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RESEARCH OF DYNAMIC PROCESSES OF THE SYSTEM “VEHICLE – TRACK” USING THE NEW METHOD OF VEHICLE WHEEL WITH METAL SCALE

BADANIE DYNAMICZNYCH PROCESÓW ZACHODZĄCYCH W UKŁADZIE „POJAZD-TOR” Z WYKORZYSTANIEM NOWEJ METODY DLA KÓŁ Z METALOWĄ ŁUSKĄ

Mathematical models of vehicle wheel with metal scales are introduced in this article. When analysing the interaction between vehicle wheel with a metal scale and rail in the system “Vehicle – Track”, the changes of the kinematic and dynamic parameters of the wheel and rail contact points in time are examined, depending on the height of the 2 mm metal scale, when the length of the metal scale is 100 mm and the speed of movement is \( V = 40 - 100 \) km/h. The results obtained after the research of the system “Vehicle – Track”, when the wheel has a metal scale, help to better understand and evaluate the impact of metal scale on wheel on dynamic loads of rail and vehicle and the regularities of their movement. The appearance of a metal scale on the wheel's surface causes technical and maintenance problems for the rolling stock. Railway standards limit the speed of movement that depends on a certain size of metal scale.

**Keywords:** rail-wheel interaction; spatial model of metal scale; contact area; impact force; friction force; vibration.

W niniejszym artykule przedstawiono modele matematyczne kół pojemnościowych z powstałej w wyniku zużycia metalową łuską. Analizując oddziaływania pomiędzy kółem pojazdu z łuszką a szyną w układzie "pojazd–tor", badano zmiany kinematycznych i dynamicznych parametrów punktów kontaktu koła z szyną zachodzące w czasie, w zależności od wysokości metalowej łuski (2 mm), przy długości łuski 100 mm i zakresie prędkości ruchu pojazdu \( V = 40 - 100 \) km/h. Wyniki uzyskane w badaniu układu "pojazd–tor" dla kół na powierzchni których powstała metalowa łuska, umożliwiają lepsze zrozumienie oraz ocenę wpływu łuski na dynamiczne obciążenia szyny i pojazdu oraz prawidłowości ruchu pojazdu. Pojawienie się metalowej łuski na powierzchni koła powoduje problemy techniczne i obsługa w utrzymaniu ruchu taboru kolejowego. Normy kolejowe ograniczają prędkość ruchu pojazdów szynowych, uzupełniając ją od rozmiaru łuski.

Słowa kluczowe: oddziaływania koło–szyna; przestrzenny model łuski metalowej; obszar kontaktu; siła uderzenia; siła tarcia; vibracja.

1. Introduction

The precise operation of modern machines and vehicles can only be ensured by complex measures, which consist of modern construction solutions for design, the use of high-quality and suitable operational and structural materials, and quality maintenance, which is done on time.

Multicriteria decision-making is widely used in all areas. Multicriteria assessment models are presented in the field of transport [24, 27]. Using the [20] multicriterion additive model, it was found that the main parameter of the technical condition of the railway track is the speed of the wagon.

Initial surface of vehicle wheel profile is symmetric. The radius of vehicle wheel is equal at all points. However, during exploitation, roughness appears on the surfaces of wheel and rail profiles, due to interaction between vehicle wheel and rail, and their different geometrical surfaces. Furthermore, the wheel profile constantly changes and becomes asymmetrical.

Damages on surfaces of wheel and rail mostly appear due to their interaction. However, they may also occur due to various manufactured inaccuracies and poor quality of machined parts. Wheel and rail damages are rarely found on initial stages of exploitation, and over time, the damages increase and may cause irreversible consequences.

To reduce the risk of train accidents, railway traffic risk management models are being developed [6], which enable railway managers to improve their traffic safety strategy by determining the priorities of the required measures.

Most of the vehicle wheel damage occurs due to braking, surface roughness, temperature differences, and other factors. Metal plasticity methods are used to evaluate the mechanical condition (determine strength and plasticity characteristics) of solid moulding wheels of the wagon [28]. The theoretical angular speeds of the loaded wheels are different due to disproportionate loads and although they are forced to move at the same speed as the wagon, the wheels are sometimes forced to tow and even slip unevenly.

The most common damage in wheel is flat, which occurs due to wheel slippage or stuck brake pads [31]. Increase in wheel damages has been examined in the article [10].

Metal scales on wheel occur due to thermos-mechanical damages. Intensive plastic metal deformation appears due to sudden braking, short-term wheel skidding and jumping, when the metal of the wheel suddenly heats up and then suddenly cools down. Sudden braking means the braking of the train in exceptional cases, in these circumstances the largest braking force is used when the air is released from the brake pipe [26].
Many metal scales can appear on the rolling surface of the wheel, which can have one or more layers and differ in height. U-shaped scales are the most common to occur. The damages of scaled wheel are measured from the surface intact to the highest point of the scale. During the exploitation, the metal scale can become even more layered and can even come off the wheel surface. The hardness of metal scale is about 900 HV, which is typical for tempered steel with high residual stress.

