1. Introduction

The reality of the production environment is inseparably connected with disruptions, which negatively impact the executed processes, thus leading to disorganisation [14]. The key uncertainty factors include the occurrence of technological machine failure. From the practical point of view, prediction of failure times is an issue of fundamental importance, as it enables implementing preventive activities in a way that does not interfere with the current production process. Failure time prediction is frequently in use in Time-Based Maintenance (TBM), and in response to the growing demand, specialised IT solutions aimed to support this strategy are developed [5, 16, 37]. It is crucial that these tools employ effective prediction algorithms, drawing from reliable historical data and thus providing the basis for a reliable analysis of machine failure and proper adjustment of maintenance activities [6, 13, 40].

The literature analysis shows that numerous studies have been devoted to the prediction of disruption in the production process. Those studies primarily concern the development of effective methods for countering failure, as well as absorb their impact [3, 33]. Preventive activities frequently correspond with the principles of Time-Based Maintenance [13, 25], as well as activities representing Conditioned-Based Maintenance [1, 30]. The development of scenarios and operational strategies is also a very popular trend [26, 27, 34, 35, 39]. Failure prediction methods proposed in the literature are categorised into several groups:

- methods based on probability distribution,
- methods using typical performance indicators,
- alternative failure prediction methods,
- methods based on real data.

The vast majority of the solutions proposed in the literature are based on probability distribution analysis [8, 15, 24, 2], which considers typical distributions and their combinations, such as: uniform distribution [17, 2], normal distribution [8] or exponential distribution [24, 30]. The primary purpose of distribution analysis is to define the...
time of failure occurrence. Solutions based on combinations of typical distributions are also proposed in the literature, for instance, in the 2010 study [15] the authors propose combinations of normal, triangular and exponential distributions to describe the problem of failure occurrence. Admittedly, most of the proposed solutions consider the problem in a purely theoretical manner, and as such, disregard the critical aspect of prediction: the use of historical data on machine failure rate. Furthermore, researchers fail to provide a sufficient justification for a given probability distribution selection.

Another trend visible in the literature is employing key performance indicators (KPIs) used in maintenance for failure prediction, such as:
- Mean Time To Failure (MTTF),
- Mean Time Between Failures (MTBF),
- Mean Time To Repair (MTTR).

The KPIs listed above are employed in numerous studies [9, 12, 21, 20], predominantly directly, in other cases indirectly – as estimators for the purpose of Weibull distribution [21]. In research, the authors follow predefined scenarios and the indicators are specified from preset ranges, which ensures that the failure events occur at a desired frequency (frequently or rarely) and are eventually analysed from the perspective of the consequences of failure occurrence [12]. Sometimes the use of KPIs is supported by the use of appropriate statistical methods [30]. The use of methods applying performance indicators typical for maintenance is substantiated by the fact that these parameters provide large amounts of information on the technological machines in use. Nevertheless, the acquisition and use of parameters in question is largely in the theoretical domain: the published studies fail to perform verification of the proposed solutions with the real data on machine failure rates [9, 20].

With respect to alternative methods of failure prediction, several solutions are particularly worth highlighting, e.g. the methods in which all machine failures are accumulated into one and evaluated by means of the MTTR and MBL (Machine Breakdown Level) parameters [18], the methods where the failure rate is determined from the analysis of the machine loading time distributions [31], those in which the prediction of machine failure is carried out with the application of artificial neural networks [4], or the well-established time series models [38]. During the verification of the proposed solutions, however, test data is employed, which, furthermore, stems from the use of simplifying assumptions adopted by the researchers.

In the works of Davenport et al. and Kempa et al. [8, 19], the authors note that performing computations on actual sets of process data is of paramount importance. These suggestions represent a novelty approach to failure prediction. They point out the necessity to develop methods focusing on the practical use of historical data on technological machine failure. Although studies implementing such solutions may be found in the specialist literature, their number is still negligible [33]. Nonetheless, they represent a clear trend in the area of failure prediction.

