless power supply (CET), SZ – primary winding of the contactless energy transfer system, UW - secondary winding of the non-contact energy transfer system, KR2 – resonant circuit capacitors on the secondary side, PD – diode rectifier, FN – drive inverter, PM – bi-directional converter for energy storage and brake resistor converter, SC – supercapacitor tray, RH – braking resistor, linear induction motor (SIL)

Fig. shows a diagram of the energy increments that can be associated with separate circuits during vehicle movement. In addition, it was assumed that:

- The energy of the CET transformer is the electrical energy of a single-phase sinusoidal non-distorted 50 kHz transformer. The increments of this type of energy are indicated by the following symbols: \( dE_Z \) – CET primary power supply, \( dE_{CET} \) – CET secondary site, \( dE_{SC} \) – supercapacitor discharge, \( dE_{DC} \) – supercapacitor charge, \( \Delta E_{CET} \) – CET system losses, \( \Delta E_{SC} \) – supercapacitor losses.

- The energy supplying the motor is the electric current of the three-phase sinusoidal variable non-deformed variable frequency controlled by the inverter. Increments of this type of energy are denoted by: \( dE_{conv} \) - motor power, \( dE_R \) - recuperation, \( \Delta E_{SIL} \) - engine loss, \( \Delta E_R \) - recuperation.

3. Model of linear induction motor

The energy characteristics of the steady-state drive system components can be described by efficiency, power factor and power loss as follows:

\[
\eta = \frac{P_m}{P_c} \quad (1)
\]

\[
\cos \phi = \frac{P}{\sqrt{P_c^2 + Q_c^2}} \quad (2)
\]

\[
P_e = P_m + \Delta P \quad (3)
\]

where: \( P_m \) – power on the shaft, \( P_c \) – total active power, \( \Delta P \) – power losses, \( Q_c \) – reactive power.

The mechanical power of the linear motor can also be defined as the product of force and velocity:

\[
P_m = F_{lin} \cdot v_{lin} \quad (4)
\]
The theoretical description of the above relationships has been applied to the peripheral model of the simplest motor, known from the static modeling of the rotary engine operating state for sinusoidally variable non-distorted current supply conditions. The diagram of electrical circuit of the model substitute is shown in Fig. 4. The parameters listed are as follows: $R_s$ - primary winding resistance, $R_r$ - secondary winding resistance, $X_s$ - secondary substitution reactance (consisting of scattering and stator and rotor reactances), $R_{Fe}$ - replacement resistance in iron (for vortex currents and hysteresis), $X_m$ - magnetization reactance. Index ‘ means the conversion of the value of the secondary side parameter to the value of the primary side. The use of such a simple model was determined through its usefulness, understood here as the ability to obtain a sufficiently small divergence in the description of the traction-energy characteristics of the engine used in the propulsion system at the laboratory. However, it should be noted that the linear motor (in contrast to the rotary motor) has an open magnetic circuit. Marginal and edge parasitic phenomena are very important in linear motors, which can be omitted in rotary motors. The magnetic gap of the linear motor is much wider than the gap in the rotary engine. The effect of these phenomena is the reduction of the efficiency of linear induction motors and increase in the demand for reactive power to produce magnetic flux. The mathematical model of linear induction motor describing these additional phenomena is a complex process which has been found in many theoretical papers [20], also in the area of control theory [2, 24, 11].

For the accepted substitution scheme, it is possible to formulate the following set of formulas:

a) Total active power consists of rotor electromagnetic power and power loss in the iron:

$$P_e = P_m + \Delta P = F_i V_s + p m \frac{U_s^2}{R_{Fe}}$$

(4)

b) Reactive power is charged to the formation of the magnetizing stream and diffusion streams:

$$Q_e = \frac{sF_i V_s}{X_m} + p m \frac{U_s^2}{X_m}$$

(5)

c) Strength is a slip function described by the Kloss formula:

$$F_s = \frac{2F_X}{s\frac{s K}{s K + s}} = \frac{2F_X s K}{s (s K + s^2)}$$

(6)

where: $V_s$ – synchronous linear speed, $m$ – number of phases, $p$ – number of poles, $U_s$ – stator voltage, $F_X$ – critical power, $s_K$ – critical slip, $s$ - slip

For the accepted method of describing the active and reactive power, the determination of efficiency (1) and power factor (2) can be reduced to:

$$\eta = \frac{V_i}{V_s + m \frac{U_s^2}{F_i R_{Fe}}}$$

(7)

$$\cos \phi = \frac{F_i V_s + m \frac{U_s^2}{R_{Fe}}}{\sqrt{F_i V_s + m \frac{U_s^2}{R_{Fe}}} + \left(\frac{sF_i V_s}{s K + m \frac{U_s^2}{X_m}}\right)^2}$$