In the case of scaled wheel, large impact forces appear in the contact between the wheel and rail, which load the rail and wheelset bearings. Wheel flats frequently appear on the rolling surface of the wheel due to metal scales.

The metal scale on wheel may form when the wheel is broken extremely. Due to large frictional forces and increased temperatures, the metal on the wheel melts and a certain layer of metal is cut off. A flat develops in the contact zone, and the molten metal flows outside the contact zone and to the surface of the wheelset. If the wheel continues to rotate, the moulded metal cools off and continues to deform when it comes in the contact with the rail. This is how the multi-layered metal scale forms on the surface of wheel (Fig. 1, Fig. 2).

Railway vehicle exploitation in Lithuanian Railways (JSC Lithuanian Railways) [22] is prohibited, when wheel has a scale with height higher than 0.5 mm in passenger cars and 1 mm in freight cars.

The authors [2-5, 27] usually choose flat as vehicle wheel damage to simplify the mathematical models, as it is the most common damage in vehicle wheel.

The contact zone of wheel and rail is described as a point and the geometry of the wheel as an analytical function in the mathematical models of interaction between wheel and rail [1, 15, 17, 18, 19, 20, 21, 22, 23].

Researchers [9, 16, 29, 30] describe variations of mass accelerations and displacements of wagon and rail in time or path length, in their studies of dynamical processes that occur in the interaction between wheel and rail. It was determined that the maximum acceleration value is obtained in the case of a wheel with a flat.

Parameters of the deformation of the isolated sinusoidal shape and the influence of the wagon speed on the vertical vibration of the body are evaluated in the dynamic models of interaction between the vehicle and track [12].

The influence of contact forces on the deformation of rolling carload wheels and rails, and the influence of this deformation on the redistribution of the contact stresses is also investigated [25].

Theoretical and practical study of the surface roughness of interaction between rail and train defined the values the surface roughness, when interacting with different rail profiles that affect the increase in exploitation time [11].

Research has been carried out [8] to improve the safe operation of trains and increase the efficiency of load assessments.

Rail condition is assessed [7,13,14] by using the movements of axle-box and bodies, estimating and analysing accelerations.

The authors of this article have failed to find other researches about metal scale in wheelset wheel.

The article consists of three chapters. In the first chapter the system „Vehicle – Track” and literature relevant to its elements are analysed. In the second chapter authors present a mathematical model of wheelset wheel with a metal scale and briefly describe the model of system „Vehicle – Track” and present a calculation algorithm for this system. The third chapter consists of the results of the calculations and discussion.

This paper presents mathematical models of a vehicle wheel with a 2 mm metal scale. The mathematical model allows examination of the interaction between wheel with a metal scale and the rail and displays its effect on dynamic loads. The new mathematical model allows evaluation of the rotation of the wheel around its longitudinal axis Y, in order to evaluate the rotation of the wheel and determine its slip on the rails. In this article, the authors seek to determine the slip of the wheel with a defect, when the defect is a metal scale and without a wheel defect. In the case of sliding the wheel in relation to the rail, frictional forces occur, and heat is released, which increases the wheel and rail temperature in the contact area.

The paper shows results of the changes in kinematic parameters of a wheel (angular velocity and acceleration, velocities, accelerations and other parameters) and dynamic parameters (forces and other parameters), depending on the geometry of metal wheel scale, movement speed and other parameters of the system “Vehicle – Track”.

2. Materials and Methods

2.1. The scope of calculations and numerical characteristics of an adhesive joint

During exploitation, the metal scale on wheel can take on various forms. Therefore, this article presents a method for shape generation of metal scale of a wheel and allows generation of a wheel profile with a metal scale if the exact geometry is known.

The mathematical model created in respect to the geometry of metal scale shown in Figure 1 is described in this section. The metal scale of wheelset wheel that consists of three layers is shown in Figure 1 (real top view), where \(N_{\text{layer}}\) are the layers of metal scale.

Metal scales are rarely included in the calculations of mathematical models of railway system “Vehicle – Track”, due to complex and different geometrical shapes (Fig. 2). There are only a few known scientific articles about wheel damage, called metal scale, and its effects on the system “Vehicle – Track”.

![Fig. 1. The metal scale of wheelset wheel, that consists of three layers (real top view)](image1)

![Fig. 2. Multi-layered metal scale of wheelset wheel](image2)
The developed mathematical model allows generating more metal scales on wheel and more layers of the metal scale (Fig. 2).