Despite the fact that several methods have been proposed, no solutions towards the practical use of historical data on the failure of technological machines have yet been developed. In addition, in the production environment the typical modus operandi is to propose implementation of extensive and high-priced monitoring systems, while in the field of TBM strategies, the data is obtained from all maintenance departments. Therefore, this study provides a novel approach to machine failure prediction in multi-machine manufacturing systems that employs an algorithm performing an in-depth, elaborate analysis of actual production data, thus enabling the prediction of future machine breakdowns and implementation of effective preventive measures. This method constitutes an alternative to those characterised in the preceding paragraphs as it makes use of data obtained from maintenance services to achieve the intended objective – identification of the potential moment of failure. The innovation of our method consists in its incorporation of elements of survival analysis theory in technological machine failure analysis enabling statistical inference based on historical data.

2. Failure prediction with elements of processing times analysis

2.1. Machining times as duration

In its essence, failure prediction is the determination of the time and degree of certainty for the occurrence of failure of a given technological machine; to this end, elements of Survival Analysis, also referred to as Duration Analysis [11, 23], may be put to use.

When employing Duration Analysis it is essential to precisely specify the essence of the studied process, which should meet the following conditions [11]:
1. Changes to the analysed unit are made between discrete states.
2. Changes of states occur at any time and are not fixed in time.
3. Changes are reversible or irreversible (relative to the form of the process).
4. Changes are predetermined by the current state of the process.
5. Certain factors affect the process – the analysis enables their detection.

Considering these determinants of the Survival Analysis, it appears that technological machine failure is a process that meets these requirements. Machine failure can occur at any time and is a change between two states – the functioning and breakdown. In addition, damage to the machine is a reversible change – once repaired, it returns to its original state, being defined by the state in which the device is. There are also a number of factors that can affect the process under scrutiny and can be identified by means of Duration Analysis [36]. In the case of machine there is a need to consider the duration time as a time of undisturbed machine operation. In the consequence, the failure time of machine can be determined. An additional advantage of this technique is the ability to determine failure patterns (time characteristics of failures), especially when the historical data do not allow the use of typical inference techniques [33].

Let $T$ be a non-negative random variable representing the time of failure (duration) of the technological machine, whose value is in the range $(0; \infty)$. In addition, $f(t)$ is a function of probability density, where $t > 0$ and $F(t)$ is a cumulative distribution function of the random variable, $T$ – a non-decreasing function that indicates that the object will experience the event in time $(0; t)$:

$$F(t) = P(T < t).$$

(1)

Based on the cumulative distribution function $F(t)$, the survival function $S(t)$ can be defined as:

$$S(t) = 1 - F(t) = P(T \geq t) = \int_{t}^{\infty} f(s) ds,$$

(2)

which gives the probability of undisturbed machine work until $t$. It, furthermore, determines the probability that a failure will not occur until $t$. The selected function is an ideal solution for the determination of patterns of correct machine operation and, as a consequence, also its failure. The survival and cumulative function are shown in Figure 1.
In order to determine the particular functions presented above, appropriate historical data describing the failure of the technological machine should be obtained and incorporated in the models. Their analysis provides a great amount of critical information that can be used in the further prediction process.

2.2. The use of historical data

To determine the failure characteristics, it is necessary to define the suitable data source, i.e. production maintenance departments – since these cells collect the information in question [3, 10]. The data on the history of maintenance and repair of technological machines in manufacturing enterprises are most commonly recorded by means of the following solutions:

– paper documentation – typically in the form of Maintenance Cards and Service Books,

– IT software coupled with dedicated spreadsheets (Fig. 2),

– data acquisition directly from technological machines, using SCADA (Supervisory Control And Data Acquisition) and MES (Manufacturing Execution Systems).

All of the data collection methods above share a common feature – each provides information that, when properly processed, can be employed in Survival Analysis for the prediction of machine failure.

The data contained in the documentation are historical failure times. For a given technological machine $M_j$, they are given as $T_{Mj}$:

$$T_{Mj} = \{t_1, t_2, ..., t_n\} \text{ [hours]}, \quad (3)$$

where: $t_i$ – $i$-th time of failure.

An example dataset for $M_1$ historical failure times is expressed by:

$$T_{M1} = \{4, 8, 20, 16, 10, 28, 43, 15, 24, 2, ...\} \text{ [hours]}. \quad (3)$$

The use of data contained in relevant datasets $T_{Mj}$ enables the determination of potential failure times of a given machine, saved in dataset $FT_{Mj}$:

$$FT_{Mj} = \{T_{Mj1}, T_{Mj2}, ..., T_{Mjn}\}, \quad (4)$$

where: $T_{Mji}$ – failure time of machine $j$,

$j$ – the number of the considered machine.

