1. Introduction

A. Background

Today’s systems are becoming more complex and more sophisticated, and the problems of system reliability are drawing an increasing attention. Common cause failures are critical risk contributors in complex technological systems as they challenge multiple redundant systems simultaneously. Common cause failures can contribute significantly to the overall system unreliability [9]. Therefore, it is important to incorporate common cause failure into the system reliability analysis. Alizadeh et al. [1] introduced the impact of common cause failure on the system reliability using Markov analysis technique. Zuo et al. [23] analyzed the system failure suffering common cause failure. Fan et al. [2] developed a new model for common cause failures considering components degradation based on mathematical framework of Stochastic Hybrid Systems. Levitin [5] adapted the universal generating function method of multistate system reliability analysis to incorporate common-cause failures. Pourali [7] presented presented the importance of considering common cause failure in reliability, availability, and maintainability analysis for industrial and commercial mission-critical facilities and high-reliability organizations. Vaurio [11] incorporated common-cause failures into system analysis by an explicit method and discussed the possible limitations and extensions. Wang et al. [13] incorporated effects of probabilistic common cause failures into system reliability analysis. Wang et al. [14] proposed an explicit method and an implicit method to analyze the reliability of systems. Xiao and Gao [15] proposed efficient simulation methods to assess the system reliability with input uncertainty. Xiao et al. [16] presented a data simulation approach to estimating the system failure probability in the presence of stochastic constraints. Yuan [17] extended the pivotal decomposition method for system availability and failure frequency from the case where components are statistically independent to that where components are also subject to common-cause failures.

Load-sharing is always an essential nature in parallel system. Huang et al. [3] presented a general closed-form expression for lifetime reliability of load-sharing redundant systems. Liu [6] developed a model to calculate the reliability of a load-sharing system which is composed of non-identical components each having an arbitrary failure time distribution. Paula et al. [8] analyzed the optimization in redundant system considering load sharing. Jiang et al. [4] formulated two load optimization models to identify the optimal loading strategy. Sutar et al. [10] modeled the load sharing phenomenon in a k-out-of-
m system through the accelerated failure time model. Wang et al. [12] presented three policies for load assignment among unequal strength components and compared three of these policies. Ye et al. [18] developed a model for a load sharing system where an operator dispatches work load to components in a manner that manages their degradation. He assumed degradation is the dominant failure type, and that the system will not be subject to sudden failure due to a shock. Yang et al. [19] proposed a novel approach for assessing a systems’ reliability with dependency structures, load sharing, and damage accumulation and reversal. Zhao et al. [20] presented a reliability modeling and analysis framework for load-sharing systems with identical components subject to continuous degradation. Zhang et al. [21] proposed a new reliability analysis method for the load-sharing k-out-of-n: F system based on the load-strength model. Zhang et al. [22] presented a two-component load-sharing system. And the failure rates of the two components are time dependent and load dependent.

### B. Motivation

Undoubtedly, above researches has contributed to the development of reliability of parallel system. Some of them propose excellent methods to calculate the mean time to failure of system, rest of them help to investigate the reliability of system under common cause failure or load-sharing failure respectively. However, lots of researches often ignore the joint of common cause and load-sharing failure in terms of the failure analysis of the parallel system with stress strength. Some parallel systems often happen simultaneously common cause and load-sharing failures. The results tend to be over positive than factual information. In this paper, for parallel systems, common cause failure model with stress strength and joint failure model of load-sharing and common cause failures are established respectively. Based on these models, the results are more approaching to the realistic situation considering the mean time to failure of parallel systems under common cause and load-sharing failures.

The rest of this paper is organized as follows. Section 2 analyzes the reliability model with stress strength under common cause failure. The reliability model under common cause and load-sharing failure is presented in section 3. Section 4 utilizes a numerical example to testify the validity of the proposed model. Finally, the conclusions of this paper are given in Section 5.

### 2. Reliability analysis with stress strength under common cause failure

Generally, \(x\) and \(y\) denote stress and strength respectively, \(f_s(x)\) and \(f_r(y)\) denote stress probability density function and strength probability density function respectively. We suppose parallel system is composed of \(n\) components. The probability of all components failure in the system is system conditional failure probability, so statistical average of system conditional failure probability under common cause failure is \(p^n_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^n dx\) where \(x \in (0, +\infty)\). We utilize the model to calculate conditional failure probability of two-dependent-component and three-dependent-component parallel system respectively, and compare ultimate consequence.