The geometry of the wheel and the metal scale on it is shown in Figure 3.

![Diagram of wheel and metal scale](image)

**Fig. 3. Computational scheme of metal scale of wheel: a) wheel profile; b) geometrical parameters of metal scale of wheel in directions \( Y_{CP} \) and \( Z_{CP} \); c) geometrical parameters of metal scale of wheel in directions \( X_{CP} \) and \( Z_{CP} \).**

The width of the \( i \)-th layer of wheel metal scale is \( 2c_{p1} \), and the length is \( 2b_{p1} \). Maximum length of metal scale is \( 2b_{p1} \). Points A and B (Fig. 3) indicate the start and end of metal scale of vehicle wheels. The geometry of metal scale is described in local coordinate system \( X_{CP}, Y_{CP}, Z_{CP} \). Points \( C_{p1}, C_{p2}, C_{p3} \) are the centre points of the metal scale layers. \( Z_{CP20}, Z_{CP30} \) (Fig. 3a and Fig. 3b) are the coordinates of centre points \( Z_{CP} \) of second and third layers, furthermore \( Z_{CP10} = 0 \).

The maximum height of metal scale \( h_{max} \) (Fig. 3a) is equal to the difference between maximum wheel radius \( R_{WII} \) and nominal wheel radius \( R_{W} \). The size of angles \( \theta_{max} \) and \( \theta_{min} \) (Fig. 3a) depends on the position of scale on the surface of the wheel and indicates the maximum and minimum angle size, when generating wheel profile with a metal scale. The position of the scale centre \( C_{p} \) can be described as centre angle \( \theta_{CP} = (\theta_{max} + \theta_{min})/2 \).

When developing the geometrical model of metal scale of vehicle wheel it is accepted that:

- The metal scale of the vehicle wheel consists of \( N_{layer} \) layers (Fig. 1).
- The profile of metal scale is generated in local coordinate system \( X_{CP}, Y_{CP}, Z_{CP} \).
- Each centre of metal scale layer can be moved in \( Z_{CP} \) axis by a value of \( Z_{CP0} \) but \( Z_{CP0} = 0 \).
- Geometrical parameters of each metal scale layer (starting from the second layer) are independent values that are selected in such way, that total profile of metal scales would be generated as accurate as possible.

It is assumed, that metal scale of vehicle wheel is between points A and B, when the centre angle \( \theta \) (Fig. 3a) varies from \( \theta_{min} \) to \( \theta_{max} \), i.e. \( \theta \in [\theta_{min}, \theta_{max}] \). The section between points A and B is divided to \( N_{P} - 1 \) intervals, where \( N_{P} \) – total number of points on the surface of generated scale.

During interpretation of the geometry of a metal scale, it is assumed that the number of layers of the calculated sheet is \( N_{layer} = 3 \) (Fig. 1, Fig. 3.b, c).

When generating a wheel profile with one scale, the perimeter of the vehicle wheel surface is divided into three: I zone, when \( 0 \leq \theta < \theta_{min} \); II zone, when \( \theta_{min} \leq \theta < \theta_{max} \); III zone, when \( \theta_{max} \leq \theta < 2\pi \).

A change of radius in vehicle wheel with metal scale is described as a function \( R_{WII}(\theta) \) in the second zone. Changes of radius in the first and third zones of vehicle wheel with metal scale can be approximated in the same way as in the second zone, when the wariness of vehicle wheel is known in each of the zones.

By knowing the variation of wheel radius \( R_{W} \) in each zone, it is possible write the function \( R_{WII}(\theta) \) of changes in vehicle wheel radius, over the whole perimeter of the wheel surface:

\[
R_{WII}(\theta) = R_{WII}(\theta) \left[ H(\theta - H(\theta - \theta_{min})) + R_{WII}(\theta) + R_{WIII}(\theta) \right] - H(\theta - 2\pi)]
\]

where \( R_{WII}(\theta) \), \( R_{WIII}(\theta) \) are known wheel radius functions in the first and third zones, \( R_{WII}(\theta) \) is the radius function in the second zone (Fig. 3a), \( H \) is Heaviside step function.

Wheel radius function \( H \) \( R_{WII}(\theta) \) in the second zone is:

\[
R_{WII}(\theta) = \sum_{i=1}^{N_{P}-1} R_{WII,i+1,\theta}(\theta) \left[ H(\theta - \theta_{j}) - H(\theta - \theta_{j+1}) \right]
\]

where \( R_{WII,i+1,\theta}(\theta) \) is radius function between points \( i \) and \( i+1 \),

\[
R_{WII,i+1,\theta}(\theta) = N_{i}(\xi)R_{WII}(\theta + \xi) + N_{2}(\xi)R_{WII} \left( \frac{\theta + \theta_{j+1}}{2} \right) + N_{3}(\xi)R_{WII}(\theta_{j+1})
\]

where \( \xi \) dimensionless coordinate, \( N_{i}(\xi) \) is shape function, \( j = 1, 2, 3 \).