For each time $T_{Mji}$ the probability of failure is given in the set $P_{Mj}$:

$$P_{Mj} = \{P_{Mj1}, P_{Mj2}, ..., P_{Mjn}\}, \quad (5)$$

where: $P_{Mji}$ – the probability of machine failure $j$, given that:

$$T_{Mji} \neq 0 \land P_{Mji} \neq 0.$$

Therefore, the result of the prediction will be the pairs $(P_{Mji}, T_{Mji})$ that define the probability and the failure time of machine $M_j$.

2.3. The proposed time-based machine failure prediction algorithm

In order to predict the probability of failure and the time of failure, a four-step algorithm was developed to analyse and properly implement the collected repair history data.

Step 1 of the proposed algorithm defines the machine for which the prediction process is carried out, as well as acquires the historical data from in the set $T_{Mj}$ (Fig. 3).

At step 2, the imported data are saved: the failure times of machine $M_j$ by means of an appropriate sequence:

$$T_{Mj} = \{t_1, t_2, ..., t_n\} \text{ [hours]}, \quad (3)$$

where: $t_i$ – $i$-th time of failure.

An example dataset for $M_1$ historical failure times is expressed by:

$$T_{M1} = \{4, 8, 20, 16, 10, 28, 43, 15, 24, 2, ...\} \text{ [hours]}. \quad (3)$$

The use of data contained in relevant datasets $T_{Mj}$ enables the determination of potential failure times of a given machine, saved in dataset $FT_{Mj}$:

$$FT_{Mj} = \{T_{Mj1}, T_{Mj2}, ..., T_{Mjn}\}, \quad (4)$$

where: $T_{Mji}$ – failure time of machine $j$,

$j$ – the number of the considered machine.

For each time $T_{Mji}$ the probability of failure is given in the set $P_{Mj}$:

$$P_{Mj} = \{P_{Mj1}, P_{Mj2}, ..., P_{Mjn}\}, \quad (5)$$

where: $P_{Mji}$ – the probability of machine failure $j$, given that:

$$T_{Mji} \neq 0 \land P_{Mji} \neq 0.$$

Therefore, the result of the prediction will be the pairs $(P_{Mji}, T_{Mji})$ that define the probability and the failure time of machine $M_j$. 
\{(t_i, d_i)\}_{1 \leq i \leq n}, t_i \in T_{Mj} \tag{6}

where: \(t_i\) – the time between successive failures, 
\(d_i\) – number of cases.

In addition, at this step the data is arranged in an increasing order 
\{\{t_i\}_{1 \leq i \leq n}\}:

\[0 < t_1 < t_2 < \ldots < t_n, \tag{7}\]

Subsequently, the acquired data are filtered and outliers (representing atypical values) removed (Fig. 4). Then, the basic statistics for the collected data (minimum, maximum, average deviation, quartile range) are determined.

Step 3 is crucial for the inference process because it is at this stage that the survival function, characterising the considered failure process of the analysed machine, is determined. By ordering machine failures according to the increasing occurrence times and by determining the number of cases for each such occurrence, the survival function of a given process is determined. The obtained function conveniently determines duration patterns (failure occurrence) and allows to determine failure characteristics of the defined machine. The application of Kaplan-Meier estimation, on the other hand, produces the survival function, determined from the relationship:

\[
\hat{S}(t) = \begin{cases} 
1, & \text{for } t < t_1 \\
\prod_{t_i \leq t} \frac{r_i - d_i}{r_i}, & \text{for } t_1 < t
\end{cases} \tag{8}
\]

where: \(r_i\) – the number of all breakdowns, given by:

\[r_i = \sum_{j=i}^{k} d_j. \tag{9}\]

Subsequently, the survival function is determined, which allows to determine (with defined probability level) the undisturbed machine operation times (Fig. 5).

The determined survival function is implemented at step 4, where the obtained results serve to determine the elements of searched sets:

- potential times of machine failure \(FT_{Mj}\),
- probability of machine failure \(PM_{Mj}\).