According to above model, conditional failure probability of two-dependent-component parallel system is \(p^2_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^2 dx\), and conditional failure probability of three-dependent-component parallel system is \(p^3_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^3 dx\). Now we need to compare them. Because \(\int_0^\infty f_r(y)dy = 1\), we could get \(0 \leq \int_0^y f_r(y)dy \leq 1\). Therefore, \(\int_0^y f_r(y)dy = 1\) when \(y\) is not more than \(x\) forever. However, \(y\) could be more than \(x\). Thus, \(0 \leq \int_0^y f_r(y)dy < 1\). Based on relative mathematical knowledge, we can get \(p^3_s < p^2_s\). Obviously, conditional failure probability of three-dependent-component parallel system under common cause failure is less than two-dependent-component parallel system, which shows that we could decrease system conditional failure probability by increasing a redundant component.

For a parallel system under common cause failure with \(n\) components, if statistical average of system conditional failure probability satisfies \(p^n_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^n dx\), where \(x\) denotes stress, three properties could be deduced.

**Property 1:** conditional failure probability of three-dependent-component parallel system under common cause failure is less than two-dependent-component parallel system, where \(x \in (0, +\infty)\).

**Proof:**

\[
p^3_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^3 dx\]

Because \(0 < f_s(x) \left[ \int_0^y f_r(y)dy \right]^3 < f_s(x) \left[ \int_0^y f_r(y)dy \right]^2\), based on relative mathematical knowledge, we could deduce \(\int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^3 dx < \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^2 dx\). That is \(p^3_s < p^2_s\).

**Property 2:** conditional failure probability of \(k\)-dependent-component parallel system under common cause failure is less than \((k-1)\)-dependent-component parallel system, where \(x \in (0, +\infty)\).

**Proof:**

\[
p^{k-1}_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^{k-1} dx\]

\[
p^k_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^k dx\] . According to property 1, we can deduce \(p^k_s < p^{k-1}_s\).

**Property 3:** system conditional failure probability approaches 0 when \(n\) approaches infinity, that is to say, \(\lim_{n \to +\infty} p^n_s = 0\).

**Proof:**

\[
p^n_s = \int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^n dx\]

because \(\int_0^\infty f_r(y)dy = 1\), \(f_s(x) \left[ \int_0^y f_r(y)dy \right]^n\) approaches infinitesimal when \(n\) approaches infinity. Thus, \(\int_0^\infty f_s(x) \left[ \int_0^y f_r(y)dy \right]^n dx \to 0\), that is to say, \(\lim_{n \to +\infty} p^n_s = 0\).

Besides, \(F_s(x)\) and \(F_r(y)\) denote stress distribution function and strength distribution function. We suppose parallel system is composed of \(n\) components. Reliability of parallel system under common cause failure is

\[
R_s = \int_{-\infty}^{+\infty} \left[1 - \left[ \int_{-\infty}^{x} f_r(y)dy \right]^n\right] f_s(x)dx = \int_{-\infty}^{+\infty} \left[1 - \left[ F_r(x) \right]^n\right] f_s(x)dx \]

[13], where \(x \in (0, +\infty)\).

According to above model, reliability of two-dependent-component parallel system is \(R_2(x) = \int_{-\infty}^{x} \left[1 - \left[ F_r(x) \right]^2\right] f_s(x)dx\), and reliability of three-dependent-component parallel system is

\[
R_3(x) = \int_{-\infty}^{x} \left[1 - \left[ F_r(x) \right]^3\right] f_s(x)dx\]. Now we need to compare them.
0 ≤ F_r(x) ≤ 1, F_r(x) = 1 when y is not more than x forever. However, y could be more than x. Thus, 0 ≤ F_r(x) < 1, according to above analysis, we can deduce \(\frac{1 - [F_r(x)]^n}{1 - [F_r(x)]} f_r(x) > \frac{1 - [F_r(x)]^n}{1 - [F_r(x)]} f_r(x)\). Based on relative mathematical knowledge, we could deduce \(R_n(2) < R_n(3)\). That is to say, reliability of three-dependent-component parallel system is more than two-dependent-component parallel system. Thus, we could deduce that increasing a redundant component would enhance system reliability.

