

Paulius SKAČKAUSKAS  
Edgar SOKOLOVSKIJ

## MODIFICATION AND RELIABILITY ESTIMATION OF VECTOR BASED DUBINS PATH APPROACH FOR AUTONOMOUS GROUND VEHICLES PATH RE-PLANNING

### POPRAWA I OCENA NIEZAWODNOŚCI OPARTEJ NA WEKTOROWEJ METODZIE TRAJEKTORII DUBINSA DLA KOREKTY TRAJEKTORII POJAZDÓW AUTONOMICZNYCH

*Due to global purposes to ensure growth of a competitive and sustainable transport system, also to solve traffic safety and environmental problems, various engineering solutions are being sought out. It can be assumed that autonomous vehicles are the technology, which will ensure the positive change in the transport system. Even though many studies successfully advanced toward realisation of autonomous vehicles, a significant amount of technical and policy framework problems still has to be solved. This paper addresses the problem of predefined path feasibility and proposes an effective methodology for a path to follow re-planning. The proposed methodology is composed of three parts and is based on the Dubins path approach. In order to modify the vector based Dubins path approach and to ensure the path feasibility, the optimisation problem was solved. A cost function with different inequality constraints was formulated. The performance and reliability of the proposed methodology were analysed and evaluated by carrying out an experimental research while using the autonomous test vehicle.*

**Keywords:** *autonomous ground vehicle, Dubins path, path re-planning, path following, performance, reliability estimation.*

*Dla zapewnienia rozwoju konkurencyjnego i zrównoważonego systemu transportowego, oraz w celu rozwiązania problemów związanych z bezpieczeństwem ruchu i środowiskiem, poszukiwane są różne rozwiązania techniczne. Można założyć, że autonomiczne pojazdy są technologią, która zapewni pozytywną zmianę w systemie transportowym. Mimo że wiele badań z powodzeniem dotyczyło realizacji autonomicznych pojazdów, należy jeszcze rozwiązać wiele problemów technicznych i prawnych. W niniejszym dokumencie poruszono problem predefiniowanej wykonalności ścieżki i zaproponowano skuteczną metodologię dla ścieżki do śledzenia ponownego planowania. Proponowana metodologia składa się z trzech części i opiera się na metodzie Dubinsa. Aby zmodyfikować metodę trajektorii Dubinsa i zapewnić optymalną trajektorię, w publikacji rozwiązano zadanie optymalizacji. Sformułowana funkcja celu z różnymi nieliniowymi ograniczeniami. Skuteczność i niezawodność proponowanej metodologii została przeanalizowana i oceniona po przeprowadzeniu eksperymentalnych badań z wykorzystaniem autonomicznego pojazdu badawczego.*

**Słowa kluczowe:** *samochód autonomiczny, trajektoria Dubinsa, przeplanowanie trajektorii, śledzenie trajektorii, skuteczność, ocena niezawodności.*

#### 1. Introduction

In research papers a number of investigations and solutions on how to increase mobility, passenger comfort, traffic safety or cut carbon emissions, etc. can be found. For example, technological innovations for traffic safety improvement are proposed in [8, 36], reliability improvement propositions of urban / commercial transport and transportation are given in [28-32], decision making methods in automotive industry are reviewed in [34] and so on. However, to achieve all of the mentioned goals, a technological breakthrough is necessary. In [18] it is stated that the main breakthrough in the transport system will be brought by the technology of autonomous vehicles. As also pointed out in [22], nowadays it is assumed that the autonomous ground vehicles (AGVs) have a great potential to be widely applied in a variety of fields, such as road transportation, agriculture, planetary exploration, military purpose and so on. Despite the scientific advances that have been made throughout the last decade, there are still a number of problems and challenges, related to reliability of the autonomous vehicles, human-vehicle interaction, path planning and following, control systems, driving stability, policy framework, etc.

To realize this essential improvement of the current transport system, all the mentioned problems must be solved, i.e., the AGVs reliability and safety must be evident. Thus, as stated in [6], reliability of the autonomous vehicles as a new aim for quality and reliability engineering can be described.

#### 2. Scientific background

One of the major factors, on which depends reliability and a successful integration of the AGVs into the transport system, is accurate and safe path following. Based on [13], the autonomous path following problem can be divided into two key steps: extraction of the desired path to follow and lateral / longitudinal control of the vehicle. According to [25], the most fundamental requirement for a path is feasibility – it must be possible for the planned path to be executed by the AGV. However, in most of the research works it is assumed that the predefined path to follow is safe and feasible and the main focus is the lateral / longitudinal control of the AGV. Such assumption becomes incorrect in real driving scenarios, when an on-board planner or a human cannot evaluate or does not have sufficient information about ob-

stacles, road network, mobility and dynamic limits of the vehicle. According to [15], a human driver does not have a precise path in mind when driving; instead, s/he would normally have a global sense how they should drive, reach the destination, avoid an obstacle, etc. Under such circumstances, safe deviation from the predefined path and the path re-planning becomes an important task. Based on researches described in different sources, three conditions can be singled out, under which the path must be re-planned: 1. Avoiding dangerous situations, like spinning out or obstacle hitting (described in [7]); 2. Sharp turns and discontinuities in the path which may compromise the integrity of the AGV (described in [35]), and as singled out in [25] – 3. Neglected constraints and mobility of the vehicle.

In various literature different algorithms can be found, which are related to the planning, re-planning and optimisation of the paths of the AGVs. In the majority of cases, the constraints for the planned, re-planned or optimised path are minimum path length, minimum energy spent to follow the path, shortest time distance, etc. One of the pioneering works about smooth path generation based on energy minimization was presented in [12] by Delingette. Delingette [12] developed a method based on the usage of intrinsic splines with polynomial curvature profile which allowed to solve general geometric constraints. Liang and Liu [23] developed the shortest path planning method, which searched for a minimal length path from all the paths generated by a linear programming optimisation. The main advantage of the developed method – it can be applied while considering backward motion capability. However, in both works [12, 23], the developed algorithms were theoretically based, experimental investigations were not performed. Arokiasami [3] proposed a vector based path generation method, which follows a geometric approach to path-generation and path-tracking and is suitable for aerial and ground autonomous vehicles. Gupta [16] used a sampling based model predictive optimisation for feasible path planning and pointed out that a single-objective path length minimization can lead to trajectories that have unnecessary energy consumption. One drawback of the used method – it consists of kinematic, dynamic and power models of the AGV and is rather complex. Castillo [9] described the use of a genetic algorithm for the point-to-point path planning, while using single-objective and multi-objective optimisation (minimization of path length and difficulty). In [11] the path planning problem was also formulated as a multi-objective problem, while focusing on the energy consumption and the path safety. Although multi-objective optimisation is more suitable and useful for difficult path planning problems than single-objective optimisation, from researches provided in [9, 11] it is notable that the complexity of multi-objective path planning problems is very important in the efficiency of the algorithm performance. Finding an optimal solution in path planning can become difficult, thus, real time application of the algorithm can be complicated. To move in the shortest and collision-free path in environments with obstacles Han and Seo [17] suggested a methodology based on a surrounding point set algorithm. An advantage of the proposed methodology is that, when narrow spaces exist in the optimal path, the proposed approach does not fail to place points on the related space. Krishnan [20] defined the path length minimization problem in polar and not in the Cartesian coordinate system. Authors in [20] state that the migration from the Cartesian coordinate system to the polar coordinate frame has improved the efficiency of the proposed algorithm, however, in order to ensure better performance, environment limitations should be addressed while using an online guidance system that can make finer corrections to the trajectory on a real time basis. Human-vehicle interaction during the path planning was referred in [33] by Receveur. Receveur [33] described a multi-criteria path optimisation, listed the criteria that need to be minimized in technical order of importance, but also stated that a human can have different preferences for the path planning. Though the algorithm described by Receveur effectively generated a close to human-like trajectory, the author still suggests that the algo-