Dimensionless coordinate \( \xi \) is:

\[
\xi = \frac{\theta - \theta_{j}}{\theta_{j+1} - \theta_{j}}, \text{ when } \xi \in [0,1].
\]

The shape functions \( N_{i}(\xi) \) are equal:

\[
N_{1}(\xi) = (2\xi - 1)(\xi - 1), \quad N_{2}(\xi) = 4\xi(1 - \xi), \quad N_{3}(\xi) = (2\xi - 1).
\]
Equation (13) is used during examination of the metal scale profile function. The mathematical model of the system “Vehicle – Track”, when the vehicle wheel is scaled, is composed of several mathematical models and designed to determine the forces acting during the interaction between rail and damaged wheel. This mathematical model assesses the speed of the vehicle, the geometric parameters of the interacting bodies, their physical and mechanical properties, and allows determination of the changes in the forces acting during contact.

The mathematical model of the system “Vehicle – Track” for wheel with a metal scale is used on two-dimensional space. It is assumed that $Y = 0$.

Many parameters that appear in the contact zone of wheel and rail, between two contacting surface points and at every time moment can be determined by using this mathematical model [5, 27]: wheel slip, friction forces, frictional torque, distributed load and other parameters. During analysis of the interaction between elements in system “Vehicle – Track”, these assumptions and conditions are considered:

- Rail deformation in $X$, $Z$ directions;
- Interaction between roadbed and rail, as an elastic foundation;
- Possible gap between the sleeper and roadbed;
- Length of wheel and rail contact and geometrical unevenness appearing on it;
- The effect of rail axial forces on rigidity (due to differences in temperature);
- Initial bending of the rails;
- Possible gap between rail and sleeper;
- Bending of rail that is between two sleepers;
- Interaction of soil layers, that is under two adjacent sleepers;
- Wheel profile with damages;
- Contact zone is examined as linear contact according to $X$ coordinate.

The system “Vehicle – Track” is examined in vertical direction. The computational scheme and its elements are shown in Figure 4. Vehicle in computational model of system “Vehicle – Track” consists of (Appendix B and Fig. 4): 1/8 wagon mass, 1/4 bogie mass $m_{bg3}$, 1/2 wheelset mass. Wheelset mass is divided into two parts: $m_{bg1}$ - wheel mass, in direct contact with the rail and $m_{bg2}$ - main wheelset mass. During interaction between the wheel and the rail, the use of the wheel mass $m_{bg1}$, which is directly in a contact with the rail, allows a more accurate assessment of the forces occurring on the wheel-rail contact and the kinematic parameters of the individual wheelset. Track in computational model of system “Vehicle – Track” consists of (Fig. 4): rail ($m_r$), sleeper ($m_s$) and railway roadbed $m_{rl}$. Roadbed consists of three layers (Appendix C): ballast ($m_{b3}$), sub-ballast ($m_{b2}$) and soil ($m_s$).

Linear (marked as ovals in Fig. 4 and non-linear (marked as triangles in Fig. 4) stiffness and damping elements are used in the systems “Vehicle – Track”.

2.3 Nonlinear dynamical computational algorithm for movement equations of system “Vehicle – Track”

System of movement equations of “Vehicle – Track” with the metal scale of wheelset wheel is equal to:

$$\begin{align*}
\dot{\mathbf{q}} &\in [\mathbf{M}]\mathbf{\ddot{q}} + [\mathbf{C}][\mathbf{\dot{q}}] + [\mathbf{K}][\mathbf{q}] = \{\mathbf{F}_{NL}(q, \dot{q}) + \{F\}}
\end{align*}$$

where $[\mathbf{M}], [\mathbf{C}], [\mathbf{K}], \{\mathbf{F}_{NL}(q, \dot{q})\}$ are mass, damping and stiffness matrices, nonlinear generalized force vector and external force vector, respectively. $\{\mathbf{q}\}, \{\mathbf{q}\}, \{\mathbf{q}\}$ are the system generalized displacements, velocities and accelerations vectors, respectively.
Nonlinear generalized force \( F_{NL}(q, \dot{q}, t) \) is extracted in the Taylor series at the point \( \{q, \dot{q}\} \):

\[
F_{NL}(q, \dot{q}) = F_{NL,k} + [K_{T,k}]\{\Delta q\} + [C_{T,k}]\{\Delta \dot{q}\},
\]

(15)

where:

\[
[K_{T,k}] = \left[ \frac{\partial [F_{NL}(q, \dot{q})]}{\partial [q]} \right],
[C_{T,k}] = \left[ \frac{\partial [F_{NL}(q, \dot{q})]}{\partial [\dot{q}]} \right] ;
\]

\(\{\Delta q\}\) and \(\{\Delta \dot{q}\}\) are increments displacements and velocities vectors, respectively.