Fig. 6 shows the principles of statistical inference based on the survival function. Predictions of failure times \(f_{Mji}\) are determined for specified probability levels \(p_i\).

Since the probability of undisturbed machine operation (\(p_i\)) is determined from the survival function, therefore, machine failure probability \(p_{Mji}\) is given by:

\[p_{Mji} = 1 - p_i, \tag{10}\]

where: \(p_{Mji}\) – machine failure probability,
\(p_i\) – undisturbed machine operation probability.

Determining the searched machine failure probability \(p_{Mji}\) enables the determination of the searched \(f_{Mji}\) and, consequently, determining the pairs \((p_{Mji}, f_{Mji})\). The calculated data are collected in sets \(P_{Mji}\) and

3. Experimental verification of the proposed algorithm

3.1. Data used in verification

The step preceding the model verification, presented below, was the acquisition and implementation of data describing the characteristics of the executed technological processes and the failure rate of technological machines. As mentioned before, the investigations reported in this study were based on actual production data, which specifically consisted of 12 production tasks performed at 12 work stations, arranged in manufacturing cells. The prevailing manufacturing process carried out in production is subtractive machining. Table 1 below lists technological processes at selected production jobs.
The actual data used in the verification process were employed in the following scope:
- technological machine failure data were used as input data for the prediction algorithm verification,
- data on executed manufacturing processes were used in simulation tests to assess the effectiveness and validity of the proposed algorithm considering real production conditions (including technological machinery failure).

### 3.2. Failure time prediction

The proposed algorithm was verified by means of an appropriate script compiled in a programming language R. The successful verification was followed by the use of the historical data in the process of statistical inferring with respect to the potential breakdown times of machines at particular workstations. The machines constituting the stock of the machine tools were labelled as follows:

- Laser 1 – machine $M_1$
- Laser 2 – machine $M_2$
- CNC press – machine $M_3$
- CNC band saw – machine $M_4$
- Metalworking station – machine $M_5$
- MIG welder – machine $M_6$
- TIG welder – machine $M_7$
- Drilling machine – machine $M_8$
- Milling machine – machine $M_9$
- Turning lathe – machine $M_{10}$
- Metal shearing machine – machine $M_{11}$
- Punching machine – machine $M_{12}$

In the paragraphs below, the exemplary execution of the verification process is presented for machine $M_6$, in which case the historical data included 121 observations.

Prior to the initiation of the prediction process, the prepared script was fed with appropriate commands – preparing the software working environment; this was followed by specifying the machine number and importing the data from the *.CSV file. By importing the data into the set $T_{M_6}$ (the variable) stored in the workspace, facilitated sorting the considered observations in ascending order, as well as filtering the data by means of the box plots (Fig. 7). In addition, basic statistics were determined (Fig. 8).

![Fig. 7. Box plots – before and after data filtering](image-url)
calculated automatically from the produced observation sequences. The result was a survival function in the form of a stepped curve at 95% confidence.

Determining the course of the searched function, \( \hat{S}(t) \), triggers the next step of the algorithm: the prediction of the failure time of the considered machine at the defined probability level (Fig. 9). As the probability of undisturbed machine operation can also be read from the chart, an additional legend with explanations was generated. In the case of calculations for the given machine \( M_6 \) (and other machines), the following probability levels were considered:

\[ p_1 = 0.75; \quad p_2 = 0.50; \quad p_3 = 0.25. \]

The values of the considered levels have been chosen so as to determine: low, medium and high level of risk of the machine being affected. Therefore:

\[ p_{M_61} = 1 - p_1 = 0.25; \quad p_{M_62} = 1 - p_2 = 0.50; \quad p_{M_63} = 1 - p_3 = 0.75; \]

Table 2. Technological machine failure times obtained from prediction

<table>
<thead>
<tr>
<th>Machine</th>
<th>( p_{M_1} = 0.25 )</th>
<th>( p_{M_2} = 0.50 )</th>
<th>( p_{M_3} = 0.75 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>8</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>( M_4 )</td>
<td>8</td>
<td>24</td>
<td>104</td>
</tr>
<tr>
<td>( M_5 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( M_6 )</td>
<td>8</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>( M_7 )</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>( M_8 )</td>
<td>8</td>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>( M_9 )</td>
<td>8</td>
<td>24</td>
<td>40</td>
</tr>
<tr>
<td>( M_{10} )</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>( M_{11} )</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>( M_{12} )</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

which is characteristic of authentic industrial conditions. This was done in a two-stage experiment:

1. Nominal production schedules were produced based on the actual production data. Next, corresponding robust schedules were prepared by implementing service times as indicated by the results of the executed algorithm.
2. The production process was modelled according to the developed schedules and examined to indicate the schedule of the shortest production completion time under the constraint of machine failure.

### 3.3. Production simulation under uncertainty

The plan of the study described in this paper assumed the verification of the introduced algorithm in the real production environment in order to validate its applicability under machine failure uncertainty.

The values of the considered levels have been chosen so as to determine: low, medium and high level of risk of the machine being affected. Therefore:

\[ \hat{S}(t) = \frac{N(t)}{N_0}, \]

As a result, sets \( p_{M_1} = \{0.25, 0.50, 0.75\} \) and \( FT_{M_6} = \{8, 24, 48\} \) [hours] were determined.

The proposed algorithm was used to the same extent in other technological machines. Due to the nature of the metalworking workstation (\( M_2 \)) the prediction process was not carried out. The calculated failure times are given in Table 2.

The results obtained from the executed algorithm were employed in the subsequent part of the verification process, consisting in the simulation of production under technological machinery failure constraint.

#### 3.3.1. Scheduling production

Different job scheduling methods to follow at individual workstations were evaluated by means of 4 established dispatching rules:

1. FCFS (First Come First Service).
2. EDD (Earliest Due Date).
3. SPT (Shortest Processing Time).
4. LPT (Longest Processing Time).

It was assumed that the products were made in 50-piece batches, and the objective function of the schedule was to minimise the makespan – \( C_{\text{max}} \).

The task scheduling tool employed in the study was LiSA, a software package for solving job scheduling problems typical of real production environments (flow-shop, job-shop or open-shop), which makes use of algorithms in imposing a set of constraints and evaluation criteria [7]. Fig. 10 shows an example schedule solved with the use of LPT dispatching rule.
Potential technological machine failure was accounted for in the schedules by the implementation of service buffers of 0.5 hours, aimed to protect schedules against disruptions and providing the necessary inspection or servicing time. Buffers were incorporated in the schedules in accordance with the indications of the algorithm (Table 2). It was assumed that failure may only occur after the processing time block (processing of jobs). Should there be a technological operation in a given place of the schedule – it would be moved right (immediately after the buffer), thus maintaining the order of tasks indicated in the nominal schedule. An example of a robust schedule with implemented service buffers is shown in Fig. 11 (buffers are represented by white blocks).

The times of completion of all jobs (makespan) in the nominal and robust schedules are presented in Table 3.

The completion times of all jobs obtained from the test schedules were elongated in every case when time buffers were incorporated. This resulted in the elongation of the objective function \( C_{\text{max}} \) in each reported case. The average time difference between the nominal and robust schedule amounted to 6.75 h. It may be, therefore, concluded that accounting for technological machine failure causes that the production will extend over approximately one additional shift. Expressed in percentage, the elongation ranged from 8.5% for the robust schedule with the LPT priority rule, to 16.7% for the FCFS schedules. The makespans of particular robust schedules are given in Fig. 12 below.

To evaluate whether the implemented buffers should be incorporated in the schedules, thus leading to the production schedule elongation, the second stage of the verification process was carried out: simulation of production under uncertainty. This step indicated which of the schedules – nominal or robust (produced by the proposed algorithm) – fulfills the objective function, i.e. minimisation of completion of all production tasks.

The completion times of all jobs obtained from the test schedules were elongated in every case when time buffers were incorporated. This resulted in the elongation of the objective function \( C_{\text{max}} \) in each reported case. The average time difference between the nominal and robust schedule amounted to 6.75 h. It may be, therefore, concluded that accounting for technological machine failure causes that the production will extend over approximately one additional shift. Expressed in percentage, the elongation ranged from 8.5% for the robust schedule with the LPT priority rule, to 16.7% for the FCFS schedules. The makespans of particular robust schedules are given in Fig. 12 below.

