In order to deal with unexpected events such as employee absenteeism and/or a demand for personnel that is higher or lower than expected, organizations need to adopt proactive and reactive scheduling strategies to protect the personnel roster and to respond to this operational variability, respectively. In this paper, we discuss a proactive approach that exploits the concept of employee substitutability to improve the flexibility of a personnel shift roster to respond to schedule disruptions. With a view to developing a DSS-driven method dedicated to competence allocation planning robust to unexpected staff absenteeism, we present the concept of the so-called robust employee competence structure. A declarative model of the concept allows to find an employee competence structure robust to a given set of disruptions while guaranteeing an admissible personnel allocation to the assumed set of tasks. Since the problem of designing such a robust structure is NP-hard, another goal of the present study is to propose a sufficient condition the fulfillment of which will guarantee the validity of calculations. Potential applications of the proposed solution are discussed using examples.

Keywords: Competence assignment, robust planning, employee competences, robust employee competence structure, employee absenteeism.

1. Introduction

The object of planning man-power needs (labor demand), as part of employment planning, is to define the competence profiles of the personnel and other individuals employed in a company. In particular, this involves defining the requirements regarding employees' knowledge, skills, abilities and behavior, determining the number of workers needed for various positions, and the scope of work that employees in each position have to perform. The quality of employment plans obtained in this process depends on the robustness of the production process to disruptions caused by unexpected events such as employee absences, machine failures, accidents at work, etc. To deal with these uncertainties, organizations must either hire a properly prepared staff of competent workers (with a certain redundancy of competences), or introduce on-line changes to the existing task schedule that will mitigate the effects of the disruptions. In this study, we consider the first of the above-mentioned measures, in which, by anticipating possible disruptions, an organization builds a staff of employees with specific competences, robust to a selected set of disruptions.

It should be noted that planning decisions regarding the allocation of production tasks (which require specific employee competences) to resources (employees with given competences) are made in dynamically changing organizational settings [7], which involve frequent changes in the scope and structure of objectives, tasks and resources. Examples of such changes include employee absenteeism (sick leaves, accidents, maternity leaves, etc.), changes in the number of jobs, staff mobility (frequent employment changes), etc. Most of them are random and cannot be anticipated well in advance. Such events are henceforth referred to as disruptions [6, 20]. If a disruption caused by an employee's absence results in a so-called competence gap, it is usually too late to bridge the gap by introducing appropriate changes (training, employment, outsourcing, etc.). While the existing literature describes
many methods for the assessment and determination of competence structures [42], there is still a scarcity of research addressing the issues of planning of structures that can guarantee the achievement of business objectives in a dynamically changing setting, i.e. structures robust to disruptions. The known methods offer no possibility of predicting disruptions and shaping competence structures robust to selected types of disruptions. A robust competence structure is understood as one that ensures the performance of tasks under specific types of disruption. In turn, allocation of competences is further understood as a process in which the competences of an employee (contractor) are adjusted to the requirements of the given task, especially as a measure to deal with a given type of disruption. It is worth noting that the problem of allocation of employees (viewed through the prism of their competences) to the individual component activities (operations, positions) of a job being performed belongs in the category of task assignment problems. Problems of this type are found in many areas of science and business, such as distribution of goods, production management, telecommunications, roster planning, etc. They all boil down to assigning a known set of tasks to a given set of agents (e.g. employees, vehicles, processors, warehouses). Different allocation problems can accentuate different objective functions that include, for example, minimizing total task completion time, minimizing costs, maximizing profit, minimizing the length of routes, etc.

Research that deals with the planning of competence structures robust to disruptions, similarly to research on robust scheduling [4], is still in its initial, conceptual phase. One of the reasons for this state of affairs is NP-hardness of this class of problems. Preliminary results of studies aimed at developing a method for synthesizing competence structures robust to a selected set of disruptions [37, 38] confirm the attractiveness of approaches based on the declarative modeling paradigm. A declarative model of a task assignment and scheduling problem allows to develop interactive methods of planning competence allocation that can be directly implemented in declarative programming environments such as ECLiPSe [46], IBM ILOG CPLEX [47] and OzMozart [48].