Then, the total system of equations (14), at the moment of time \( t + \Delta t \), is equal to:

\[
[M]\{\ddot{q}_{t+\Delta t}\} + [C]\{\dot{q}_{t+\Delta t}\} + [K]\{q_{t+\Delta t}\} - [C_T]\{\Delta \dot{q}_{t+\Delta t}\} - [K_T]\{\Delta q_{t+\Delta t}\} = F_{NL}(q_{t+\Delta t}, \dot{q}_{t+\Delta t}) + F(t)
\]

(16)

where \( \Delta t \) is integration time step; \( t \) is time.

By applying Newmark and Newton–Raphson methods, the total system for linear algebraic equations is solved in each of \( k \)-th iteration:

\[
[A_{t+\Delta t,k}]{\Delta q}_{k} = -\{\Delta \ddot{q}_{t+\Delta t}\}, \quad \text{or} \quad \{p_{t+\Delta t,k}\} = [A_{t+\Delta t,k}]\{\Delta q\} + \{p_{t+\Delta t,k}\} = 0
\]

(17)

where:

\[
[A_{t+\Delta t,k}] = \left[ \frac{1}{\beta \Delta t^2}[M] + \frac{1}{\beta \Delta t}[C] - [C_{T,k}] \right] - \left[ \frac{1}{\gamma \Delta t}[K] - [K_{T,k}] \right]
\]

\[
\{p_{t+\Delta t,k}\} = \left[ \frac{1}{\beta \Delta t^2}[M]\{\ddot{q}_{t+\Delta t}\} + \left[ \frac{1}{\beta \Delta t}[C]\{\dot{q}_{t+\Delta t}\} + [K]\{q_{t+\Delta t}\} - [F_{NL,k}] \right] - [F(t + \Delta t)] \right]
\]

where: \( \beta, \gamma \) are the Newmark coefficients \((\gamma = 1/2, \beta = 1/4)\).

The computational algorithm of nonlinear system “Vehicle – Track” is presented in Fig. 5.

3. Results and discussion

3.1. Initial data of research of the system “Vehicle – Track”, when the wheel has a metal scale

The purpose of the research is to determine the interaction between wheel and rail, show how the wheel and rail movement changes and introduce the impact of geometrical parameters of metal scale on the dynamical loads occurring during the wheel-rail contact, by using the mathematical model of the system “Vehicle – Track”, when the wheel has a metal scale. The system “Vehicle – Track” is analysed, when the vehicle wheel has radius \( R_W = 0.495 \) m and has a metal scale, is moving on the rail (R-65) at different speeds \((V = 40, 60, 80,\) km/h), Fig. 4.

![Fig. 4. Computational model of element interaction of system “Vehicle – Track”](image)

![Fig. 5. Dynamical, nonlinear movement equation solving algorithm of system “Vehicle – Track”](image)

![Fig. 6. Alteration of wheel radius, when the wagon is moving at speed \( V = 40 \) km/h, metal scale width \( L_p = 100 \) and at different heights of metal scale \( h_{\text{max}} = 2 \) mm, at time interval from 1.36 s to 1.38 s](image)
100 km/h), when the length of scale is \( L = 100 \text{ mm} \) and the maximum height of scale is \( h_{\text{max}} = 2 \text{ mm} \).

The data of system “Vehicle – Track”, used in the calculations is published in previous author’s works [5, 27] and shown in Appendix A. Integration time step is \( \Delta t = 5 \times 10^{-6} \text{ s} \). A profile of a vehicle with scaled wheel is described using Fourier transformation, number of harmonics is \( NH = 401 \). The calculations assume, that average value of friction coefficient is \( \mu = 0.135 \), obtained from experiments.
carried out by the authors [5, 31]. Friction coefficient between the vehicle wheels in relation to the rail must not be lesser than 0.09 - 0.12, otherwise a wheel slip may occur, due to its sticking.

3.2. Results and discussion of research of the system “Vehicle – Track”, when the wheel has a metal scale

Dynamical characteristics of the wheel may alter due to damages in vehicle wheel. The developed model [5], allows a detailed analysis of kinematical and dynamical characteristics of system “Vehicle – Track”. All parameters of calculations that are shown below are averaged according to the length of the contact.

The dependence of vehicle wheel radius $R_W$ on time and height of metal scale of the wheel, when the movement speed is $V = 40$ km/h, is shown in Figure 6.