For a parallel system under common cause failure with n components, if system reliability satisfies:

\[ R_n = \int_{x}^{\infty} \left[ 1 - \left[ \int_{y}^{\infty} f_r(y)dy \right]^n \right] f_r(x)dx = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^n \right] \right] f_r(x)dx, \]

where \(x\) and \(y\) denote stress and strength respectively, three properties could be deduced.

**Property 4:** Reliability of three-dependent-component parallel system under common cause failure is more than two-dependent-component parallel system, where \(x \in (0, +\infty)\).

\[ R_n(2) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^2 \right] \right] f_r(x)dx, \]

\[ R_n(3) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^3 \right] \right] f_r(x)dx. \]

Because \(F_r(x) < 1\), we could get \([F_r(x)^n] < [F_r(x)^2]\), and we could deduce \(1 - [F_r(x)^2] < 1 - [F_r(x)^3]\). Thus, we could take a further step to deduce \(1 - [F_r(x)^2] f_r(x) < 1 - [F_r(x)^3] f_r(x)\). Based on relative mathematical knowledge, we get \(R_n(2) < R_n(3)\).

**Property 5:** Reliability of k-dependent-component parallel system under common cause failure is more than (k-1)-dependent-component parallel system, where \(x \in (0, +\infty)\).

\[ R_n(k - 1) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^{k-1} \right] \right] f_r(x)dx, \]

\[ R_n(k) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^k \right] \right] f_r(x)dx. \]

According to derivation way of property 4, we could deduce \(R_n(k - 1) < R_n(k)\).

**Property 6:** Parallel system reliability approaches 1 when \(n\) approaches infinity, that is to say, \(\lim_{n \to \infty} R_n(n) = 1\).

\[ R_n(n) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^n \right] \right] f_r(x)dx, \]

\(\left[ F_r(x)^n \right] \leq 1\) and \(R_n(n) = \int_{x}^{\infty} \left[ 1 - \left[ F_r(x)^n \right] \right] f_r(x)dx \neq 1\) when \(n\) approaches infinity.

### 3. Reliability analysis under load-sharing and common cause failures

We assume a system is composed of three same components. All components share whole system load and failure rate of each component is \(R_3\), when system works normally. Failure rate will become \(R_2\) with one component failed. When two components fail, failure rate will become \(R_1\). When there is one component working in the system, the common cause failure rate is \(R_3\), when there are two components working in the system, the common cause failure rate is \(R_2\), and when all of the three components are working normally, the common cause failure rate is \(R_1\). We have merely one maintenance device which repairs randomly one failed component once, and other failed components must wait until last one has worked normally. With one component failed, \(\mu_1\) denote mean time to maintenance and maintenance rate respectively. With two components failed, \(\mu_2\) denote mean time to maintenance and maintenance rate respectively. With three components failed, \(\mu_3\) denote mean time to maintenance and maintenance rate respectively. According to the above assumptions, we can describe the state transition figure of three-dependent-component parallel system under common cause and load-sharing failure as Fig. 1.

**Fig. 1. state transition under common cause and load-sharing failures**

As is shown in Fig. 1, based on state transition figure, we establish transition intensity matrix for calculation of system mean time to failure, and \(A\) denotes transition intensity matrix:

\[ A = \begin{pmatrix} -\mu_1 & \mu_1 & 0 & 0 \\ \lambda_2 + \lambda_3 & -\lambda_2 + \lambda_3 + \mu_2 & \mu_2 & 0 \\ \lambda_2 & 2\lambda_2 & -\lambda_2 + 2\lambda_2 + \mu_3 & \mu_3 \\ \lambda_3 & 2\lambda_3 & 3\lambda_3 & -(6\lambda_3 + \lambda_2 + \lambda_3) \end{pmatrix} \]

The state 0 is absorbing state, therefore, we need to omit all elements in the system that is related to state 0. And \(B\) denotes a transition intensity matrix:

\[ B = \begin{pmatrix} -(\lambda_1 + \lambda_2 + \mu_2) & \mu_2 & 0 \\ 2\lambda_2 & -(\lambda_2 + 2\lambda_2 + \mu_3) & \mu_3 \\ 3\lambda_3 & 3\lambda_3 & -(6\lambda_3 + \lambda_2 + \lambda_3) \end{pmatrix} \]