Section 2 presents the state of the art of task assignment planning in conditions connected with unexpected employee absenteeism. In Section 3, a reference model is proposed which can be used to search for competence structures that allow to develop competence allocation plans robust to the set of anticipated types of disruption, such as absences of individual employees. Based on this model, a procedure for the assessment and synthesis of competence structures robust to disruptions is presented in Section 4. Section 5 reports computational experiments performed in the IBM ILOG CPLEX environment, which illustrate the possibilities of applying the proposed method. The conclusions and directions for further research are discussed in Section 6.

2. Background

In the literature of the subject, competences are defined in various ways. In [9], competence is understood as the general capability based on knowledge, experience, values, and dispositions which a person has developed through engagement in educational practices. Competences are also defined as a set comprising theoretical knowledge, practical skills and behaviors that enable successful task performance [25]. In [43], competences are construed as a set of patterns of behavior required for proper performance of tasks or functions. In this present study, competences are understood (in accordance with [25]) as a set of knowledge, skills, experience, and qualifications that allow one to carry out one’s assigned tasks.

Activities aimed at identifying, acquiring, developing and retaining employee competences that enable an organization to achieve its business objectives [23] fall within the scope of human resources management, in particular competence-based management. Personnel assignment problems, which are part of human rotations management HRM, basically boil down to the allocation of tasks (division of work among employees with appropriate competences) and scheduling of work (division of tasks and defining the time windows in which they are to be performed) with a view to maximizing/minimizing the organization’s selected quality criteria, e.g., production efficiency, order completion time, robustness to disruptions, etc.

The process of planning the assignment of shop floor personnel can be divided into the following stages: strategic (long-term) planning of personnel structure aimed at supplying adequate staff capacity (e.g. ensuring that the required personnel competences are available when needed) to match the planned production capacity, tactical (medium-term) structure planning focused on the allocation of specific tasks to employees (task/job assignment), and operational (short-term) planning of allocation of current tasks to available employees (assignment scheduling) [1, 8]. The literature describes many decision support methods and models for competence assessment, identification of competence gaps, prototyping changes to the competence structure, etc. [2, 17, 22, 27, 28, 35]. Functionalities of this type are available in some commercial IT tools such as TETA HR, KARO HRMS, Comarch HRM, Asseco Softlib HR, etc. [44, 45, 49, 50].

A factor that determines the quality of generated job schedules and task assignments is their robustness to disruptions caused by [41] uncertainty of demand, uncertainty of arrival connected with unpredictable work-load (prolonged machine maintenance time, unknown number of patients in a hospital, etc.), and uncertainty of capacity related to employees’ health problems, machine failures, etc. Common approaches to improving the robustness of task assignments use either reactive scheduling (in which the existing schedule is modified to accommodate the identified disruption) or proactive scheduling (in which robust personnel rosters and schedules are constructed taking into account different types of disruption) [13, 21].

One commonly used approach to improving the robustness of task assignments is to introduce time buffers or capacity buffers. Time buffers (most often additional time windows for the completion of delayed tasks) are used in project management in situations involving uncertain job durations [18] or unexpected delays in task completion [13, 14, 39]. In turn, so-called capacity buffers (surplus resources), also referred to as reserve personnel (reserve crew, reserve resources, etc.) are often used in services, e.g. passenger transport, school services, hospital services, etc. [11] where common disruptions include events such as employee sickness [11, 30] or technical failures [10, 12, 19, 32, 33, 36, 40]. One interesting approach which assumes that a system should necessarily have surplus resources (financial, material, human), is the solution presented in [2], which allows to determine a competence structure that minimizes the risk of non-performance of tasks (brought on by a specific type of disruption).