rithm should be improved by periodically recalculating the optimal trajectory. It can be seen that the path planning, re-planning and optimisation problems have been studied using various methodologies which were theoretically and experimentally validated using various car-like mobile robots or other AGVs. Davoodi [11] noticed that since there are many types of robots and AGVs with different abilities and constraints, it is nearly impossible to provide a unique exact definition for the expression “optimal path” in the path planning context. For example, a typical vehicle cannot instantaneously move orthogonally to the wheel plane. The nonholonomic and other constraints, associated with their movement and mechanical design, need to be considered. Furthermore, as mentioned above, the main focus in such research works is the minimization of selected parameters, with less or no concern to the path feasibility, which leads to undesired deviations from the path and other negative effects. That is why one of the major issues associated with the path planning and the feasibility of the path is related to the mobility of the vehicle. Because of these reasons, the aim of this work is to develop a reliable and effective methodology for re-planning of sharp turns in a path, predefined by an on-board planner or a human, while focusing on the constraints and the mobility of the vehicle.

### 3. Methodology

In this paper the proposed methodology is composed of three parts. First of all, to develop a methodology for the re-planning of sharp turns in a path, while focusing on the constraints and the mobility of the vehicle, a bicycle-like kinematic model of the AGV is considered. Secondly, the vector based Dubins path approach for the path re-planning is given and explained. And finally, in order to increase the path feasibility and safety, a cost function with inequality constraints for the vector based Dubins path approach is proposed.

#### 3.1. Kinematic vehicle model

In the considered kinematic model, it is assumed that the two wheels on the front and on the rear axles are collapsed into a single wheel, respectively located at the midpoints of both axles. The front wheel can be steered, and the orientation of the rear wheel is fixed. The main feature of the kinematic model of the AGV is the presence of the nonholonomic constraints. According to [1], the bicycle-like kinematic models of the AGVs are widely used because of their low parameter dependency and the usage of such models is a quite standard assumption in the literature where analysis of different control strategies is performed [2] or new path planning and following methods as in [4, 5], are developed. It is assumed that the AGVs have planar motion, therefore the vector of the generalized Cartesian coordinates of the centre of mass is:

$$Q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \quad (1)$$

where  $x$  and  $y$  are the position coordinates of the centre of mass;  $\theta$  is the vehicle orientation with respect to the fixed frame  $(X, Y)$  of the Cartesian coordinates.

Considering the variables indicated in the Eq. (1), the kinematic model of the vehicle can be represented as:

$$\begin{cases} \dot{x} = v \cdot \cos \theta \\ \dot{y} = v \cdot \sin \theta, \\ \dot{\theta} = \frac{v}{L} \tan \delta \end{cases} \quad (2)$$

where  $L$  is the AGV wheelbase;  $\delta$  is the steering angle of the front wheels;  $v$  is the linear velocity of the vehicle.

The average steering angle of the front wheels can be described geometrically as:

$$\delta = \tan^{-1}\left(\frac{L}{r}\right), \quad (3)$$

where  $r$  is the radius of the path of the vehicle.

In the methodology proposed below, the kinematic model is hereinafter used for expressing the desired movement coordinates, while evaluating the constraints of the vehicle accordingly.

### 3.2. Vector based Dubins Path Approach

Based on research works [14, 19, 26], in which path planning problems with a turning radius motion constraint are addressed, it can be stated that the Dubins path approach for the AGVs, often referred to as the Dubins vehicle, is one of the most effective, widely used and modified classic methods for the optimal path planning. When applying this method in the general case, finding the optimal path involves checking for 6 possibilities, which consist of concave or convex circle segments and / or straight line segments. In this work, the purpose of applying the Dubins path approach is not to generate a large number of paths that eventually will be discarded, but, based on the path waypoints / intersections predefined by an on-board planner or a human, to solve the optimisation based path planning problem and find the optimal Dubins path possibility for path re-planning. The waypoints predefined by an on-board planner or a human can be converted into Dubins path by inserting a filleted circular arc near the intersection waypoints. A general idea of the vector based Dubins path approach for path re-planning, further used in this work, originally was presented in [19] for unmanned aerial vehicles. The scheme of a path re-planning example using the vector based Dubins path approach is presented in Fig. 1.

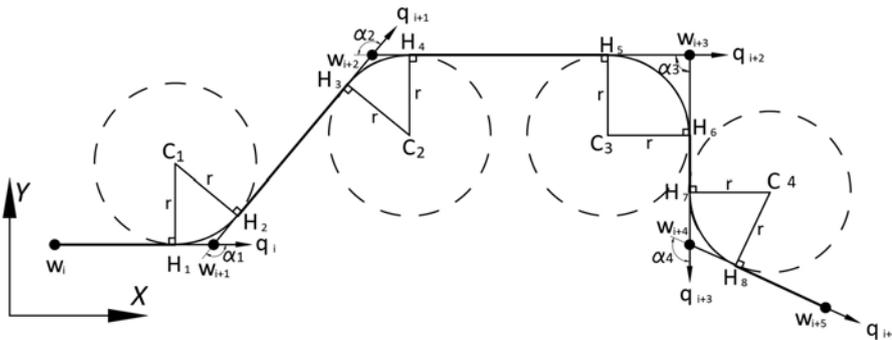


Fig. 1. Scheme of path re-planning using the vector based Dubins path approach

In the provided scheme (Fig. 1)  $w_i$  are referred to as path waypoints predefined by a human or an on-board planner ( $x$  and  $y$  coordinates) which are connected by the straight line segments and form a global path – path with sharp turns to follow. The AGVs movement directions in straight line segments are shown by unit vectors  $q_i$ , which are expressed:

$$q_i = \frac{w_{i+1} - w_i}{\|w_{i+1} - w_i\|}, \quad (4)$$

$$q_{i+1} = \frac{w_{i+2} - w_{i+1}}{\|w_{i+2} - w_{i+1}\|}. \quad (5)$$

Based on [19], since the AGV needs a command to switch from the straight to a circular moving segment, the coordinates, where the transition happens, have to be defined. To define these coordinates, the angles  $\alpha_i$  of the sharp turns have to be expressed:

$$\alpha_i = \cos^{-1}(-q_i^T \cdot q_{i+1}). \quad (6)$$

Additionally, the transition distances  $k_i$  from the predefined waypoints  $w_i$  to the transition points  $H_i$  have to be found:

$$k_i = -\left(\frac{r}{\tan \frac{\alpha_i}{2}}\right) q_{i+1}. \quad (7)$$

Then, the transition points (coordinates)  $H_i$  where the movement changes from the straight line into a circular regime are expressed:

$$H_i(w_{i+1} - k_i, q_i) \quad (8)$$

$$H_{i+1}(w_{i+1} + k_i, q_{i+1}). \quad (9)$$

The coordinates of the circular arcs centre points – Dubins circles  $C_i$ , in which the AGV moves during the turning-circular regime, are calculated:

$$C_i = w_{i+1} - \left(\frac{r}{\sin \frac{\alpha_i}{2}}\right) \frac{q_i - q_{i+1}}{\|q_i - q_{i+1}\|}. \quad (10)$$

The same approach is used for all the waypoints / intersections in the predefined path.

### 3.3. Modification of Vector based Dubins Path Approach

According to [21], the Dubins path approach for the AGVs is based on three assumptions: 1. Vehicle moves at a constant velocity; 2. Vehicle cannot move in reverse, and 3. Vehicle has a minimum turning radius. From [37] it is clear that, with the mentioned assumptions, the steady state cornering condition is considered.

In research works such as [14, 19, 25, 27], where the Dubins path approach is applied and modified, the assumption that a vehicle always moves in a minimum turning radius is considered to be an advantage of this method. In this paper, the assumption that a vehicle always moves in a minimum turning radius is deemed not as an advantage but as a disadvantage. This proposition can be explained by the fact that the steering angle of the front wheels change is not a discrete function. After reaching the transition points where moving has to switch from the straight line into the turning-circular regime, the steering angle of the front wheels at the same time point does not change from neutral to the desired and vice versa. The change of the vehicle steering angle of the front wheels depends on the angular velocity of the steering wheel turning, the steering ratio and other

mechanical constraints. The same proposition can be applied to steer-by-wire systems. This means that at the beginning of the turning-circular regime, the actual steering angle of the front wheels is not equal to the predefined steering angle of the front wheels and, because of this reason, the undesired deviations from the predefined path appear and are always increasing. If the turning radius is minimum, then the amount of time during which the desired value of the steering angle is reached will be longer, and, respectively, the undesired deviations will be growing a longer amount of time and will be larger. That is why it is needed to optimise the turning radius, evaluating all the mentioned variables. To solve this problem and modify the vector based Dubins path approach, a cost function, which ensures the least possible deviations from the predefined straight line segments, is formulated:

$$\min f = \sum(x - x_p)^2 + \sum(y - y_p)^2 + \sum(\theta - \theta_p)^2, \quad (11)$$

where  $x_p, y_p, \theta_p$  respectively, are the coordinates of the predefined path.

Because the optimised parameter is turning radius  $r$ , also seeking to evaluate the nonholonomic constraints of the AGV, Eq. (11) must be rearranged. The rearrangement is done based on the kinematic AGV model, by inserting Eq. (3) into Eq. (2), integrating the obtained expression and inserting the result into Eq. (11). After rearranging, the cost function expression is obtained:

$$\min f = \sum(x - x_p)^2 + \sum(y - y_p)^2 + \sum \left( \frac{v \cdot \tan \left( \tan^{-1} \left( \frac{L}{r_i} \right) \right) \cdot t_i}{\sqrt{1 + \frac{l_r^2 \cdot \tan^2 \left( \tan^{-1} \left( \frac{L}{r_i} \right) \right)^2}{L}}} - \theta_p \right)^2, \quad (12)$$

where  $r_i$  is the optimised turning radius for every sharp turn in the re-planned path;  $l_r$  is the distance from the centre of mass to the rear axle;  $t_i$  is the demanded movement time from one transition point to the next transition point.

In the non-modified Dubins path approach it is assumed that the turning radius  $r$  for every re-planned sharp turn is the same (Fig. 1, Fig. 4, part b). Seeking to achieve better re-planning results, in the proposed methodology it is considered that, based on the straight line segments lengths, vehicle velocity, etc., the turning radius  $r_i$  is optimised separately for every sharp turn, which is re-planned. Consequently, based on the statement mentioned above, that the proposed cost function has to be applied separately to every sharp turn that is being re-planned, it is clear that Eq. (12) is only suitable for the re-planning of the first sharp turn. For the rest of the sharp turns re-planning, Eq. (12) has to be appended:

$$\min f = \sum(x - x_p)^2 + \sum(y - y_p)^2 + \sum \left( \frac{v \cdot \tan \left( \tan^{-1} \left( \frac{L}{r_i} \right) \right) \cdot t_i}{\sqrt{1 + \frac{l_r^2 \cdot \tan^2 \left( \tan^{-1} \left( \frac{L}{r_i} \right) \right)^2}{L}}} + \theta_{i-1} - \theta_p \right)^2, \quad (13)$$

where  $\theta_{i-1}$  is the vehicle orientation at the previous transition point.

In this methodology, lower and upper bounds for the turning radius optimisation problem, in which iterations must stay, are defined as a maximal steering angle of the front wheels  $-\delta_{max} \leq \delta \leq \delta_{max}$ , which cannot be exceeded due to technical vehicle limits / capacity. Respectively, the lower and upper bounds for Eq. (12) and Eq. (13) are  $-r_{max} \leq r \leq r_{max}$ . Seeking to ensure path integrity and safety, also to ensure the reliability and simplicity of the method, two different inequality constraints are proposed. In the case of the first inequality constraint, to utilise all of the six optimal Dubins path possibilities, such inequality constraint is formulated:

$$\sqrt{\frac{r_i^2 \cdot q_i(x)}{\tan^2 \left( \frac{\alpha_i}{2} \right)} + \frac{r_i^2 \cdot q_i(y)}{\tan^2 \left( \frac{\alpha_i}{2} \right)}} - \frac{q_{i-s}}{2} \leq 0, \quad (14)$$

where  $q_i(x, y)$  respectively, are the length of the corresponding unit vector in the  $x$  and  $y$  directions;  $q_{i-s}$  is the length of the shorter straight line segment, connected to the waypoint / sharp turn, which is re-planned.