To evaluate whether the implemented buffers should be incorporated in the schedules, thus leading to the production schedule elongation, the second stage of the verification process was carried out: simulation of production under uncertainty. This step indicated which of the schedules – nominal or robust (produced by the proposed algorithm) – fulfills the objective function, i.e. minimisation of completion of all production tasks.

![Fig. 11. Production schedule including service buffers](image1)

The times of completion of all jobs (makespan) in the nominal and robust schedules are presented in Table 3.

The completion times of all jobs obtained from the test schedules were elongated in every case when time buffers were incorporated. This resulted in the elongation of the objective function \( C_{\text{max}} \) in each reported case. The average time difference between the nominal and robust schedule amounted to 6.75 h. It may be, therefore, concluded that accounting for technological machine failure causes that the production will extend over approximately one additional shift. Expressed in percentage, the elongation ranged from 8.5% for the robust schedule with the LPT priority rule, to 16.7% for the FCFS schedules. The makespans of particular robust schedules are given in Fig. 12 below.

To evaluate whether the implemented buffers should be incorporated in the schedules, thus leading to the production schedule elongation, the second stage of the verification process was carried out: simulation of production under uncertainty. This step indicated which of the schedules – nominal or robust (produced by the proposed algorithm) – fulfills the objective function, i.e. minimisation of completion of all production tasks.

![Fig. 12. Makespan \( C_{\text{max}} \) – completion time of all jobs](image2)

### Table 3. Obtained values of \( C_{\text{max}} \)

<table>
<thead>
<tr>
<th>Dispatching rule</th>
<th>Completion time of all jobs – makespan ( C_{\text{max}} ) [hours]</th>
<th>Elongation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCFS</td>
<td>43.68, 52.44</td>
<td>16.7%</td>
</tr>
<tr>
<td>EDD</td>
<td>42.59, 49.42</td>
<td>13.8%</td>
</tr>
<tr>
<td>SPT</td>
<td>48.92, 55.75</td>
<td>12.3%</td>
</tr>
<tr>
<td>LPT</td>
<td>49.10, 53.69</td>
<td>8.5%</td>
</tr>
</tbody>
</table>

3.3.2. Production simulation under machine failure constraint

The second stage of the experiment was carried out in the Enterprise Dynamics simulation environment, which is one of the leading solutions in simulating various processes. This platform enables representing a range of processes, including production, storage, supply chain management, transport systems, and its capacity for modelling, simulation and visualisation earmarks it for controlling dynamic processes [14, 16, 22]. Putting to use the available elements of the environment, a model was made for the production execution analysis in the considered production system (Fig. 13).

![Fig. 13. The production system model developed in the ED environment](image3)

Given the failure rate of technological machines, MTTF and MTTR values were defined for each of them, by modifying the properties of a given block. The MTTF parameter values were defined using uniform probability distribution so that the failures occurred at any time – from the commencement of processing jobs on a machine until its completion. The MTTR parameter was determined by gamma distribution, as it was indicated to be the best fitting by the results from the statistical analysis of historical data on machine repair times. The MTTF and MTTR parameters for individual machines are presented in Table 4. Note that due to the ED simulation environment – the times describing the distribution parameters were given in seconds.

The model developed for the purpose of this study included the modification of job orders on particular machines (in accordance with the schedules implementing the particular dispatching rules FCFS, EDD, SPT and LPT).

When assessing the results of simulations, the following stability indicators were used:

- elongation of completion time of all jobs \( \Delta C_{\text{max}} \) given by:

\[
\Delta C_{\text{max}} = C_{\text{max}} - C'_{\text{max}} ,
\]

where:

- \( C_{\text{max}} \) – elongation of completion time of all jobs,
- \( C'_{\text{max}} \) – nominal schedule makespan.
\( C'_\text{max} \) – actual (executed) schedule makespan.

– relative elongation of makespan \( E_{C_{\text{max}}} \), determined from the relationship:

\[
E_{C_{\text{max}}} = \frac{C_{\text{max}}}{C'_\text{max}},
\]

where: \( E_{C_{\text{max}}} \) – relative elongation of makespan.