It is easy to notice that by applying solutions that use the concept of a buffer one can enhance the robustness of the plans one is constructing (both assignment and schedule plans), but at the expense of increasing the costs of keeping redundant staff. Studies [16, 24, 26, 29] have shown that resource redundancy affects the efficiency of an organization (understood as the organization’s ability to perform tasks despite the occurrence of disruptions). However, the authors of those works have not performed a quantitative assessment of the impact of the competencies of the existing staff on the quality of the processes carried out in an organization and their robustness to disruptions.

3. Modeling of competence allocation

3.1. A motivational example

A company uses a cyclic multi-item batch flow production system to complete three production orders a day: \{J_1, J_2, J_3\} – Fig. 1.
Each order is comprised of a set of tasks (jobs) $J_i: J_1 = \{Z_1, Z_2\}$, $J_2 = \{Z_3, \ldots, Z_{10}\}$, $J_3 = \{Z_{11}, \ldots, Z_{14}\}$, executed in a given technological order, job durations $J_i$, and a job schedule determined by the critical path – Fig. 2. For example, order placement tasks $J_1$ are executed along the route marked in blue, and their duration times are: 3h for $Z_1$, 2h for $Z_2$, 5h for $Z_3$, 2h for $Z_4$, and 2h for $Z_5$. The order processing schedule assumes that the orders can be completed within 10 hours (10h for $J_1$, $J_2$ and 9h for $J_3$).

In the context of the above specification, let us consider the following question: Can the available staff of employees $\{P_1, \ldots, P_6\}$ process the given orders $\{J_1, J_2, J_3\}$ within the given deadline (order completion time)?

In the case under consideration, to assess the robustness of the earlier adopted competence structure (Table 1) one has to answer the following question: Is competence structure $G$ robust to the absence of one employee? Or, put differently, is it possible to create a job assignment such that jobs are executed in accordance with the schedule from Fig. 2 and that working time limits are obeyed for all available employees? As an illustration, a job assignment for the case of an absence of employee $P_3$ is shown in Fig 4.

The absence of this employee means that his/her duties (execution of job $Z_7$) have to be taken over by employee $P_5$ (only this employee has the competence to complete job $Z_7$). Part of the duties of $P_1$ (job $Z_{11}$) are taken over by employee $P_6$. Such an assignment of jobs does allow the staff to complete all orders but within a period exceeding 10 hours (order $J_1$ is completed after 11h) and with workload of employee $P_6$ exceeding the permissible 8h. A similar analysis of other cases of employee absence shows that the processing time limit (deadline) of 10h is exceeded in each case. If the deadline is exceeded for each case of employee absence, this means that the competence structure $G$ is not robust to this type of disruption. In other words, faced with the absence of one employee, the company cannot guaran-
A generalized version of the question formulated earlier in this section takes the following form: What should a competence structure robust to a disruption caused by the absence of one of the employees be like? Or, put differently, which employee should acquire what competences for the competence structure to become robust to the given type of disruption? It is assumed that each employee can acquire competences needed for the completion of each job \( Z_1,...,Z_{14} \) which means that so-called competence barriers are not considered [23].

The problem of synthesis of competence structures robust to a selected set of disruptions formulated in this way is an NP-hard problem. It is easy to see that the search space in the analyzed case \( (m=6 \text{ employees, } n=14 \text{ jobs and } c=18 \text{ fixed competences}) \) contains \( 2^{66} \) potential competence structure variants. The high computational complexity of the problem in hand \( f(m,n,c)=O(2^{mn-c}) \) requires the use of advanced computational techniques and methods (such as declarative programming techniques [3]) which allow to search large data structures.

In connection with the above, the synthesis problem of competence structures robust to a selected set of disruptions can be formulated as follows: given is an organization/firm/production company with human capital represented by the competence structure of the personnel (employees). Known are the organization’s business objectives and the set of tasks it carries out. The goal is to find a set of personnel development actions and decisions which should be taken to make the competence structure robust to the selected type of disruption. For a problem defined in this way, it is necessary to find a model and a time-effective method of synthesis of robust competence structures. In other words, the research problem can be solved by finding an answer to the following question: Does there exist a model and a method of constructing competence structure robust to selected disruptions caused by employee absenteeism, loss of qualifications, etc.?  