Wheel sliding on the deformed rail causes friction forces in contact zone. Dependency of normal forces $F_N$, friction forces $F_T$ around the wheel longitudinal axis $Y$ and on time, when vehicle wheel has $h_{max} = 2$ mm metal scales height and the vehicle is moving at the speed of 40 km/h, are shown in Figure 7.

Normal force $F_N$ increases in the contact zone of wheel with metal scale and rail, when the metal scale is in the contact zone.

The value of this force depends on the size of the metal scale (Fig. 7 a) and the movement speed. When the movement speed of wheel is $V = 40$ km/h and the maximum height of metal scale is $h_{max} = 2$ mm, the maximum normal force $F_N$ is equal to 300 kN.

Therefore, due to the wheel slip that appears in the contact zone (Fig. 8) and the normal force $F_N$ acting in the contact, the friction force $F_T$ must appear.
In Figure 8 it can be seen that acceleration of wheel (mass \( m_{bg2} \)) depends on the movement speed of the wheel. When the movement speed \( V \) changes (40 km/h, 60 km/h, 80 km/h, 100 km/h), the maximum acceleration of the wheel \( a_{bg2} \) alters to 350 m/s\(^2\), 510 m/s\(^2\), 750 m/s\(^2\), 1400 m/s\(^2\), and the minimum acceleration alters to 80 m/s\(^2\), 150 m/s\(^2\), 250 m/s\(^2\), 490 m/s\(^2\), respectively.

Parameters of interaction between wheel with metal scale and rail are dependent on contact zone of the rail. The closer the contact zone is to the sleeper, the bigger the rail stiffness and parameters of interaction are.

The vehicle mass \( m_{bg} \) (when \( i = 1, 2 \)) acceleration \( a_{bg} \) depend on time and contact zone, when vehicle moves at speed \( V = 40 \) km/h and wheel is with metal scale, which’s height is \( h_{max} = 2 \) mm are shown in Figure 9.

After a comparison of mass \( m_{bg} \) accelerations (when \( i = 1, 2 \)) \( a_{bg} \) in time \( t = 0.08 \) s, 1.37 s, 1.66 s, it can be seen that, contact between wheel and rail is closer to sleeper at time \( t = 1.66 \) s. Wheel mass accelerations acquire highest values at this time. Contact between wheel and rail is most distant from the sleeper at time \( t = 1.37 \) s and has the smallest mass acceleration. Dependency of vehicle mass \( m_{bg} \) accelerations \( a_{bg} \) (when \( i = 1, 2 \)) on time, when wheel vehicle has a metal scale, which height is \( h_{max} = 2 \) mm and the vehicle movement speed is \( V = 40 \) km/h.

Variations in time of the angular velocity of wheel \( \Omega \) and angular acceleration \( \Omega^\prime \) when the wheel vehicle has a metal scale, which height is \( h_{max} = 2 \) mm, at different moving speeds are shown in Figure 10 a, c and moving at speed \( V = 40 \) km/h are shown in Figure 10 b, d.

In Figure 10, it can be seen that average angular velocity \( \Omega \) and acceleration \( \Omega^\prime \) of wheel with a metal scale decreases in the contact zone and its decrease depends on the movement speed.

Loads on sleepers of forces occurring in the contact zone of wheel with metal scale and rails, on sleeper when wheel movement speed is \( V = 40 \) km/h and metal scale (\( h_{max} = 2 \) mm), are shown in Figure 11.
The results of this work confirm the statement that a wheelset wheel with a metal scale increases the load on the wheelset, the axle box bearings, the rail, and reduces the durability and safe movement of the wheelset wheel and the rail. The developed method and obtained results of the research give a deeper insight into the problems of the wheel with metal scale and rail, as well as the analysis of the forces and moments involved in the contact, depending on the parameters of the system “Vehicle – Track” and the movement speed of the vehicle.

The developed method allows determination of the forces and moments occurring in the contact between wheel with metal scale and rail. It also allows to adjust the loads on wheel bearings, the permissible wagon speed, depending on the geometry of the wheel with metal scale, and determine the heat release in the contact, the speed and size of the wheel and rail wear, the reduction in ride comfort, etc. By using this method, it is possible to create monitoring systems for damaged wheels with metal scales.

The symmetrical profile of vehicle wheel changes during exploitation, due to changes in its radius. This mathematical method allows examining the changes in the wheel profile, if a flat, metal scale, other damages, or if other bodies appear on the surface of the wheel.

The main drawback of this research is that the zone of railway wheel with metal scale and rail is examined as a contact line, but the geometrical model of the metal scale is three-dimensional. The object of the research is the system „Vehicle-Track“; so the main focus was on description of contact geometry of rail and wheel with metal scale and on determination of contact forces and moments occurring in the contact zone. Research of interaction between surfaces of wheel and rail are provided later.