Table 5 shows the results of the simulation under the SPT dispatching rule. For each simulation, the obtained stability indicators confirmed the effectiveness and applicability of the proposed algorithm. Both the values of elongation of completion time of all jobs, \( \Delta C_{\text{max}} \), and the relative elongation of makespan, \( E_{C_{\text{max}}} \), showed that the schedule accounting for potential technological machine failure indicates a more feasible completion time of all jobs.

For other simulated conditions, the applicability of the solutions proposed in this publication was also confirmed, as validated by mean of the performance indicators from individual simulations listed in Table 6.

The obtained values clearly indicate that the schedule incorporating service buffers gives a more feasible completion time of all jobs.

### Table 4. Technological machine failure times obtained from the prediction results

<table>
<thead>
<tr>
<th>Machine</th>
<th>MTTF</th>
<th>MTTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>Uniform(0; 66323)</td>
<td>Gamma(3075; 1.62)</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>Uniform(0; 31691)</td>
<td>Gamma(2700; 2.07)</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>Uniform(0; 57877)</td>
<td>Gamma(2491.8; 2.79)</td>
</tr>
<tr>
<td>( M_4 )</td>
<td>Uniform(0; 12013)</td>
<td>Gamma(2773.2; 1.88)</td>
</tr>
<tr>
<td>( M_5 )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( M_6 )</td>
<td>Uniform(0; 85475)</td>
<td>Gamma(3421.2; 2.43)</td>
</tr>
<tr>
<td>( M_7 )</td>
<td>Uniform(0; 30024)</td>
<td>Gamma(3352.8; 1.96)</td>
</tr>
<tr>
<td>( M_8 )</td>
<td>Uniform(0; 80687)</td>
<td>Gamma(2377.2; 2.45)</td>
</tr>
<tr>
<td>( M_9 )</td>
<td>Uniform(0; 24012)</td>
<td>Gamma(2884.8; 1.64)</td>
</tr>
<tr>
<td>( M_{10} )</td>
<td>Uniform(0; 60624)</td>
<td>Gamma(2694; 1.85)</td>
</tr>
<tr>
<td>( M_{11} )</td>
<td>Uniform(0; 756)</td>
<td>Gamma(3169.8; 2.16)</td>
</tr>
<tr>
<td>( M_{12} )</td>
<td>Uniform(0; 19800)</td>
<td>Gamma(3015; 1.78)</td>
</tr>
</tbody>
</table>

### Table 5. Stability indicators – order of jobs according to the SPT rule

<table>
<thead>
<tr>
<th>Sim. No.</th>
<th>Executed schedule (simulation) ( C'_{\text{max}} ) [hours]</th>
<th>Elongation and relative elongation of completion times of all jobs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>nominal schedule</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C_{\text{max}} ) [hours]</td>
</tr>
<tr>
<td>1</td>
<td>56.10</td>
<td>-7.18</td>
</tr>
<tr>
<td>2</td>
<td>53.88</td>
<td>-4.96</td>
</tr>
<tr>
<td>3</td>
<td>54.09</td>
<td>-5.17</td>
</tr>
<tr>
<td>4</td>
<td>56.91</td>
<td>-7.99</td>
</tr>
<tr>
<td>5</td>
<td>52.60</td>
<td>-3.68</td>
</tr>
<tr>
<td>6</td>
<td>55.50</td>
<td>-6.58</td>
</tr>
<tr>
<td>7</td>
<td>56.43</td>
<td>-7.51</td>
</tr>
<tr>
<td>8</td>
<td>55.88</td>
<td>-6.96</td>
</tr>
<tr>
<td>9</td>
<td>53.48</td>
<td>-4.56</td>
</tr>
<tr>
<td>10</td>
<td>54.04</td>
<td>-5.12</td>
</tr>
<tr>
<td>11</td>
<td>58.31</td>
<td>-9.39</td>
</tr>
<tr>
<td>12</td>
<td>52.97</td>
<td>-4.05</td>
</tr>
<tr>
<td>13</td>
<td>54.20</td>
<td>-5.28</td>
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<td>14</td>
<td>55.33</td>
<td>-6.41</td>
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<td>15</td>
<td>55.98</td>
<td>-7.06</td>
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<tr>
<td>16</td>
<td>56.01</td>
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<tr>
<td>17</td>
<td>53.53</td>
<td>-4.61</td>
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<tr>
<td>18</td>
<td>56.51</td>
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<td>56.49</td>
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<td>57.52</td>
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<td>23</td>
<td>54.86</td>
<td>-5.94</td>
</tr>
<tr>
<td>24</td>
<td>55.04</td>
<td>-6.12</td>
</tr>
<tr>
<td>25</td>
<td>54.83</td>
<td>-5.91</td>
</tr>
</tbody>
</table>
Figures 14 and 15 summarise the obtained values of the considered indicators, which further confirm the applicability of the proposed algorithm.