### 3.2. A reference model

Further deliberations, illustrating how competence structures robust to the absence of one employee can be synthesized, are based on the following model:

**Sets:**
- \( Z_i = \text{set of jobs indexed by } i = 1,...,n \)
- \( P_k = \text{set of employees indexed by } k = 1,...,m \)

**Parameters:**
- \( I_i = \text{duration of the } i\text{-th job } Z_i \) (in hours)
- \( s_k^i = \text{minimum number of working hours (lower working time limit) of the } k\text{-th employee } (s_k \in \mathbb{N}) \) when the \( j\text{-th employee is absent} \)
- \( z_k^j = \text{maximum number of working hours (upper working time limit) of the } k\text{-th employee } (z_k \in \mathbb{N}) \) when the \( j\text{-th employee is absent} \)
- \( w_{a,b} = \text{a parameter that specifies whether jobs } Z_a \text{ and } Z_b \text{ can be performed by the same employee (the jobs are mutually exclusive)}: \)
  - \( w_{a,b} = 1 \) when jobs \( Z_a \text{ and } Z_b \) are mutually exclusive
  - \( w_{a,b} = 0 \) in the remaining cases

**Decision variables:**
- \( G = (g_{k,j}|k = 1...m; j = 1...n) \), where \( g_{k,j} \) stands for employees’ competences to perform jobs; \( 0 \leq g_{k,j} \leq 1 \)
- \( R^* = \text{expected robustness of competence structure, } R^* \in [0,1] \)

### Table 1. Personnel competence structure \( P_1,...,P_6 \)

<table>
<thead>
<tr>
<th>( G )</th>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>( Z_3 )</th>
<th>( Z_4 )</th>
<th>( Z_5 )</th>
<th>( Z_6 )</th>
<th>( Z_7 )</th>
<th>( Z_8 )</th>
<th>( Z_9 )</th>
<th>( Z_{10} )</th>
<th>( Z_{11} )</th>
<th>( Z_{12} )</th>
<th>( Z_{13} )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
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<td>( P_2 )</td>
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<td>( P_3 )</td>
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<td>( P_4 )</td>
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<td>( P_6 )</td>
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</table>
the $k$-th employee has the competences to perform the $i$-th job.

$R$ : measure of robustness of competence structure $G$ to the absence of one employee $R \in [0,1]$. $R = 0$ – stands for lack of robustness, i.e. each absence results in unassigned jobs; $R = 1$ – stands for full robustness, i.e. regardless of which employee is absent, all jobs are assigned to available staff. For example:

- value $R = 0.25$ means that the competence structure ensures allocation of tasks in one-quarter of the possible cases of absence of one employee,
- value $R = 0.5$ means that the competence structure ensures allocation of tasks in half of the possible cases of absence of one employee,

$G^j$ : a competence structure obtained for a situation in which the $j$-th employee $G^j = (g^j_{k,i})|k = 1\ldots(m-1); i = 1\ldots n)$ is absent from his/her scheduled duty.

$X^j$ : job assignment in the situation when the $j$-th employee is absent, defined as $X^j = (x^j_{k,i})|k = 1\ldots(m-1); i = 1\ldots n)$, where $x^j_{k,i} \in \{0,1\}$:

$$ x^j_{k,i} = \begin{cases} 1 & \text{when job } Z\_i \text{ has been assigned to employee } P_k \\ 0 & \text{in the remaining cases} \end{cases} $$

$c^j$ : an auxiliary variable that specifies whether assignment $X^j$ satisfies the given constraints. The value of variable $c^j \in \{0,1\}$ depends on variables: $c^j_{i,j}$, $c^j_{k,j}$, $c^j_{k,k}$ which specify whether constraints (3), (4), (5) are satisfied.