(8)

4. Map of linear induction motor model efficiency

Table 1 shows the linear induction motor ratings of a 1:4 vehicle [12, 25]. Table 2 shows the variation of the parameters for the change in the nominal width of the air gap in a permissible construction range of 2-4 mm. Parameters of the peripheral motor model were identified by analysing the waveforms of the theoretical and laboratory-measured characteristics. This was done using the least distance method. The calculated parameters are presented in Table 3. The theoretical characteristics obtained are shown in Fig. 5: a) mechanical, b) efficiency, c) power factor.

Table 1. SIL rated characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of inductor</td>
<td>L</td>
<td>m</td>
<td>0.27</td>
</tr>
<tr>
<td>Number of pairs of poles</td>
<td>p</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Number of phases</td>
<td>m</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Rated gap</td>
<td>$g_m$</td>
<td>mm</td>
<td>3</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>$U_1$</td>
<td>V</td>
<td>230</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>$f_n$</td>
<td>Hz</td>
<td>45</td>
</tr>
<tr>
<td>Rated linear speed</td>
<td>$V_n$</td>
<td>[m/s]</td>
<td>3.375</td>
</tr>
<tr>
<td>Rated power</td>
<td>$P_{cn}$</td>
<td>W</td>
<td>433</td>
</tr>
<tr>
<td>Active power</td>
<td>$P_{in}$</td>
<td>[W]</td>
<td>966</td>
</tr>
<tr>
<td>Apparent power</td>
<td>$S_{1n}$</td>
<td>[VA]</td>
<td>7250</td>
</tr>
</tbody>
</table>

Table 2. Parameters of continuous motor operation in conditions of changing the length of the air gap in the range of 2-4 mm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip</td>
<td>$s_n$</td>
<td>-</td>
<td>0.143</td>
<td>0.160</td>
<td>0.167</td>
<td>0.180</td>
<td>0.1901</td>
</tr>
<tr>
<td>Linear speed</td>
<td>$V_{in}$</td>
<td>[m/s]</td>
<td>3.470</td>
<td>3.402</td>
<td>3.375</td>
<td>3.32</td>
<td>3.28</td>
</tr>
<tr>
<td>Continuous force</td>
<td>$F_{in}$</td>
<td>[N]</td>
<td>166</td>
<td>148</td>
<td>128</td>
<td>116</td>
<td>103</td>
</tr>
<tr>
<td>Power on the shaft</td>
<td>$P_{in}$</td>
<td>[W]</td>
<td>575</td>
<td>504</td>
<td>433</td>
<td>385</td>
<td>338</td>
</tr>
<tr>
<td>Active power</td>
<td>$P_{in}$</td>
<td>[W]</td>
<td>11.48</td>
<td>10.53</td>
<td>9.66</td>
<td>10.06</td>
<td>10.39</td>
</tr>
<tr>
<td>Apparent power</td>
<td>$S_{in}$</td>
<td>[VA]</td>
<td>7250</td>
<td>7250</td>
<td>7250</td>
<td>7250</td>
<td>7250</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta_n$</td>
<td>-</td>
<td>0.50</td>
<td>0.48</td>
<td>0.448</td>
<td>0.38</td>
<td>0.32</td>
</tr>
<tr>
<td>Power factor</td>
<td>$\cos \phi_n$</td>
<td>-</td>
<td>0.158</td>
<td>0.146</td>
<td>0.133</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>Energy factor</td>
<td>$\eta_n \cos \phi_n$</td>
<td>-</td>
<td>0.079</td>
<td>0.070</td>
<td>0.060</td>
<td>0.053</td>
<td>0.047</td>
</tr>
</tbody>
</table>
Fig. 6 shows a map of the efficiency and power factor of the motor for the gap width of 3 mm. The parameters listed in Table 3 provide maps for the remaining slot widths.

5. Model of contactless power supply

The basic element of the contactless power supply system is the high frequency CET transformer, whose primary and secondary windings are maintained in the resonant state - using additional capacitors \((X_1 = X_2 = 0)\) [3, 9, 13, 14, 22]. The circuit diagram used to describe the steady-state condition of the sine wave voltage supply [4, 21] is shown in Fig. 7.