In this inequality constraint case, the transition distances  $k_i$  from the predefined waypoints  $w_i$  cannot be less than half of the shorter straight line segment, connected to the waypoint / sharp turn, which is re-planned. That is why, depending on the shortest straight line segments lengths, all of the six optimal Dubins path possibilities are available.

During the optimisation of every turning radius  $r_i$ , it is necessary to evaluate that the steering angle of the front wheels change is not a discrete function. It is accepted that, in the kinematic model, the steering angle of the front wheels change is described as a time varying function:

$$\delta_i(t) = \frac{\pm d \cdot \omega \cdot t}{S_R}, \quad (15)$$

where  $d$  is the direction of the steering angle of the front wheels (negative – to the right, positive – to the left);  $\omega$  is the angular velocity of the steering wheel turning;  $t$  is time;  $S_R$  is the steering ratio.

When using the described methodology and separately optimising every turning  $r_i$ , the transition distances  $k_i$  from the predefined waypoints  $w_i$ , the transition points  $H_i$ , the coordinates of the circular arcs centre points  $C_i$  also have to be recalculated separately while applying the already described vector based Dubins path approach. Because the steering angle of the front wheels change is not a discrete function, in order to ensure feasibility of the re-planned path, the turning of the steering wheel must be started not at the exact same time as the first transition point  $H_i$  is reached. The transition point  $T_1$  for the start of the turning of the steering wheel is expressed:

$$T_1 = \sqrt{(H_1(x) - w_1(x))^2 + (H_1(y) - w_1(y))^2} - \frac{\tan^{-1} \left( \frac{L}{r_1} \right) \cdot v \cdot S_R}{\omega}, \quad (16)$$

where  $H_1(x, y)$  respectively, are the  $x$  and  $y$  coordinates of the recalculated first transition point;  $w_1(x, y)$  respectively, are the  $x$  and  $y$  coordinates of the first waypoint.

Based on Eq. (16), the second inequality constraint case can be formulated by revising Eq. (14):

$$\sqrt{\frac{r_i^2 \cdot q_i(x) + r_i^2 \cdot q_i(y)}{\tan\left(\frac{\alpha_i}{2}\right)^2 + \tan\left(\frac{\alpha_i}{2}\right)^2}} - \left( \frac{q_{i-s}}{2} - \frac{\tan^{-1}\left(\frac{L}{r_i}\right) \cdot v \cdot S_R}{\omega} \right) \leq 0. \quad (17)$$

In this inequality constraint case, the transition distances  $k_i$  from the predefined waypoints  $w_i$  will be less than half of the shorter straight line segment, connected to the waypoint / sharp turn, which is re-planned. Then, only four of the optimal Dubins path possibilities will be utilised, which necessarily involves the concave / convex circle segments connection with the straight line segments.

Based on each optimised turning radius  $r_i$ , the time bounds for every steering angle of the front wheels change, when the constant value of the steering angle must be maintained, is expressed as:

$$T_{UB\_i} = \frac{\tan^{-1}\left(\frac{L}{r_i}\right) \cdot S_R}{\omega}. \quad (18)$$

At the transition point, where the moving of the AGVs has to shift from the turning-circular regime into the straight line regime, the time bounds for the steering angle of the front wheels change from desired to neutral are also determined by applying Eq. (18).

Taking into consideration various researches, described in [9, 11, 14, 24], which analyse different algorithms for path optimisation, it can be stated that genetic algorithm as a robust popular heuristic search method has been extensively used to solve various single-objective and multi-objective path planning problems in discrete and continuous spaces by many authors. Thus, in this paper, to solve the described optimisation problem, a genetic algorithm was used.

#### 4. Experimental research

In order to investigate which inequality constraint of the proposed methodology is more effective and reliable, also to execute the experimental research, the described modified vector based Dubins path approach for the path re-planning was designed in the *MATLAB/Simulink* software package. For the lateral AGV control purposes, the kinematic-based controller described in [1] was used and also designed in the *MATLAB/Simulink* software package. Basic level of vehicle autonomy, i.e., autonomous steering, was implemented by using an automated steering device, developed in the *Vilnius Gediminas technical university*, with *Arduino* microcontroller (software and hardware), mounted on the test vehicle (Fig. 2). During the experimental research, the modified vector based Dubins path approach for the path re-planning and the kinematic-based controller developed in *MATLAB / Simulink* software were connected in real time with the *Arduino* microcontroller of the automated steering device while using the universal asynchronous receiver-transmitter (UART) based communication system. To satisfy the real time communication condition and to not overload the data buffers of the used communication system, the serial signal sending and receiving rates were 10 Hz. The step size of the mathematical opera-

tions solver used in the controller was fixed at 0.1 s. In [10] a similar method to achieve the basic level of vehicle autonomy, while using the *Arduino* microcontroller, is described.

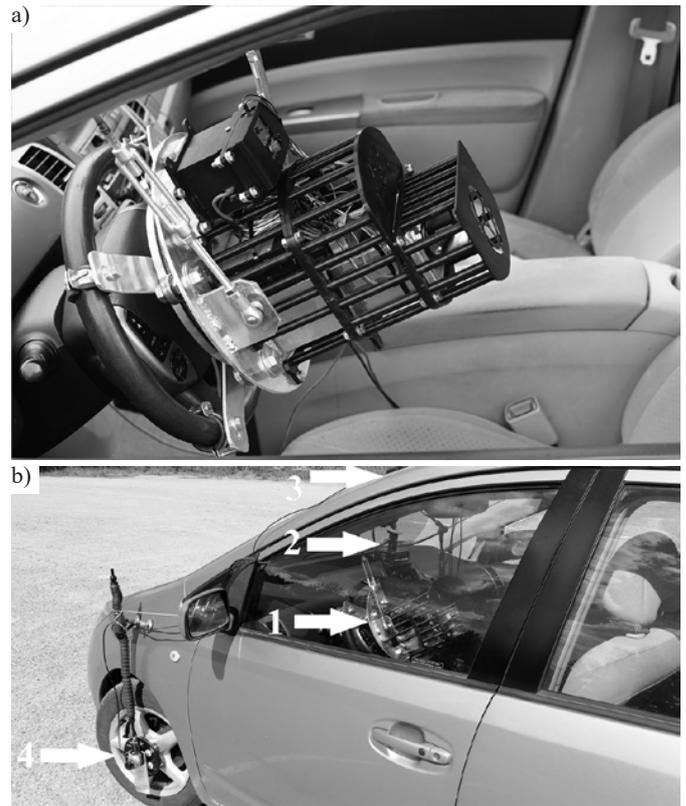


Fig. 2. Experimental research: a – automated steering device mounted on the test vehicle; b – autonomous test vehicle with 1 – automated steering device, 2 – GPS tracker, 3 – GPS antenna, 4 – angular velocity sensor

Seeking to perform reliability analysis of the proposed methodology, an experimental research was executed while re-planning the same path, predefined by a human, using different proposed inequality constraints (Fig. 3). The length of the longer side of the predefined path was 45 m, the length of the shorter side of the predefined path was 32 m. The angular velocity of the front wheels, the angle of the steering wheel, and the steering angle of the front wheels were the input parameters in the used controller. The actual movement coordinates were recorded with a GPS tracker, that was installed in the autonomous vehicle (Fig. 2, part b), upon the frequency of 100 Hz.