Constraints:

1. Construction of competence structures for situations when the $j$-th employee is absent from his scheduled duty:

$$ g^j_{k,i} = \begin{cases} g_{k,i} & \text{when } k < j \\ S_{(k+1)i} & \text{when } k \geq j \end{cases} \quad (1) $$

2. Jobs can only be performed by employees who have appropriate competences:

$$ x^j_{k,i} = 0, \quad \text{ gdy } g^j_{k,i} = 0, \text{ for } k = 1\ldots(m-1); i = 1\ldots n; \quad j = 1\ldots m. \quad (2) $$

3. Job $Z\_i$ is assigned to exactly one employee:

$$ \sum_{k=1}^{m-1} x^j_{k,i} = 1 \Rightarrow c^j_{i,j} = 1, \text{ for } i = 1\ldots n; \quad j = 1\ldots m. \quad (3) $$

4. Workload of the $k$-th employee should be no less than the lower working time limit $s^j_k$:

$$ \sum_{i=1}^{n} x^j_{k,i} \cdot l_i \geq s^j_k \Rightarrow c^j_{k,k} = 1, \text{ for } k = 1\ldots(m-1); \quad j = 1\ldots m. \quad (4) $$

5. Workload of the $k$-th employee should not exceed the upper working time limit $s^j_k^*$:

$$ \sum_{i=1}^{n} x^j_{k,i} \cdot l_i \leq s^j_k, \text{ for } k = 1\ldots(m-1); \quad j = 1\ldots m. \quad (5) $$

6. Performance of mutually exclusive jobs:

$$ x^j_{k,d} + x^j_{k,h} \leq 1, \text{ when } w_{d,h} = 0, \text{ for } k = 1\ldots(m-1); i = 1\ldots n; \quad j = 1\ldots m. \quad (6) $$

7. Robustness of the competence structure:

$$ R = \frac{LP}{m}, \quad (7) $$

$$ R \geq R^*^\ast, \quad (8) $$

$$ LP = \sum_{j=1}^{m} c^j, \quad (9) $$

$$ c^j = \prod_{i=1}^{n} \prod_{k=1}^{m} c^j_{i,j} \prod_{k=1}^{m} c^j_{k,k}. \quad (10) $$

The concepts of competence structure and job assignment are represented in the model by decision variables $G$, $G^j$ and $X^j$. Job assignment $X^j$ which satisfies constraints (2)–(6) is referred to as an admissible assignment in the situation of an absence of the $j$-th employee. In this context, the questions considered previously can be narrowed down to: Does there exist a competence structure $G$ that can guarantee robustness $R \geq R^\ast$ in the event of an absence of one employee?

### 3.3. Problem formulation

An answer to the question above can be searched for using brute-force methods (e.g. the branch and bound method). The literature provides advanced declarative programming techniques which allow to reduce the calculation time compared to that required by exact methods. One such technique is constraint programming/constraint logic programming (CP/CLP) [31]. It is a set of techniques used to solve combinatorial problems, such as the assignment problem considered in the present work, and many others, e.g. the problems of vehicle routing, batching, warehousing, and scheduling. The essence of constraint programming is to solve problems formulated as constraint satisfaction problems (CSP) [5, 34].

The search for robust competence structures can be modeled using the CSP formalism, which allows to implement the proposed model directly in commercially available constraint programming environments, such as IBM ILOG CPLEX, Gurobi, ECLPS®, Oz Mozart, and others, which are a subclass of declarative programming environments. In contrast to procedural (imperative) modeling techniques, approaches based on declarative modeling allow to formulate models taking into account the specific needs and requirements of a given version of a problem. In reference to the CSP formulated in this work, any change in the structure of orders, organization and staff will only require a correction/
change in the set of constraints without affecting the implemented constraint propagation and variable distribution mechanisms.

In other words, what is sought is a solution that guarantees a given level \( R \) of robustness. In general, a CSP defined in this way can be treated as an optimization problem. In such cases, the search focuses on determining the minimum competence structure \( G_{OPT} \) (e.g. one that meets the criterion of minimum number of competence changes).