Marked: \(R_1\) – primary winding resistance, \(R_s'\) – secondary winding resistance reduced to primary side, \(R_r'\) – receiver resistance reduced to primary side, \(L_s\) – inductance of primary winding, \(L_{s'}\) – inductance of secondary winding.
The power allocated at the receiver is determined by the formula:

\[ P_{\text{out}} = (I_2')^2 R_0' \]  (9)

The power output from the source is the sum of the power output at all resistances of the circuit:

\[ P_{\text{in}} = P_{\text{out}} + \Delta P = I_1^2 R_1 + (I_2')^2 (R_2' + R_0') \]  (10)

Electrical efficiency is defined by the formula:

\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{(I_1^2 R_1 + (I_2')^2 (R_2' + R_0'))}{(I_1^2 R_1 + (I_2')^2 R_0')} \]  (11)

The reactive power of the system is only taken to produce a magnetizing stream:

\[ Q_{\text{in}} = (I_M')^2 X_M' \]  (12)

The power factor is defined by the formula:

\[ \cos \phi = \frac{P_{\text{in}}}{S_{\text{in}}} = \frac{(I_1^2 R_1 + (I_2')^2 (R_2' + R_0'))}{\sqrt{(I_1^2 R_1 + (I_2')^2 R_0')^2 + ((I_M')^2 X_M')^2}} \]  (13)

The output voltage of the contactless power system is determined by the formula:

\[ E_0 = E_2' + Z_{\text{sw}}' I_2' \]  (14)

where: \( E_0 \) – is idle voltage, \( Z_{\text{sw}}' \) – is internal impedance of the power supply.

The formula for external characteristic describes the cosine pattern, which after transformations takes the following form:

\[ E_0^2 = (E_2')^2 + (Z_{\text{sw}}' I_2')^2 + 2(E_2')(Z_{\text{sw}}' I_2') \cos \psi_w \]  (15)

where additionally: \( \psi_w \) – phase angle between current and voltage in internal impedance.

Based on the calculated design calculations and measurements of CET transformer parameters for the vehicle power supply, the following parameters of the peripheral model were adopted: \( M = 5 \times 10^{-6} \text{ H} \), \( R_1 = 18 \times 10^{-7} \text{ } \Omega \), \( R_2 = 20 \times 10^{-3} \text{ } \Omega \), \( \nu = 1/18 \), \( f = 50 \times 10^3 \text{ Hz} \) [19]. The required maximum power of the motor \( P_{\max} = 2 \times 10^3 \text{ W} \) is developed at a voltage of \( U_s = 230 \text{ V} \), under voltage conditions of \( E_1 = 280/18 \text{ V} \).

Fig. 8 shows the characteristics of a contactless power supply system: external \( E_2'(I_2) \) designated \( E_2 \) (converted to secondary side) and efficiency \( \eta(I_2) \) designated \( \eta \). Two selected work points are indicated in the drawing: \( P_{\text{in}} \) – the load point of the system with the active power corresponding to the rated operating conditions of the motor with a 3mm magnetic slot (966 W – table 1) and \( P_{\max} \) – the maximum load point with the active power.

6. Theoretical analysis of the energy properties of the power supply system for the assumed operating conditions of the laboratory vehicle.

The starting point for the analysis of the energy consumption of the vehicle drive system is the analysis of demand for power (energy) mechanical, popularly known as traction power. The diagram of the applied traction power calculation method is shown in Fig. 9 [18], where: Speed – predetermined instantaneous speed, Route – predetermined path and path profile, \( P_K \) – increase in kinetic energy over time, \( P_V \) – power to overcome aerodynamic motion resistance, \( F_R \) \( v_{\text{lin}} \) – power to overcome the forces of additional resistance (corners). In the presented method the power component of the motion resistance is the function of the arc radius \( R \) and the velocity \( v_{\text{lin}} \) determined interapolatively on the basis of the results obtained through simulation [15] using the vehicle dynamics models [15, 16]. Fig. 10 shows a diagram of a laboratory track, where the symbols H mark the stops, K – switches, L – track connections. Fig. 11 depicts the interstitial distribution travel speed in the H4 – H2 fragment (as a function of the road), determined from the solutions of the computational model of the structure shown in Fig. 9. The dashed line indicates speed limits on the corners. The limitations of traction forces in the form of the characteristics shown in Fig. 6 and the influence of the forces of motion resistance (basic from speed and additional curvature of the trajectory) are taken into account in determining the course of speed. Fig. 12 shows the driving force of the SIL motor (as a function of time) on the analysed road segment.
The above forces and velocities are the basis for determining the power supply of the SIL motor. The calculation takes into account the efficiency of the motor function of the map shown in Fig. 6. The calculation of power output is shown in Fig. 13. The graphs are marked: 1 - the instantaneous power of SIL motor, - 2 the instantaneous power to be delivered from the PD rectifier (fig.1) when a supercapacitor is used in the system. The negative values of power in diagram 1 correspond to the possibility of applying the electric braking of the vehicle. According to the diagram of Fig. 2, the recuperation process is based on the accumulation of braking energy in the supercapacitor and the equalization of the starting power. The calculations take into account the share of the energy loss in the supercapacitor system.