During the path following, an approximately constant velocity of 2.5–3.05 m/s of the test vehicle was maintained. The velocity of the test vehicle was controlled by a human supervisor for safety reasons.

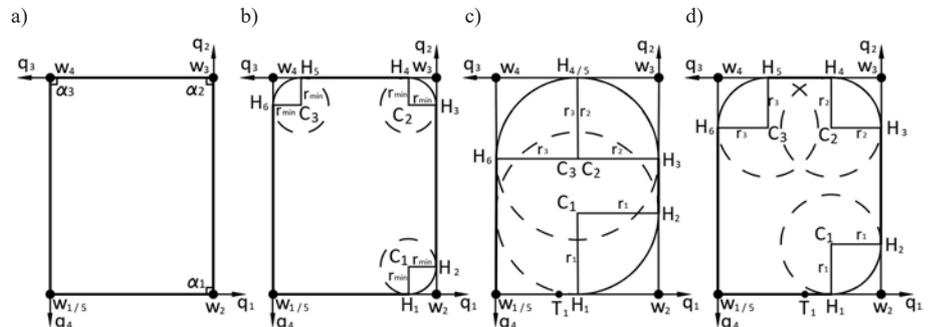


Fig. 3. Basic schemes of the predefined and re-planned paths: a – the predefined path; b – the path re-planned while using the non-modified Dubins path approach; c – the path re-planned while using Eq. (14); d – the path re-planned while using Eq. (17)

The human supervisor did not interact with the steering wheel at all. The experimental researches were done under equal conditions in an enclosed lot on a dry asphalt, without vertical slope surface at the temperature of 15–20°C. To ensure quantitative reliability analysis of the developed methodology, overall, 32 experimental drives, i.e., 16 experimental drives per each inequality constraint case, were performed.

## 5. Results and discussion

During all the experimental drives in the first inequality constraint case, the recorded moving trajectories were similar (Fig. 4, part b), and, respectively, during all the experimental drives in the second inequality constraint case, the recorded moving trajectories were also similar (Fig. 5, part b). Although in Fig. 4, part b and Fig. 5, part b some inaccuracies can be seen in the recorded moving trajectories, actual deviations or other undesired errors did not occur during the path following. The inaccuracies that can be seen in the recorded moving trajectories can be explained by the low performance of the used GPS tracker in some parts of the path. Due to these reasons, seeking to clearly describe the performance and reliability of the proposed algorithm, for further investigation of the proposed methodology, a specific experimental drive from each inequality constraint case was selected.

In Fig. 4 (part a) and Fig. 5 (part a) the values of the steering angle of the front wheels are presented, which were predefined while using the described modified Dubins path approach with different inequality constraints. Respectively, the actual values of the steering angle of the front wheels are also presented, which were recorded in real time as feedback data for the used kinematic-based controller during both cases of the experimental drives. For proper understanding of the results given in Fig. 4 and Fig. 5, it must be pointed out that during the moving in the turning-circular regime, the change of the steering angle of the front wheels was linear. Discretisation of the feedback signal is seen due to the selected serial signal receiving rate value (10 Hz), i.e., the frequency of the feedback data receiving into the data buffer. Discretisation of the feedback signal would be less visible if the serial signal receiving rate value was higher, however, in that case, the real time communication condition would not be satisfied because of the overloaded data buffer of the used *Arduino* microcontroller. Similarly, due to the selected serial signal receiving rate value and the discretised feedback signal, in both figures (Fig. 4, part a and Fig. 5, part a) it can be seen that there is a slight delay ( $\sim 0.5$  s) between the predefined values and the recorded feedback data values of the actual

steering angle of the front wheels. Though, when the feedback signal is delayed, the control signal is formulated based on the delayed data and, respectively, is inaccurate, the significance of the mentioned delay to the path tracking in any case of the experimental drives was not observed.

The re-planning of the first sharp turn (point  $w_2$ , Fig. 3) does not clearly visibly reflect the difference between the re-planned first turning radius  $r_1$ , while using different proposed inequality constraints. However, as it should be, the turning radius  $r_1$  (16.0 m), which was re-planned while using the case of the first inequality constraint (Eq. 14), is larger than the turning radius  $r_1$  (14.8 m), which was re-planned while using the case of the second inequality constraint (Eq. 17). This can be explained by the assumption proposed above that, in the case of the first inequality constraint, seeking to use all of the six optimal Dubins path possibilities, the transition distance  $k_1$  from the predefined waypoint  $w_2$  cannot be less than half of the shorter straight line segment, connected to the waypoint / sharp turn (from  $w_1$  till  $w_2$ ), which is re-planned ( $w_2$ ). This means that the transition point  $H_1$  cannot be closer to the predefined waypoint  $w_2$  than the middle point of the straight line segment from  $w_1$  until  $w_2$ . In the case of the second inequality constraint, as already described, seeking to use only four of the optimal Dubins path possibilities, the transition point  $H_1$  must be closer to the predefined waypoint  $w_2$  than half of the shorter straight line segment, connected to the waypoint / sharp turn, which is re-planned ( $w_2$ ). Thus, in the first re-planned sharp turn, and, respectively, in other re-planned sharp turns, moving in the turning-circular regime starts later while using the case of the second inequality constraint. Although the turning radiuses are different, from the presented results (Fig. 4 and Fig. 5) it can be seen that in both inequality constraint cases, the re-planned first sharp turn is feasible for the AGV.