We have \(C = [q_0(0) q_2(0) q_3(0)], D = [0 \ 0 \ -1]\), where state transition equation is \(CB = D\). Therefore, we could get the following equation:

\[ \begin{pmatrix} \lambda_1 + \lambda_2 + \mu_2 \\ 2\lambda_2 \\ 3\lambda_3 \end{pmatrix} = [0 \ 0 \ -1] \]

Considering the complexity of equation and the accuracy of calculation, we can get \(q_0(0), q_2(0), q_3(0)\) by using the math software. Then the mean time to failure of three-dependent-component parallel system is \(MTTF_3 = \{q_0(0) + q_2(0) + q_3(0)\}\). But the solution is too complex, we cannot use it to get some useful message, so we should make some assumptions to simplify the solution.

**Assumptions 1:** No matter how many components are working in the system, \(R_3\) denotes common cause failure rate.
Assumptions 2: The failure rate decrease linearly with the decline of the quantity of the components which are working in the system, this is, if \( \lambda_1 = \lambda_2 \), then we will get \( \lambda_2 = 2 \lambda_1 \) and \( \lambda_3 = 3 \lambda_2 \).

However, the difference between the mean time to failure of four-dependent-component parallel system and three-dependent-component parallel system is more complex, even if it has been simplified, so it is hardly to find the same regular. Through the assumption we have made, we also can simplify the result of \( q_1(0) \), \( q_2(0) \), \( q_3(0) \), they are:

\[
q_1(0) = \frac{\lambda_1^2 - 7 \lambda_1 \lambda_2 + 3 \mu \lambda_1 + 24 \mu^2 + 9 \mu \lambda_2}{2 \lambda_1^3 + 20 \lambda_1 \lambda_2^2 + 8 \mu \lambda_1^2 + 25 \mu \lambda_1 \lambda_2 + 72 \lambda_1 \mu + 3 \lambda_2^2 \mu - 9 \lambda_2 \mu^2 + 9 \mu^2}
\]

\[
q_2(0) = \frac{3 \lambda_1 \lambda_2 + 2 \mu \lambda_1 + 9 \mu \lambda_2 + 9 \lambda_2^2}{2 \lambda_1^3 + 20 \lambda_1 \lambda_2^2 + 8 \mu \lambda_1^2 + 25 \mu \lambda_1 \lambda_2 + 72 \lambda_1 \mu + 3 \lambda_2^2 \mu - 9 \lambda_2 \mu^2 + 9 \mu^2}
\]

\[
q_3(0) = \frac{\lambda_2^3 + 7 \lambda_1 \lambda_2 + 3 \mu \lambda_2 + 24 \mu^2 + 9 \mu \lambda_1}{2 \lambda_1^3 + 20 \lambda_1 \lambda_2^2 + 8 \mu \lambda_1^2 + 25 \mu \lambda_1 \lambda_2 + 72 \lambda_1 \mu + 3 \lambda_2^2 \mu - 9 \lambda_2 \mu^2 + 9 \mu^2}
\]

Proof: Under the premise that all of the three rates are positive, we can get \( Y_{1(\lambda_1)} = Y_{2(\lambda_2)} = Y_{3(\lambda_2)} \), so we can say the failure rate \( \lambda_1 \) has the most influence to the system when there is only a component working.

Property 8: The common cause failure rate \( \lambda_1 \) has the most influence on the system when there are three components working in, has the second most influence on it when there are two components working in, and has the least influence on the system when there is only a component working.

Property 9: The maintenance rate \( \mu \) influences the system when there is only a component working in more than when there are two components working in.