![Fig. 11. Distribution of speed as a function of road while passing on the interstitial section of road H4-H5](image)

![Fig. 12. Waveform of driving force SIL as a function of time on the section of road H4-H5](image)

7. Theoretical analysis of the energy properties of the power supply system for the assumed operating conditions of the vehicle with actual dimensions.

One of the locations considered for the construction of the PRT is the city of Rzeszow. At this stage of the discussions, the type of electric motors used for the drive and the way the vehicle will be powered for that location are not yet determined. However, traction calculations show that a train of actual size and a max. speed of 50 km/h will require a drive with a nominal capacity of approx. 16 kW. Brushless motors located in the wheels of a vehicle which are already used in the car’s ECO pre-processor, can be used to implement the drive, for example. The power loss maps of this engine are shown in publication [17]. Equally good is the use of 5-phase induction motors in wheel hubs. This type of engine dedicated to automotive applications was recently presented by HCP at the Hannover Fair. In the already completed ECO Mobility project, a linear induction motor design was implemented [12]. The performance map of this motor is shown in Fig. 15. Fig. 16 shows the external characteristics and performance characteristics of the contactless power system designed in this solution. Fig. 17 shows a diagram of the track line of one of the route sections in the planned location. Fig. 18 shows the driving speeds of the forcing drive - minimum time at this section of the road. Speed limits result from both the limits of the maximum permissible axial acceleration on the curvature of the tracks and the speed limits for direct driving conditions. The exponential acceleration and motion delays result from the possibility of obtaining maximum traction forces (described in the form of the F-V relation in Fig. 15). Fig. 19 shows the course of traction forces on the analysed section of the road, and Fig. 20 shows the power flow patterns. In this figure, the waveforms are shown in two possible variants of the vehicle power supply solution: black diagram - waveform without additional power source, blue - with supercapacitor system for braking energy return (recupera-

![Fig. 13. Power of the SIL power supply. Diagrams are: 1 - without energy storage on the vehicle, 2 - in a system with supercapacitor](image)

![Fig. 14. Power cycle of the laboratory vehicle power supply on the CET contactless power supply terminals. Diagrams are marked: 1 - without energy storage on the vehicle, 2 - in the system with supercapacitor](image)

![Fig. 15. SIL performance charts for vehicles with actual dimensions at a gap width of 12 mm](image)

![Fig. 16. Contactless power supply diagrams: external E2(I2) and efficiency η(I2)](image)
Fig. 17. Diagram of the track line of the first section of the route for the
designed location of the PRT track in Rzeszow

Fig. 18. Distribution of speed as a function of road during the passage of the
designated section in the city of Rzeszow

Fig. 19. Distribution of SIL driving force as a function of the road section of
the designated city of Rzeszow

Fig. 20. Power step at the output of the PD rectifier (fig. 1). Diagrams are:
1 - without energy storage on the vehicle, 2 - in the system with super-
capacitor

Fig. 21. The power of the vehicle power supply on the terminals of the con-
tactless power supply CET for the section in the city of Rzeszow. Dia-
grams are: 1 - without energy storage on the vehicle, 2 - in the system
with supercapacitor

8. Conclusions

The conducted analysis of the PRT drive and the laboratory tests
of vehicles on a test scale showed that dynamic contactless power
may be considered as one of the methods of supplying electricity to
vehicles of actual size. This type of power supply is ideal for small
vehicles moving in urban infrastructure, where safety considerations
are particularly important. Results of analyses and studies also show
that low power linear induction motors are characterized by low val-
ues of energy coefficients (both efficiency and power factor). The
16kW engine designed for the vehicle with actual dimensions is also
characterized by poorer operating conditions than rotary motors. The
use of linear motors also excludes the possibility of moving this PRT
vehicle on public roads intended for wheeled vehicles. For this rea-
son, the second generation of currently-developed PRT vehicles will
be equipped with rotary motors. These vehicles will be able to have
a contactless power supply system as a rail system, supplemented by
electrochemical energy storage on board the vehicle. During the rail
traffic, the contactless power supply system will provide the energy
required to track the path and the energy to charge the electrochemical
battery, which will be a source of power during the period of moving on
the public road.

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