The difference in the re-planned path, while using different inequality constraints, becomes more visible when second and third sharp turns are being re-planned. Because, in the re-planning of the second and the third sharp turns, the shorter straight line segment coincide – from  $w_3$  till  $w_4$ , in the case of the first inequality constraint, the transition points  $H_4$  and  $H_5$  also coincide (Fig. 3, part c). Due to the reasons that all six optimal Dubins path possibilities can be used and that the transition points  $H_4$  and  $H_5$  also coincide, at the joint transition point  $H_{4/5}$  the steering angle of the front wheels is constant. Seeking to move in the re-planned path, the constant value of the steering angle (0.16 rad) is maintained until the transition point  $H_6$ , which defines the change of moving from the turning-circular regime to the straight line regime (Fig. 4, part a). While the case of the first inequality constraint was used, four changes of the steering angle

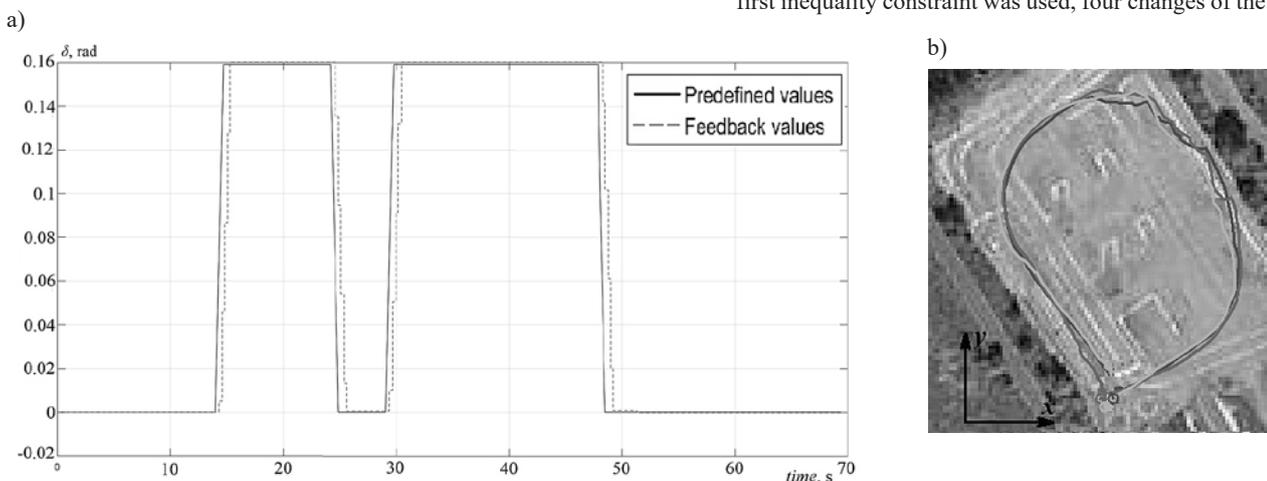


Fig. 4. Data of the performed experimental drives while using Eq. (14): a – the predefined values and the experimental drive feedback data values of the actual steering angle of the front wheels; b – example of the performed experimental drives while moving in a path, re-planned while using Eq. (14)

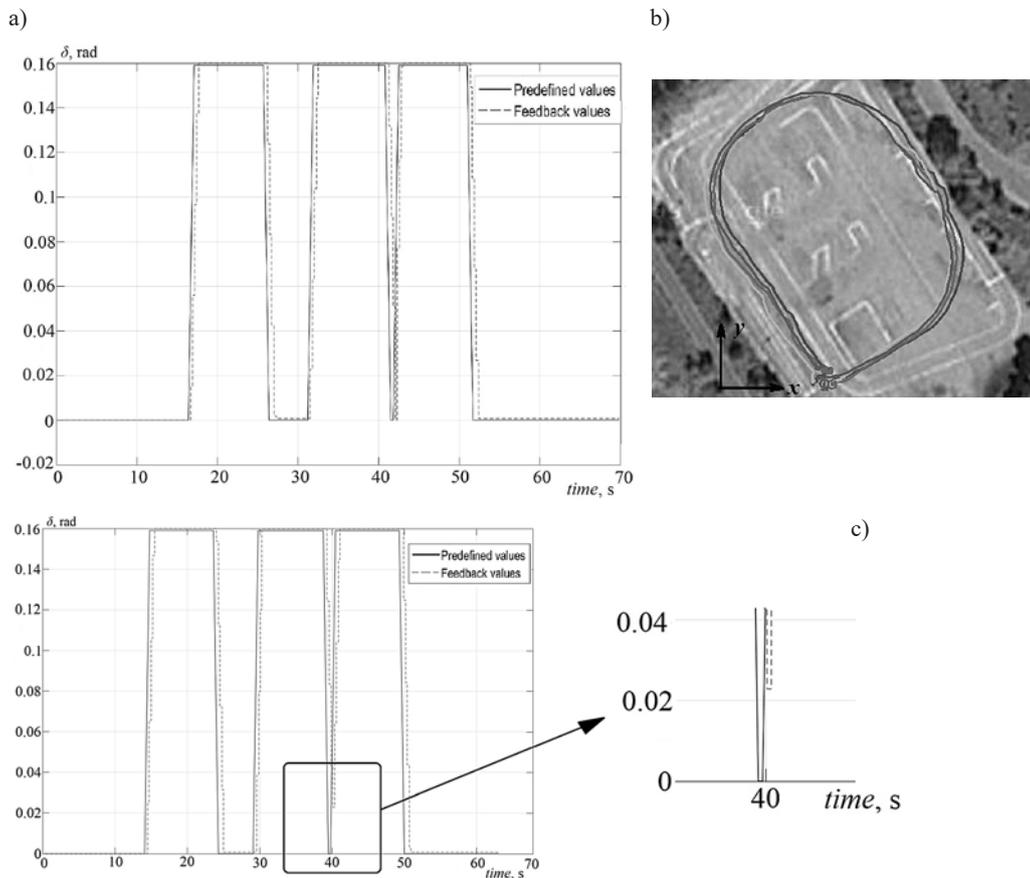


Fig. 5. Data of the performed experimental drives while using Eq. (17): a – the predefined values and the experimental drive feedback data values of the actual steering angle of the front wheels; b – example of the performed experimental drives while moving in a path, re-planned while using Eq. (17); c – the experimental drive feedback data values of the actual steering angle of the front wheels when front wheels were not returned to neutral position

of the front wheels were performed and the re-planned trajectory was feasible for the AGV (Fig. 4). In the case of the second inequality constraint, due to the assumptions mentioned above, the transition points  $H_4$  and  $H_5$  do not coincide. That is why, after reaching the transition point  $H_4$ , the front wheels are returned to the neutral position seeking to move in the straight line regime and, after reaching the transition point  $H_5$ , the turning of the steering wheels is being performed again to move in the turning-circular regime (Fig. 5, part a). While the case of the second inequality constraint was used, six changes of the steering angle of the front wheels were performed (Fig. 5). However, during the analysis of the results it was observed that when the moving in straight line segment time, due to the AGV velocity, is shorter than time, determined by using Eq. (18), then there is no possibility to accurately return the front wheels into the neutral position (Fig. 5, part c). When the AGV velocity was lower and the moving in the straight segment time was longer than time, determined by using Eq. (18), the front wheels were returned into the neutral position, as predefined (Fig. 5, part a). Based on the results, which are presented in Fig. 4 and Fig. 5, it can be stated that the reliability and performance of the proposed modified vector based Dubins path approach is proper and effective – both inequality constraints can ensure a feasible re-planned path, however, there is an additional condition. When the moving in the straight line regime time, due to the AGV velocity, is shorter than time, necessary to reach the predefined steering wheels angle, which is determined by using Eq. (18), the condition to ensure re-planned path feasibility is not satisfied. Seeking to clearly describe the reliability and possible use of the proposed ine-