The release of heat is studied in the presence of wheel and rail interactions at contact points, the weariness of contacting bodies is studied with and without lubricants (fluid), and also the effect of the rail and wheel wheels on the friction forces and the wearing of the bodies are studied in the research. The increasing speeds of the railway transport show the importance of the problem of contact between the wheel with scale and the rail.

To ensure effective maintenance loads handing over to rail, forces that influence the rail are needed to establish. This method and research results can be used to calculate the interaction between track and other transport vehicles with solid (metal) wheels that have metal scale.

4. Conclusion

The developed spatial mathematical model of wheel with a metal scale (2-13) allows to divide the vehicle wheel surface perimeter into three zones during the generation of scaled wheel profile, also it is used to generate the shape of metal scale and if the geometry of scale is known – to generate the wheel profile with a metal scale.

The mathematical model allows evaluation of the wheel rotation around its longitudinal axis Y, in order to evaluate the rotation of the vehicle wheel and determine its slip on the rails.

It is determined, that when the height of metal scale is \( h_{\text{max}} = 2 \text{ mm} \) and the length of scale is \( L = 100 \text{ mm} \), the maximum normal contact forces \( F_{N} \) is 300 kN, when the speed of moving wheelset alters \( V = 40 \text{ km/h} \), static wheel load is 100 kN and average wheel radius is \( R_w = 0.495 \text{ m} \).

When there is a metal scale on the wheel, which has a height of that \( h_{\text{max}} = 2 \text{ mm} \) and length of \( L = 100 \text{ mm} \), the slipping of wheel on rail increases, maximum slipping speed alters from 0.0437 m/s \( (V = 40 \text{ km/h}) \). Wheel slipping on rail causes friction forces in the contact, which increase the weariness of wheel and rail. The friction force in the contact \( F_f \) is 75 kN \( (h_{\text{max}} = 2 \text{ mm} \, V = 40 \text{ km/h}) \). Part of mechanical power is converted to heat per unit of time, due to that, the temperature of wheel scale gets higher and the metal scale can be heavily worn.
Rail is loaded with short-term distributed load in the contact zone of wheel with metal scale and rail. The deforming rail transfers the load onto the sleepers. It is determined, that the load on the wheel-rail contact zone loads up to four sleepers, when the distance between sleepers is 0.54 m.

The distributed load that acts on the contact zone of wheel with metal scale and rail, loads the wheel, therefore kinematical parameters of wheel, bogie and wagon alter. Because of metal scale that has formed in vehicle wheel, maximum acceleration of wheel is 336 m/s² (\( h_{\text{max}} = 2 \text{ mm}, \ V = 40 \text{ km/h} \)), maximum acceleration of bogie is 50 m/s² and maximum acceleration of wagon alter is 1.3 m/s². The wheelset with scaled wheel not only increases wheelset wheel loads, but also increases loads of bogie, wagon and the wagon’s cargo.

The method allows examination of dynamical processes occurring in the system „Vehicle – Track”, when the wheel is with a metal scale and the rail profiles are constantly changing.

**References**


Appendix A

Table A1. Data calculations of the system “Vehicle-Track”