4. Numerical example

In this section, we will have an analysis about a parallel system of three components under common cause and load-sharing failure. This section mainly studies the effect of single variance on the reliability of parallel system. We assume the reliability parameters are \( \lambda_1 = 3.12 \times 10^{-3} \), \( \lambda_2 = 2.9 \times 10^{-3} \), \( \lambda_3 = 3.12 \times 10^{-3} \), \( \lambda_4 = 3.12 \times 10^{-2} \), \( \mu_{MTTF_s} = 16 h \), \( \mu = 4.12 \times 10^{-2} \). We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

\[
MTTF_s = \int_0^\infty \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} \, dt = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} \left[ t \right]_0^\infty = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}
\]

\[
MTTF_s = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}
\]

\[
MTTF_s = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}
\]

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MTTF_s = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}
\]

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MTTF_s = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}
\]
Fig. 2 describes the effect $a_\lambda$ variation on $s_{MTTF_2}$ and $s_{MTTF_3}$. Firstly, $MTTF_2 = 1935.4$ h, $a_\lambda$ have no effect on mean time to failure of two-dependent-component parallel system. Secondly, mean time to failure of three-dependent-component parallel system is negatively correlated with $a_\lambda$. Thirdly, $3s_{MTTF_3} > s_{MTTF_2} \times 1.233 \times 10^{-3}$, so the three-dependent-component parallel system is prior to two-dependent-component parallel system when $a_\lambda \in [0,1.233 \times 10^{-3}]$.

(2) The effect of $m_\lambda$ variation on $s_{MTTF}$ $a_\lambda$ is defined as independent variable, and its range of values is $[0,5 \times 10^{-3}]$. Dependent variable is $s_{MTTF}$. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

$MTTF_2 = \frac{1}{\lambda_i + \lambda_f + 2\lambda_m + \mu_i - \frac{2\lambda_m + 8.830 \times 10^{-2}}{1.060 \times 10^{-2}\lambda_m + 2.604 \times 10^{-2}}}$

$MTTF_3 = \frac{1}{\lambda_i + \lambda_f + 2\lambda_m + \mu_i - \frac{2\lambda_m + 8.830 \times 10^{-2}}{1.060 \times 10^{-2}\lambda_m + 2.604 \times 10^{-2}}}$

Now we describe the effect $m_\lambda$ variation on $s_{MTTF}$ in Fig. 3. Firstly, mean time to failure of three-dependent-component and two-dependent-component parallel system is negatively correlated with $m_\lambda$. Secondly, $3s_{MTTF_3} = s_{MTTF_2} \times 1.832 \times 10^{-3}$, so the three-dependent-component parallel system is prior to two-dependent-component parallel system when $m_\lambda \in [1.832 \times 10^{-3}, 5 \times 10^{-3}]$.

(3) The effect of $\lambda_f$ variation on $s_{MTTF}$ $a_\lambda$ is defined as independent variable, and its range of values is $[2 \times 10^{-3}, 1 \times 10^{-2}]$. 

Fig. 2. Effect of $a_\lambda$ variation on $MTTF_i$

Fig. 3. Effect of $m_\lambda$ variation on $MTTF_i$
As is shown in Fig. 4, the three-dependent-component parallel system is prior to two-dependent-component parallel system when 
\[ \lambda_f \in \left[ 2 \times 10^{-3}, 1 \times 10^{-2} \right] \).

(4)The effect of \( \lambda_c \) variation on \( MTTF_3 \)

\[ \lambda_c \text{ is defined as independent variable, and its range of values is } \left[ 0,1 \times 10^{-3} \right]. \text{ Dependent variable is } MTTF_3. \text{ We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure:} \]

\[
MTTF_3(2)=\rho_1(0)+\rho_2(0)=\frac{\lambda_f+2\lambda_w+\mu_1}{(\lambda_f+2\lambda_w)(\lambda_f+\lambda_f+\mu_1)}-2\lambda_w\mu_1+\frac{\lambda_f+8.73 \times 10^{-2}}{4.30 \times 10^{-3}\lambda_f+2.619 \times 10^{-5}}
\]
\[
MTTF_3(3)=\rho_1(0)+\rho_2(0)+\rho_3(0)=\frac{6\lambda_d\lambda_f+12\lambda_d\lambda_f+2\lambda^2+2\lambda_f+2\lambda_f+2\lambda_f+2\lambda_f+2\lambda_f+\lambda_f\mu_1+\mu_1\mu_k}{(\lambda_f+2\lambda_w)+2\mu_1}[\lambda_f+2\mu_1)]\left[\lambda_f+2\mu_1\right)-3\lambda_f(\lambda_f+\lambda_f+2\mu_1)+\mu_1(12\lambda_f(\lambda_f+4\mu_1)+\lambda_f\mu_1)
\]
\[
MTTF_3(2)=\rho_1(0)+\rho_2(0)+\rho_3(0)=\frac{1.26 \times 10^{11}+1.08 \times 10^{-1}}{5.307 \times 10^{-3}\lambda_f+3.273 \times 10^{-7}}
\]

Now we describe the figure of the effect \( \lambda_c \) variation on \( MTTF_3(2) \) and \( MTTF_3(3) \) in Fig. 5.