The model of the synthesis problem \( CS \ (11) \) presented in Fig. 5 illustrates the procedure of finding a competence structure \( G \) with a given level \( (R \geq R^*) \) of robustness. A specific level of robustness can be obtained due to the introduction of decision variables \( G^1, \ldots, G^m \) which represent the substructures of structure \( G \) for the particular cases of one-employee absence. Full robustness \( (R = 1) \) is reached when there exists structure \( G \), for which each substructure \( G^j \) guarantees a job assignment \( X^j \) that meets constraints (2)–(6) \((c^j = 1)\).

In other words, the solution to problem \( CS \ (11) \) is a competence structure \( G \) that guarantees timely completion of jobs for all cases of one-employee absences.

### 4. Computer-aided planning of competence structures robust to disruptions caused by employee absenteeism

\( CS \ (11) \), discussed in the previous section, had been developed for the needs of synthesis of (minimum) competence structures robust to one-employee absences. In the general case, the number of absent employees may be larger than one and other types of disruptions may be considered. In this approach, the robustness of a competence structure should be treated as a parameter dependent on the type of disruption, which can be assessed by various different measures. For example, the robustness of a competence structure can be expressed as:

- a measure of robustness to the absences of single employees (7):
  - number of absences for which there exists a job assignment that guarantees timely completion of orders relative to all possible cases of absenteeism,
- a measure of robustness to an employee’s loss of qualifications (competences):
  - number of cases of lost qualifications for which there exists a job assignment that guarantees timely completion of orders relative to all possible cases of loss of qualifications,
- a measure of robustness to changes in the job structure (changes in number of jobs, technological routes, etc.): number of changes in the job structure for which there exists a job assignment that guarantees timely completion of orders relative to all possible cases of job structure change,
- and so on.

Fig. 6 illustrates the method of evaluation and synthesis of competence structures robust to disruptions (in relation to the measures presented above). The method assumes that an organization has access to information about its employees (competence structure) and jobs to be performed, and that the constraints describing the relations between these categories of information are known.

Decision makers are aware of the possibility of occurrence of a specific set of disruptions. The proposed decision support tool primarily allows to answer the question regarding the analysis (evaluation) of the robustness of the competence structure to a selected set of disruptions. When the answer is positive (the competence structure is flexible enough to allow the available personnel to complete all jobs), the company can proceed to execute its orders without fear of disruption. When the answer is negative (the competence structure is not flexible enough to allow the available personnel to complete all jobs), the decision-maker can use the method proposed in this paper to look for an answer to the question regarding the synthesis of the competence structure (solution to \( CS \ (11) \)), i.e. to search available data (in this case competence structures) to find a job assignment that

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**Fig. 5. Model of CSP (11), a problem of synthesis of robust competence structures**

The structure of the proposed model that includes a set of decision variables and a set of constraints that relate those variables to one another in a natural way allows to formulate the problem in hand as a CSP and implement it in a constraint programming environment:

\[
CS \ (11) = \left( \mathcal{V}, \mathcal{D}, \mathcal{C} \right)
\]

where:

- \( \mathcal{V} = \{ G, G^1, \ldots, G^m, X^1, \ldots, X^m, R \} \) - a set of decision variables which includes: competence structure \( G \), competence substructures \( G^j \) for cases when the \( j \)-th employee is absent, corresponding job assignments \( X^j \), and robustness \( R \).
- \( \mathcal{D} \) – a finite set of decision variable domains \( \{ G, G^1, \ldots, G^m, X^1, \ldots, X^m, R \} \).
- \( \mathcal{C} \) – a set of constraints specifying the relationships between the competence structure and its robustness (constraints 1–10).