quality constraints, it must be noted that both inequality constraints are suitable for path re-planning in unstructured roads, i.e., in cases where the turning radius is not constrained by external factors, like obstacles, road lines, etc. For example, while investigating the performance of various autonomous ground vehicles controllers, etc. Respectively, the proposed inequality constraints are not suited for a path, which is based on a real road network, re-planning. That is because the inequality constraints are built upon the length of the shorter straight line segment. To further improve the proposed methodology, with a view to ensure its use for re-planning of a path based on a real road network, the possibility to use both inequality constraints, which would be based not on the length of the shorter straight line segment but on road network limitations, must be considered and, respectively, developed.

## 6. Conclusion

In this work the development and reliability estimation of a methodology for re-planning of sharp turns in a path, while focusing on the constraints and the mobility of the vehicle, is presented. The conclusions obtained from the work are as follows:

1. Vector based Dubins path approach has been modified by applying a cost function with two different inequality constraints, this way developing a methodology for re-planning of sharp turns in a path, predefined by an on-board planner or a human. Because the non-modified Dubins path approach is based on the assumption that the AGV has a minimum turning radius, the developed methodology allows to solve the optimisation problem and to find an optimal and reliable turning radius, which would ensure path feasibility, while taking into consideration the primary path with sharp turns, the AGV velocity, the angular velocity of the steering wheel turning, the steering ratio and the wheelbase of the AGV.
2. To ensure quantitative reliability analysis and estimation of the developed methodology, experimental drives were performed. It was observed that, during the experimental drives, the real time communication condition was satisfied, while the developed methodology was used for path re-planning. Thus, it can be stated that the developed methodology is effective and does not require many computational resources.
3. During all the experimental drives in the first inequality constraint case, and, respectively, during all the experimental drives in the second inequality constraint case, actual deviations or other undesired errors did not occur during the path following. However, during the result analysis it was estimated that the reliability and performance of the developed methodology, while using different proposed inequality constraints, were not similar. When the first inequality constraint

was used for path re-planning, the reliability and performance of the developed methodology was proper and effective in all the cases of the experimental drives. Thus, it can be stated that the developed methodology with the first inequality constraint can reliably ensure a feasible re-planned path to follow. However, it was found out that the developed methodology with the second inequality constraint can reliably ensure a feasible re-planned path to follow only in the cases, when the moving in the straight line regime time, due to the AGV velocity, is longer than time, necessary to reach the predefined steering wheels angle. Such inaccuracy can lead to undesired deviations from the path and other negative effects, when the distance between the waypoints is respectively large.

In conclusion it can be stated that the developed methodology is reliable and effective, however it is important to improve the proposed methodology by considering the modification of both inequality constraints by basing them not on the length of the shorter straight line segment but on road network geometry. By realizing such an improvement, the developed methodology would not only be useful while seeking to ensure the path feasibility for the performance evaluation of various autonomous ground vehicles controllers, but also while developing high performance path planning algorithms for a safe, reliable and successful integration of the autonomous ground vehicles into the transport system.

## References

- Alcala E, Puig, V, Quevedo, J, Escobet, T, Comasolivas R. Autonomous Vehicle Control Using a Kinematic Lyapunov-Based Technique with LQR-LMI Tuning. *Control Engineering Practice* 2018; 73: 1–12, <https://doi.org/10.1016/j.conengprac.2017.12.004>
- Alcala E, Sellart L, Puig V, Quevedo J, Saludes J, Vazquez D, Lopez, A. Comparison of Two Non-Linear Model-Based Control Strategies for Autonomous Vehicles. *24th Mediterranean Conference on Control and Automation* 2016: 846–851, <https://doi.org/10.1109/MED.2016.7535921>
- Arokiasami W A, Vadakkepat P, Tan K C, Srinivasan D. Vector Directed Path Generation and Tracking for Autonomous Unmanned Aerial / Ground Vehicles. *IEEE Congress on Evolutionary Computation (CEC)* 2016: 1375–1381, <https://doi.org/10.1109/CEC.2016.7743949>
- Bascetta L, Cucci D A, Mattuucci M. Kinematic trajectory Tracking Controller for an All-Terrain Ackermann steering Vehicle. *IFAC-PapersOnLine* 2016; 49(15): 13–18, <https://doi.org/10.1016/j.ifacol.2016.07.600>
- Bayar G, Bergerman M, Konukseven E I, Koku A B. Improving the Trajectory Tracking Performance of Autonomous Orchard Vehicles Using Wheel Slip Compensation. *Biosystems Engineering* 2016; 146: 149–164, <https://doi.org/10.1016/j.biosystemseng.2015.12.019>
- Brombacher A. (Re)liability of Self-driving Cars. An Interesting Challenge!. *Quality and Reliability Engineering International* 2014; 30(5): 613–614, <http://dx.doi.org/10.1002/qre.1707>
- Brown M, Funke J, Erlien S, Gerdes J C. Safe Driving Envelopes for Path Tracking in Autonomous Vehicles. *Control Engineering Practice* 2017; 61: 307–316, <https://doi.org/10.1016/j.conengprac.2016.04.013>
- Bullough J D, Skinner N P. High visibility reflective sign sheeting materials: field and computational evaluations of visual performance. *Transport* 2018; 33(2): 344–352, <http://dx.doi.org/10.3846/16484142.2016.1230889>
- Castillo O, Trujillo L, Melin P. Multiple Objective Genetic Algorithms for Path-planning Optimization in Autonomous Mobile Robots. *Soft Computing* 2007; 11(3): 269–279, <https://doi.org/10.1007/s00500-006-0068-4>
- Coombes M, Eaton W, MacAree O, Chen W H. Development of a Generic Network Enabled Autonomous Vehicle System. *UKACC International Conference on Control* 2014: 621–627, <https://doi.org/10.1109/CONTROL.2014.6915211>
- Davoodi M, Panahi F, Mohades A, Hashemi S N. Clear and Smooth Path Planning. *Applied Soft Computing* 2015; 32: 568–579, <https://doi.org/10.1016/j.asoc.2015.04.017>
- Delingette H, Hebert M, Ikeuchi K. Trajectory Generation with Curvature Constraint Based on Energy Minimization. *Proceedings of IEEE / RSJ International Workshop on Intelligent Robots and Systems* 1991: 6 p., <https://doi.org/10.1109/IROS.1991.174451>
- Fu Y, Li H, Kaye M. Design and Lyapunov Stability Analysis of a Fuzzy Logic Controller for Autonomous Road Following. *Mathematical Problems in Engineering* 2010: 20 p., <https://doi.org/10.1155/2010/578406>
- Gottlieb Y, Manathara J G, Shima T. Multi-Target Motion Planning Amidst Obstacles for Autonomous Aerial and Ground Vehicles. *Journal of Intelligent & Robotic Systems* 2017: 1–22, <https://doi.org/10.1007/s10846-017-0684-5>
- Gu T, Dolan J M. On-Road Motion Planning for Autonomous Vehicles. *Intelligent Robotics and Applications* 2012: 588–597, [https://doi.org/10.1007/978-3-642-33503-7\\_57](https://doi.org/10.1007/978-3-642-33503-7_57)
- Gupta N, Ordóñez C, Collins E G. Dynamically Feasible, Energy Efficient Motion Planning for Skid-Steered Vehicles. *Autonomous Robots* 2017; 41(2): 453–471, <https://doi.org/10.1007/s10514-016-9550-8>
- Han J, Seo Y. Mobile Robot Path Planning with Surrounding Point Set and Path Improvement. *Applied Soft Computing* 2017; 7: 35–47, <https://doi.org/10.1016/j.asoc.2017.03.035>
- Kala R. *On-Road Intelligent Vehicles. Motion Planning for Intelligent Transportation Systems*. Kidlington: Elsevier, 2016.
- Kikutis R, Stankūnas J, Rudinskas D, Masiulionis T. Adaptation of Dubins Paths for UAV Ground Obstacle Avoidance When Using a Low Cost On-Board GNSS Sensor. *Sensors* 2017; 17(10): 23 p., <https://doi.org/10.3390/s17102223>
- Krishan J, Rajeev U P, Jayabalan J, Sheela D S. Optimal Motion Planning Based On Path Length Minimisation, Robotics and Autonomous Systems 2017; 94: 245–263, <https://doi.org/10.1016/j.robot.2017.04.014>
- LaValle S M. *Planning Algorithms*. Cambridge University Press: 2006.
- Li X, Sun Z, Cao D, Liu D, He H. Development of a new integrated local trajectory planning and tracking control framework for autonomous ground vehicles. *Mechanical Systems and Signal Processing* 2017; 87: 118–137.
- Liang T C, Liu J S. A Bounded-Curvature Shortest Path Generation Method for Cal-Like Mobile Robot Suing Cubic Spiral. *Proceedings of 2004 IEEE / RSJ International Conference on Intelligent Robot and Systems* 2004: 2819–2824, <https://doi.org/10.1109/IROS.2004.1389836>
- Liang Y, Xu L. Global Path Planning for Mobile Robot Based Genetic Algorithm and Modified Simulated Annealing Algorithm. *Proceedings of the First ACM/SIGEVO Summit on Genetic and Evolutionary Computation* 2009: 303–308, <https://doi.org/10.1145/1543834.1543875>
- Macharet D G, Neto A A, Neto V F, Campos M F M. Nonholonomic Path Planning Optimization for Dubins' Vehicles. *IEEE International Conference on Robotics and Automation* 2011: 4208–4213, <https://doi.org/10.1109/ICRA.2011.5980239>