Firstly, mean time to failure of three-dependent-component and two-dependent-component parallel system is negatively correlated with \( \lambda_m \). Secondly, \[ MTTF_3(3)>MTTF_3(2), 0 \leq \lambda_c < 5 \times 10^{-4} \]
\[ MTTF_3(3)=MTTF_3(2), 5 \times 10^{-4} \leq \lambda_c \leq 1 \times 10^{-3} \]

so the three-dependent-component parallel system is prior to two-dependent-component parallel system when \( \lambda_c \in \left[ 0,5 \times 10^{-4} \right] \).
4.2. The effect of each maintenance rate on mean time to failure of parallel system

(1) The effect of $\mu_a$ variation on $\text{MTTF}_a$

$\mu_a$ is defined as independent variable, and its range of values is $[0, 8.3 \times 10^{-2}]$. Dependent variable is $\text{MTTF}_a$. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

$$\text{MTTF}_a(2) = r_1(0) + p_1(0) = \frac{\lambda_a + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_a + 2\lambda_m)(\lambda_a + \lambda_f + \mu_a) - 2\lambda_m\mu_k} = 1935.4\text{ h},$$

$$\text{MTTF}_a(3) = q_1(0) + q_2(0) + q_3(0) = \frac{9.196\mu_a + 6.773 \times 10^{-2}}{4.589 \times 10^{-3}\mu_a + 3.491 \times 10^{-3}}.$$

In Fig. 6, the mean time to failure of three-dependent-component parallel system is weakly positive correlation with $\mu_a$. When $\mu_a = 0$, $\text{MTTF}_a(3) = \text{MTTF}_a(2)$. The three-dependent-component parallel system is prior to two-dependent-component parallel system when $\mu_a \in [0, 8.3 \times 10^{-2}]$.

(2) The effect of $\mu_k$ variation on $\text{MTTF}_k$

$\mu_k$ is defined as independent variable, and its range of values is $[5.2 \times 10^{-3}, 0.125]$. Dependent variable is $\text{MTTF}_k$.

$$\text{MTTF}_k(2) = r_1(0) + p_1(0) = \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k) - 2\lambda_m\mu_k} = \frac{\mu_k + 9.3 \times 10^{-3}}{3 \times 10^{-2}\mu_k + 2.279 \times 10^{-3}},$$

$$\text{MTTF}_k(3) = q_1(0) + q_2(0) = \frac{6\lambda_c\lambda_f + 3\lambda_c\lambda_f + 2\lambda_c\lambda_f + 4\lambda_c\lambda_f + 2\lambda_c\lambda_f + 2\lambda_c\lambda_f + 2\lambda_c\lambda_f + 2\lambda_c\lambda_f + 2\lambda_c\lambda_f + \mu_k}{(\lambda_c + \lambda_f)(\lambda_c + \lambda_f) - 2\lambda_m\mu_k} = \frac{1.252\mu_k + 1.164 \times 10^{-2}}{3.756 \times 10^{-3}\mu_k + 2.760 \times 10^{-3}}.$$

From Fig. 7, the mean time to failure of three-dependent-component and two-dependent-component parallel system is positive correlation with $\mu_k$. The three-dependent-component parallel system is
prior to two-dependent-component parallel system when 
\[ \mu_k \in \left[ 5.2 \times 10^{-7}, 0.125 \right] \].

5. Conclusions

This paper presents parallel system model under common cause and load-sharing failures. According to this model, mean time to failure of three-dependent-component and two-dependent-component parallel systems is calculated. Besides, we calculate and discuss the conditional failure probability and reliability of three-dependent-component and two-dependent-component parallel system model under common cause failure. The reliability of three-dependent-component parallel system model under common cause failure is more than two-dependent-component. We could observe that mean time to failure of three-dependent-component parallel systems is not always longer than two-dependent-component. Hence, some measures could be taken to control the range of variables to ensure mean time to failure of three-dependent-component parallel systems is more than two-dependent-component.

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