<table>
<thead>
<tr>
<th>Definition</th>
<th>Notation</th>
<th>Definition</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masses of ballast:</td>
<td></td>
<td>Mass in contact</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mass of the sleeper</td>
<td></td>
</tr>
<tr>
<td>$m_1$ = 800 kg</td>
<td></td>
<td>Static load</td>
<td>$F_s = 100$ kN</td>
</tr>
<tr>
<td>$m_2$ = 465 kg</td>
<td></td>
<td>1/8 car body mass</td>
<td>$m_{bg4} = 8743$ kg</td>
</tr>
<tr>
<td>$m_3$ = 200 kg</td>
<td></td>
<td>1/4 bogie mass</td>
<td>$m_{bg3} = 700$ kg</td>
</tr>
<tr>
<td>Damping coefficients of ballast:</td>
<td></td>
<td>1/2 wheel set mass</td>
<td>$m_{bg2} = 640$ kg</td>
</tr>
<tr>
<td>$c_{s_{10}}$ = 90 kNs/m</td>
<td></td>
<td>Mass in contact</td>
<td>$m_{bg1} = 110$ kg</td>
</tr>
<tr>
<td>$c_{s_{12}}$ = 70 kNs/m</td>
<td></td>
<td>Stiffness coefficient of the car body</td>
<td>$k_{bg34} = 2.55$ MN/m</td>
</tr>
<tr>
<td>$c_{s_{23}}$ = 60 kNs/m</td>
<td></td>
<td>Stiffness coefficient of the bogie</td>
<td>$k_{bg23} = 6.5$ MN/m</td>
</tr>
<tr>
<td>$c_{s_{34}}$ = 50 kNs/m</td>
<td></td>
<td>Stiffness coefficient of the wheel set</td>
<td>$k_{bg12} = 5$ GN/m</td>
</tr>
<tr>
<td>$c_{sl_{11},i,j}$ = 10 kNs/m</td>
<td></td>
<td>Damping coefficient of the car body</td>
<td>$c_{bg4} = 10$ kNs/m</td>
</tr>
<tr>
<td>$c_{sl_{22},i,j}$ = 13 kNs/m</td>
<td></td>
<td>Damping coefficient of the bogie</td>
<td>$c_{bg3} = 100$ kNs/m</td>
</tr>
<tr>
<td>$c_{sl_{33},i,j}$ = 15 kNs/m</td>
<td></td>
<td>Damping coefficient of the wheel set</td>
<td>$c_{bg2} = 50$ kNs/m</td>
</tr>
<tr>
<td>$k_{s_{01}}$ = 180 MN/m</td>
<td></td>
<td>Damping coefficient of mass in contact</td>
<td>$c_{bg1} = 44.2$ kNs/m</td>
</tr>
<tr>
<td>$k_{s_{12}}$ = 170 MN/m</td>
<td></td>
<td>Wheel radius</td>
<td>$R_W = 0.495$ m</td>
</tr>
<tr>
<td>$k_{s_{23}}$ = 160 MN/m</td>
<td></td>
<td>Elastic modulus of the wheel</td>
<td>$E_W = 210$ GPa</td>
</tr>
<tr>
<td>$k_{s_{34}}$ = 150 MN/m</td>
<td></td>
<td>Exponent</td>
<td>$n = 3/2$</td>
</tr>
<tr>
<td>$k_{sl_{11},i,j}$ = 15 MN/m</td>
<td></td>
<td>Maximal penetration velocity</td>
<td>$\delta_{max} = 10$ m/s</td>
</tr>
<tr>
<td>$k_{sl_{22},i,j}$ = 16 MN/m</td>
<td></td>
<td>Mass inertia moment of wheelset</td>
<td>$I_{WY} = 65$ kNm</td>
</tr>
<tr>
<td>$k_{sl_{33},i,j}$ = 17 MN/m</td>
<td></td>
<td>Restitution coefficient</td>
<td>$e = 0.65$</td>
</tr>
<tr>
<td>Spacing between the sleepers centres</td>
<td>$L_{sl} = 0.5435$ m</td>
<td>Poisson’s coefficient of the wheel</td>
<td>$\eta_W = 0.30$</td>
</tr>
<tr>
<td>Mass of the sleeper</td>
<td>$m_{sl} = 140$ kg</td>
<td>Friction parameters:</td>
<td></td>
</tr>
<tr>
<td>Width of a railway sleeper</td>
<td>$h_{sl} = 0.15$ m</td>
<td>$\mu_X = 0.14$</td>
<td></td>
</tr>
<tr>
<td>Height of a railway sleeper</td>
<td>$h_{sl} = 0.12$ m</td>
<td>$\mu_X = 0.11$</td>
<td></td>
</tr>
<tr>
<td>Number of finite elements between two sleepers</td>
<td>10</td>
<td>$T_y = -2.50$ s / m</td>
<td></td>
</tr>
<tr>
<td>Pad damping coefficient</td>
<td>$c_{pad} = 45$ kNs/m</td>
<td>$k_s = 800$ s / m</td>
<td></td>
</tr>
<tr>
<td>Pad stiffness coefficient</td>
<td>$k_{pad} = 140$ MN/m</td>
<td>Contact length of wheel with rail</td>
<td>$L_{contact} = 100$ mm</td>
</tr>
<tr>
<td>The Second moment of the area of the rail about Z axis</td>
<td>$J_{RZ} = 5.69 \cdot 10^{-6}$ m$^4$</td>
<td>Length of metal scale</td>
<td>$2b_{by} = L_D = 0.1$m</td>
</tr>
<tr>
<td>The Second moment of the area of the rail about Y axis</td>
<td>$J_{RY} = 3.54 \cdot 10^{-5}$ m$^4$</td>
<td>Maximum heights of metal scale</td>
<td>$h_{max} = 0.001$ m</td>
</tr>
<tr>
<td>Elastic modulus of the rail</td>
<td>$E_R = 206$ GPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Poisson’s coefficient of the rail $\nu_R = 0.30$

Cross-sectional area of the rail $A_R = 82.9 \times 10^{-4}$ m$^2$

Rail density $\rho_R = 7850$ kg/m$^3$

$h_{max2} = 0.003$ m

$h_{max3} = 0.005$ m

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