To solve \( PS \ (11) \), it is enough to find such values of decision variables \( G \) (personnel competence structure), \( X^j \) (job assignment) and \( R \) (robustness to absenteeism of one employee), determined by domains \( \mathcal{D} \), for which all the constraints of set \( \mathcal{C} \) are satisfied.
meets specific expectations (e.g. robustness of the competence structure to the selected set of disruptions). When the answer is positive, the decision-maker obtains a set of alternative competence structures that guarantee the organization’s robustness to a selected set of disruptions. On the basis of this set of admissible alternative structures, he/she can make decisions regarding issues such as further development of the staff. It is the decision maker’s responsibility to choose the most favorable variant (one that meets a criterion of his/her choice). A negative answer informs the decision-maker that it is impossible in the given organization to build a competence structure robust to the selected set of disruptions. Given this information, he/she may consider changing (increasing) working time limits, employing new staff, outsourcing to temporary workers, etc.

The proposed method, which uses a model of CS (11) and the mechanisms of Matlab and Gurobi programming environments, was verified in a series of computational experiments described in the section below.

5. Computational experiments

Given is the production system from Fig. 1, in which orders are executed by a staff of employees \( \{P_1, \ldots, P_n\} \). Orders are processed according to the schedule from Fig. 2. In the schedule, operations executed in the same time window are mutually exclusive. Information about which operations exclude one another in time (values of variable \( w_{a,b} \)) is given in Table 2. For example, because jobs \( Z_7 \) and \( Z_{12} \) (which require competence \( \bullet \)) are scheduled in the same time window (hours 3–5), they must be performed by different employees.

As shown in Fig. 4, competence structure \( G \) (Table 1) is not robust to an absence of a single employee. The method proposed in the present paper (Figure 6) can be used to synthesize a competence structure robust to a given type of disruption, i.e. to answer the following question: Does there exist a competence structure \( G \) that can guarantee full robustness (\( R = 1 \)) in the situation when one employee is absent from duty?

To answer this question one needs to solve CS (11), which contains competence structure \( G \) from Table 1 and parameters of the model from Fig. 1. The problem was implemented in the GUROBI environment (Intel i7-4770, 8GB RAM). The first admissible solution was obtained in less than 1s. The space of admissible solutions was searched for solutions that met the criterion of the minimum number of changes to the competence structure:

\[
L(G) = \sum_{k=1}^{n} \sum_{i=1}^{m} g_{k,i}.
\]

Table 2. Values of variable \( w_{a,b} \) determined by the schedule from Fig. 2

<table>
<thead>
<tr>
<th>( w_{a,b} )</th>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( z_3 )</th>
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The minimum structure $G_{opt}$, for which $R = 1$ is presented in a graphic form in Table 3. The value of $L(G_{opt})$ is 29, which means that employees must improve their qualifications by acquiring a total of 8 new competences (Table 3): employee $P_1$ should acquire competence $\square$, $P_2$ competence $\bigcirc$, $P_3$ competences $\square$ and $P_4$ competence $\bigcirc$.

Acquisition of these competences will guarantee full $(R = 1)$ robustness of the competence structure to the absence of any given staff member. Fig. 7 shows job assignments that guarantee timely completion (10h) of orders regardless of which employee is absent.

As an alternative to the training of available staff, a robust competence structure can be obtained by employing additional staff. A decision-maker who chooses this option has to ask him/herself the question of how many employees with what competences should be hired? In our case, synthesis of a competence structure (i.e. finding a solution to $CS$ (11)), performed under the assumption that one additional employee $P_5$ was available, yielded the solution presented in Table 4 (calculation time <1s). As it can be seen, full robustness $(R = 1)$ of the structure can be achieved only if the new employee $P_5$ has the ability to perform all jobs (7 competences).

The method was verified in a series of experiments involving different numbers of employees (5–15) and different numbers of tasks (16–32). The calculations were carried out to determine the time needed to synthesize a competence structure robust $(R = 1)$ to the absence of (any) one of the employees. The results are shown in Table 5. It is easy to notice that in cases in which the size of the structure does not exceed 10 employees and 32 jobs, a robust structure can be found in less than 1,000 seconds. Our future work will focus on implementing the proposed model in the environments of other optimization packages: IBM ILOG CPLEX, OzMozart, etc. The computational module developed in this study can be used as a software overlay for commercially available decision support systems used in human resources management.