26. Macharet D G, Neto A A, Neto V F, Campos M F M. Dynamic Region Visit Routing for Vehicles with Minimum Turning Radius. *Journal of Heuristics* 2017; 24(1): 83–109, <https://doi.org/10.1007/s10732-017-9359-4>
27. Medeiros A C, Urrutia S. Discrete Optimization Methods to Determine Trajectories for Dubins' Vehicles. *Electronic Notes in Discrete Mathematics* 2010; 36: 17–24. <https://doi.org/10.1016/j.endm.2010.05.003>
28. Millo F, Cubito C, Rolando L, Pautasso E, Servetto E. Design and development of an hybrid light commercial vehicle. *Energy* 2017; 136: 90–99, <http://dx.doi.org/10.1016/j.energy.2016.04.084>
29. Młynarski S, Pilch R, Smolnik M, Szybka J. Methodology of network systems reliability assessment on the example of urban transport. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2018; 20(2): 278–283, <http://dx.doi.org/10.17531/ein.2018.2.14>.
30. Mousavi M, Pakniyat A, Wang T, Boulet B. Seamless dual brake transmission for electric vehicles: design, control and experiment. *Mechanism and Machine Theory* 2015; 94: 96–118, <https://doi.org/10.1016/j.mechmachtheory.2015.08.003>
31. Pielecha I, Cieślak W, Szalek A. Operation of electric hybrid drive systems in varied driving conditions. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2018; 20 (1): 16–23, <http://dx.doi.org/10.17531/ein.2018.1.3>.
32. Praviļonis T, Sokolovskij E, Surblys V. The analysis of deformations of the frame in a basic minibus and a low floor minibus. *Mechanics* 2017; 23(3): 365–369, <http://dx.doi.org/10.5755/j01.mech.23.3.14480>
33. Receveur J B, Victor S, Melchior P. Multi-Criteria Trajectory Optimization for Autonomous Vehicle, *IFAC Papers Online* 2017; 50(1): 12520–12525. <https://doi.org/10.1016/j.ifacol.2017.08.2063>
34. Renzi C, Leali, F, Di Angelo L. A review on decision-making methods in engineering design for the automotive industry. *Journal of Engineering Design* 2017; 28(2): 118–143, <http://doi.org/10.1080/09544828.2016.1274720>
35. Reuter J. Mobile Robots Trajectories with Continuously Differentiable Curvature: An Optimal Control Approach. *Proceedings of the IEEF / RSJ Intl. Conference on Intelligent Robots and Systems* 1998: 38–43. <https://doi.org/10.1109/IROS.1998.724593>
36. Surblys V, Ślaski G, Pikosz H. The usage of a laser height sensors for estimating road unevenness profile. *The archives of automotive engineering = Archiwum motoryzacji*. Warszawa: Wydawnictwo Naukowe Przemysłowego Instytutu Motoryzacji 2018; 79(1): 95–106, <http://dx.doi.org/10.14669/AM.VOL.79.ART7>
37. Žuraulis, V.; Sokolovskij, E.; Matijošius, J. The opportunities for establishing the critical speed of the vehicle on research in its lateral dynamics. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2013; 15(4): 312–318.

---

**Paulius SKAČKAUSKAS**

**Edgar SOKOLOVSKIJ**

Department of Automobile Engineering

Vilnius Gediminas Technical University

J. Basanavičiaus 28, LT-03224 Vilnius, Lithuania

E-mail: paulius.skackauskas@vgtu.lt, edgar.sokolovskij@vgtu.lt

---