From the results of the verification and analytical works, it can be seen that the algorithm under scrutiny indicates a more feasible production completion time in the conditions allowing for the risk of technological machinery failure. This is evidenced, for instance, by the fact that for the robust schedule, the $\Delta C_{max}$ indicator values are close to 1, while the value of the indicator $E_{C_{max}}$ is approximate to 0, which means that the makespans of production in the robust schedules are consistent with those obtained as a result of production simulation.

4. Summary and conclusions

Machine failure prediction has been widely investigated in numerous scientific studies. Various approaches have been proposed for the determination of information regarding the failure of technological machines. Reliable and well-developed preventive maintenance job schedules are critical to effective maintenance, particularly in the case of Time-Based Maintenance strategies.

This paper focuses on the development of a prediction algorithm using typical historical data recorded by maintenance departments.

The proposed algorithm is an alternative solution to failure prediction, whose innovation, and primary advantage, consists in the implementation of Kaplan-Meier estimation to determine the characteristics of failure occurrence in time for individual technological machines of the production system, which in turn supports TBM activities. In light of these key features of the proposed prediction tool, it becomes clear that the collection of reliable data on machine failure becomes of crucial importance; it is only the adequate historical data sample size and quality that may produce reliable and factual results.

Our algorithm responds to and represents the tendency for the growing implementation of IT tools in the work of maintenance departments. Considering its potential scope of applications, it was developed as a computer program so that it is compatible with other established solutions. The verification of the proposed algorithm allowed to determine the potential failure times of technological machines. For the considered machines determined failure times were different, which means that each of them has its own failure occurrence characteristics. That confirmed the rightness and need of the TBM strategy implementation in the technical objects maintaining. The obtained data are also extremely important in the aspect of production under uncertainty. The simulation tests carried out in the second part of the publication prove that the use of the results of the proposed algorithm in the production planning allows to obtain stability of processes and determine deadlines close to the real end time of production.

The investigation works reported in this paper confirm the effectiveness of the developed prediction algorithm and indicate the need for the preventive measures to provide information on machine failure in order to improve the stability of executed processes.

Acknowledgments

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Table 6. Mean values of the considered performance indicators

<table>
<thead>
<tr>
<th>Priority rule</th>
<th>Executed schedule (simulation)</th>
<th>Elongation and relative elongation of completion times of all jobs</th>
<th>nominal schedule</th>
<th>robust schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{max}$ [hours]</td>
<td>$\Delta C_{max}$ [hours]</td>
<td>$E_{C_{max}}$ [-]</td>
<td>$C_{max}$ [hours]</td>
</tr>
<tr>
<td>FCFS</td>
<td>49.87</td>
<td>43.68</td>
<td>-6.19</td>
<td>0.88</td>
</tr>
<tr>
<td>EDD</td>
<td>47.90</td>
<td>42.59</td>
<td>-5.31</td>
<td>0.89</td>
</tr>
<tr>
<td>SPT</td>
<td>55.12</td>
<td>48.92</td>
<td>-6.20</td>
<td>0.89</td>
</tr>
<tr>
<td>LPT</td>
<td>53.14</td>
<td>49.10</td>
<td>-4.04</td>
<td>0.92</td>
</tr>
</tbody>
</table>

References

1. Albrice D, Branch M. A Deterioration Model for Establishing an Optimal Mix of Time-Based Maintenance (TbM) and Condition-Based Maintenance (CbM) for the Enclosure System. Fourth Building Enclosure Science & Technology Conference (BEST4), Kansas City, Missouri, April 13–15, 2015.


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