6. Conclusions

The proposed method of synthesizing competence structures robust to selected sets of disruptions allows to plan the allocation of production jobs (that require specific employee competences) to resources (employees with the given competences) in situations in which the disruptions are caused by employee absenteeism. According to this method, to build a competence structure robust to unforeseen disruptions, it is necessary to determine what additional (redundant) competences contractors need to have to compensate for competences lost as a result of employee absenteeism. The proposed measure of robustness of competence structures allows interactive, on-line synthesis of structures with a given level of robustness, in particular robustness to absences of single employees. Constraint programming techniques allow to extend and adapt the reference model developed in the present study to other areas of decision support which require the use of

Table 3. Minimum competence structure robust to the absence of one employee

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<tr>
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* colored fields represent newly acquired competences (relative to the structure from Table 1)
managerial decision-making support tools, for instance designing the competence structure of academic staff, recruiting panels of experts for reviewing project applications, proposing variants of the composition of medical teams, etc.

Being NP-hard, the problems under consideration must be solved using heuristic methods, e.g. ones that implement the declarative programming paradigm. The constraint satisfaction model $CS$ (11) and the method of synthesizing competence structures robust to one-employee absences based on this model use a redundant number of decision variables representing competence-depleted structures (structures that arise as a consequence of occurrence of various disruption scenarios). The redundancy of the set of decision variables, on the one hand, increases the computational complexity of the problem, but, on the other, allows to find solutions with a given level of robustness (especially in the case of fully robust competence structures). The experiments have shown that the method can be effectively used to solve small-scale problems in organizational units of up to 10 employees and 32 tasks (“effectively” means here that a robust competence structure can be synthesized on-line in under 1,000 s). It may be possible to increase the scale of the problems solved by using hybrid methods [42] dedicated to models that use sparse data structures (in the model under consideration, the competence structure contains mostly “0” values). Implementation of this type of techniques will be one of the directions of our future research.

The results of these present studies will also be verified using selected extensions of the constraint satisfaction problem that take into account other measures of robustness to disruptions, such as the measure of robustness to loss of employee qualifications (competences), changes to the order structure, simultaneous (and/or sequential) absence of several employees, etc. Depending on the results we obtain, our further research will focus on the construction of an interactive system for planning competence structures robust to disruptions to be used in human resources management. Implementation of this type of functionalities in ERP systems will enable early detection of needs and quick prototyping of alternative decisions in the area of management of staff competences. Such a solution will allow managers to make personnel decisions on-line in response to employee absenteeism and/or staffing fluctuations, legislative changes, changes in the scope of production orders, etc. It will also enable the development of other derivative methods of human resources management, such as methods of supporting the organization and planning of teamwork in situations in which the available workers have to step in for the absent colleagues.

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* computer parameters: Intel i7-4770, 8GB RAM
References
38. Szwarc E, Bocewicz G, Bach-Dąbrowska I. Planning a teacher staff competence structure robust to unexpected personnel absence. Manufacturing Modelling, Management and Control (MIM); Berlin 2019 (in print).
44. http://comarch.pl
45. http://dmz.pl
46. http://eclipse.org
47. http://ibm.com

Eryk SZWARC
Grzegorz BOCIEWICZ
Zbigniew BANASZAK
Faculty of Electronics and Computer Science
Koszalin University of Technology
ul. Śniadeckich 2, 75-453 Koszalin, Poland

Jarosław WIKAREK
Institute of Management and Control Systems
Kielce University of Technology
Al. 1000-lecia Państwa Polskiego, 25-314 Kielce, Poland

e-mail: eryk.swarc@tu.koszalin.pl, bocewicz@ie.tu.koszalin.pl,
zbigniew.banaszak@tu.koszalin.pl, j.wikarek@tu.kielce.pl